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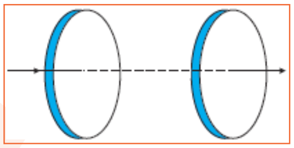
8.1 **Figure shows a capacitor made of two circular plates each of radius 12 cm, and separated by 5.0 cm. The capacitor is being charged by an external source (not shown in the figure). The charging current is constant and equal to 0.15A.**



(a) Calculate the capacitance and the rate of change of potential difference between the plates.

(b) Obtain the displacement current across the plates.

(c) Is Kirchhoff's first rule (junction rule) valid at each plate of the capacitor? Explain.

SOLUTION:

Given – Radius of each circular plate (r) = 12 cm = 0.12 m, Distance between the plates (d) = 5 cm = 0.05 m, Charging current (I) = 0.15 A.

(a) Capacitance of the system is given by,

$$C = \frac{\varepsilon_0 A}{d} \qquad \text{(Area of the plate (A) = r}^2\text{)}$$

$$C = \frac{\varepsilon_0 \pi r^2}{d}$$

$$C = \frac{(8.854\times10^{-12})\times(3.14)\times(0.12)^2}{(0.05)}$$

$$C = 80.032 \times 10^{-12}\,\text{F} = 80.032\text{pF}$$

Charge on each plate is given by,

$$q = CV$$

$$V = \frac{q}{C}$$

Rate of change of potential difference between the plates is:

$$\frac{dV}{dt} = \frac{1}{C}\frac{dq}{dt}$$

$$\frac{dV}{dt} = \frac{I}{C} \qquad \left(I = \frac{dq}{dt}\right)$$

$$\frac{dV}{dt} = \frac{0.15}{8.01\times10^{-12}}$$

$$\frac{dV}{dt} = 1.875 \times 10^{9}\ \text{V/s}$$

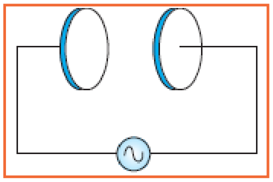
(b) Yes, Kirchhoff's first law is indeed applicable to each plate of a capacitor, as $I_d = I$. This ensures that the current remains continuous and constant across each plate.

8.2 A parallel plate capacitor made of circular plates each of radius R = 6.0 cm has a capacitance C = 100 pF. The capacitor is connected to a 230 V ac supply with a (angular) frequency of 300 rad/s.

(a) What is the rms value of the conduction current?

(b) Is the conduction current equal to the displacement current?

(c) Determine the amplitude of B at a point 3.0 cm from the axis between the plates.



SOLUTION:

Given – Radius (r) = 6 cm = 0.06 m, Capacitance (C) = 100 pF, Supply voltage (V) = 230 V, Angular frequency (ω) = 300 rad/s.

(a) rms value of conduction current is given by,

$$I = \frac{V}{X_C} \qquad \left(X_C = \frac{1}{\omega C}\right)$$

$$I = V \times \omega C$$

$$I = (230) \times (300) \times (100 \times 10^{-12}) = 6.9 \times 10^{-6}\ \text{A}$$

$$I = 6.9\ \mu\text{A}$$

Therefore, the rms value of current is $6.9\,\mu$A.

(b) Yes. Conduction current = Displacement current.

(c) Magnetic field is given by,

$$B = \frac{\mu_0 I_0 r}{2\pi R^2}$$

Where, $\mu_0 =$ Free space permeability $= 4\pi \times 10^{-7}\ \text{NA}^{-2}$

$I_0 =$ Maximum value of current $= \sqrt{2}1$ and

$r =$ Distance between the plates from the axis $= 3.0\,cm = 0.03$m

$$\therefore \qquad B = \frac{(4\pi \times 10^{-7}) \times (0.03) \times) \sqrt{2} \times 6.9 \times 10^{-6})}{2\pi \times (0.06)^2}$$

$$B = 1.63 \times 10^{-11}\ \text{T}$$

Therefore, the magnetic field at that point is 1.63×10^{-11} T.

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8.3 What physical quantity is the same for X-rays of wavelength 10^{-10} m, red light of wavelength 6800 Å and radio waves of wavelength 500 m?

SOLUTION:

Given – Wavelength of X-ray (λ) = 10^{-10} m, Wavelength of red light (λ_{Red}) = 6800 Å, Wavelength of radio waves (λ_{Radio}) = 500 m.

The speed of light in a vacuum, 3×10^8 m/s, remains constant and is independent of the wavelength.

8.4 A plane electromagnetic wave travels in vacuum along z-direction. What can you say about the directions of its electric and magnetic field vectors? If the frequency of the wave is 30 MHz, what is its wavelength?

SOLUTION:

An electromagnetic wave traveling in a vacuum along the Z-direction has its electric field (E) and magnetic field (B) confined to the X-Y plane. These fields are mutually perpendicular to each other and to the direction of wave propagation.

Frequency of the wave (v) = 30 MHz, Speed of EM wave in vacuum (c) = 3×10^8 m/s.

Wavelength of the wave is:

$$\lambda = \frac{c}{v} = \frac{3\times10^8}{30\times10^6}$$

$$\lambda = 10 \text{ m}$$

Therefore, the wavelength of the wave is 10 m

8.5 A radio can tune in to any station in the 7.5 MHz to 12 MHz band. What is the corresponding wavelength band?

Solution:

Given – Frequency band from 7.5 MHz to 12 MHz

Need to find – Corresponding wavelength band.

Minimum frequency (v_{min}) = 7.5 MHz = $7.5\times10^6\,Hz$

Speed of light (c) = 3×10^8 m/s

Corresponding wavelength (λ_{min}) is given by,

$$\lambda_{min} = \frac{c}{v_{min}} = \frac{3\times10^8}{7.5\times10^6}$$

$$\lambda_{min} = 40 \text{ m}$$

Minimum frequency (v_{max}) = 12 MHz = $12\times10^6\,Hz$

Speed of light (c) = 3×108 m/s

Corresponding wavelength (max) is given by,

$$\lambda_{max} = \frac{c}{v_{max}} = \frac{3\times10^8}{12\times10^6}$$

$$\lambda_{min} = 25 \text{ m}$$

Therefore, the wavelength band of the radio is 40 m to 25 m.

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8.6 A charged particle oscillates about its mean equilibrium position with a frequency of 10^9 Hz. What is the frequency of the electromagnetic waves produced by the oscillator?

SOLUTION:

The frequency of an electromagnetic wave generated by the oscillator is equal to the frequency of the charged particle oscillating about its mean position, which is 10^9 Hz.

8.7 The amplitude of the magnetic field part of a harmonic electromagnetic wave in vacuum is B_0 = 510 nT. What is the amplitude of the electric field part of the wave?

SOLUTION:

Given – Amplitude of magnetic field part: B_0 = 510 nT

Need to find – Amplitude of electric field part.

Amplitude of electric field of the electromagnetic wave is given by,

$$E_0 = cB_0 \qquad (\because c = 3\times10^8 \text{ m/s})$$

$$E_0 = (3\times10^8)\times(510\times10^{-9}) = 153\text{N / C}$$

Therefore, the amplitude of electric field of the electromagnetic wave is 153N/C.

8.8 Suppose that the electric field amplitude of an electromagnetic wave is E_0 = 120 N/C and that its frequency is ν = 50.0 MHz. (a) Determine, B_0, ω, *k*, and *λ*. (b) Find expressions for E and B.

SOLUTION:

Given – Amplitude of electric field (E_0) = 120 N/C, Frequency (ν) = 50.0 MHz,

(a) Amplitude of magnetic field is:

$$B_0 = \frac{E_0}{c} = \frac{120}{3\times10^8}$$

$$B_0 = 4\times10^{-7}\,\text{T} = 400nT$$

Angular frequency of the source is given by,

$$\omega = 2\pi\nu = 2\pi\times50\times10^6$$

$$\omega = 3.14\times10^8\, rad\,/\,\text{s}$$

Propagation constant is given by,

$$k = \frac{\omega}{c} = \frac{3.14\times10^8}{3\times10^8}$$

$$k = 1.05 rad\,/\,\text{m}$$

Wavelength of the wave is given by,

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$$\lambda = \frac{c}{v} = \frac{3\times10^8}{50\times10^6}$$

$$\lambda = 6m$$

Therefore, the magnitude of magnetic field is 400 nT, angular frequency of the source is $3.14\times10^8\,rad/s$, propagation constant is $1.05\,rad/m$, and wavelength of the wave is 6 m.

(b) If the wave propagates in the positive xx-direction, the electric field vector will point in the positive y-direction, and the magnetic field vector will point in the positive z-direction. This orientation arises because the propagation direction, electric field, and magnetic field vectors are all mutually perpendicular to each other.

Equation of electric field vector is given by,

$$\vec{E} = E_0 \sin(kx - \omega t)\hat{j}$$

$$\vec{E} = 120\sin\left[1.05x - 3.14\times10^8 t\right]\hat{j}$$

Equation of magnetic field vector is given by,

$$\vec{B} = B_0 \sin(kx - \omega t)\hat{k}$$

$$\vec{B} = \left(4\times10^{-7}\right)\sin\left[1.05x - 3.14\times10^8 t\right]\hat{k}$$

8.9 The terminology of different parts of the electromagnetic spectrum is given in the text. Use the formula E = hν (for energy of a quantum of radiation: photon) and obtain the photon energy in units of eV for different parts of the electromagnetic spectrum. In what way are the different scales of photon energies that you obtain related to the sources of electromagnetic radiation?

SOLUTION:

We know that the energy of a photon is given by,

$$E = hv = \frac{hc}{\lambda}$$

Where, h (Plank's constant) = 6.6×10^{-34} Js, c (Speed of light) = 3×10^8 m/s, Wavelength of radiation = λ

Frequency of radiation = v.

Now,

$$E = \frac{6.6\times10^{-34}\times3\times10^8}{\lambda} = \frac{19.8\times10^{-26}}{\lambda}\text{J}$$

$$E = \frac{19.8\times10^{-26}}{\lambda\times1.6\times10^{-19}} = \frac{12.375\times10^{-7}}{\lambda}eV$$

The given table lists the photon energies for different parts of an electromagnetic spectrum for different λ.

λ (m)	10^3	1	10^{-3}	10^{-6}	10^{-8}	10^{-10}	10^{-12}
E (eV)	12.375×10^{-10}	12.375×10^{-7}	12.375×10^{-4}	12.375×10^{-1}	12.375×10^{1}	12.375×10^{3}	12.375×10^{5}

The photon energies corresponding to different parts of the spectrum of a source provide information about the spacing of the energy levels within the source.

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8.10 In a plane electromagnetic wave, the electric field oscillates sinusoidally at a frequency of 2.0 × 10^{10} Hz and amplitude 48 Vm^{-1}.
(a) What is the wavelength of the wave?
(b) What is the amplitude of the oscillating magnetic field?
(c) Show that the average energy density of the E field equals the average energy density of the B field. [c = 3 × 108 ms^{-1}.]

SOLUTION:

Given – Frequency (v) = 2.0 × 10^{10} Hz, Amplitude (V_0) = 48 V/m

(a) Wavelength of the wave is:

$$\lambda = \frac{c}{v} = \frac{3\times10^8}{2\times10^{10}}$$
$$\lambda = 0.015m$$

(b) Magnetic field strength is:

$$B_0 = \frac{E_0}{c} = \frac{48}{3\times10^8}$$
$$B_0 = 1.6\times10^{-7}\text{T}$$

(c) Energy density of the electric field is:

$$U_E = \frac{1}{2}\epsilon_0 E^2 \quad \ldots\ldots\ldots\ldots\ldots (i)$$

Energy density of the magnetic field is:

$$U_B = \frac{1}{2\mu_0}B^2 \quad \ldots\ldots\ldots\ldots\ldots (ii)$$

The relation between E and B is:

$$E = cB \quad \ldots\ldots\ldots\ldots\ldots\ldots (iii)$$

We know that,

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$, Put this value in equation (iii) we get,

$$E = \frac{1}{\sqrt{\epsilon_0 \mu_0}}B$$

Squaring both sides we get,

$$E^2 = \frac{1}{\epsilon_0 \mu_0}B^2 \Rightarrow \epsilon_0 E^2 = \frac{B^2}{\mu_0} \quad \ldots\ldots\ldots\ldots\ldots (iv)$$

Multiplying both sided of equation (iv) by we get,

$$\frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}\frac{B^2}{\mu_0}$$

From equation (i) and (ii) we get,

$$U_E = U_B$$

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