

CHAPTER 9

RAY OPTICS AND OPTICAL INSTRUMENTS

VEDA
ACADEMY

CLASS 12TH

NCERT EXERCISE AND SOLUTIONS - PHYSICS

- 9.1** A small candle, 2.5 cm in size is placed at 27 cm in front of a concave mirror of radius of curvature 36 cm. At what distance from the mirror should a screen be placed in order to obtain a sharp image? Describe the nature and size of the image. If the candle is moved closer to the mirror, how would the screen have to be moved?

SOLUTION:

Given—Size of candle (h_o) = 2.5 cm, Object distance (u) = 27 cm, Radius of curvature (R) = 36 cm.

$$\text{Focal length (f)} = \frac{\text{Radius of curvature (R)}}{2} = \frac{-36}{2} = -18\text{cm}$$

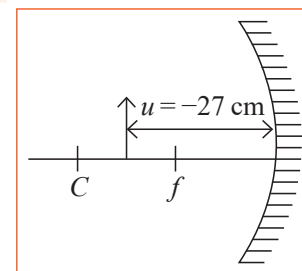
i.e. For concave mirror, object is kept between centre of curvature (C) and focus (f), so the image should be real, inverted and beyond C. For sharp image, screen should be placed at the position of image.

By using mirror formula,

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \Rightarrow \frac{1}{(-18)} = \frac{1}{v} + \frac{1}{(-27)} \quad \text{or} \quad \frac{1}{v} = \frac{1}{27} - \frac{1}{18}$$

$$\frac{1}{v} = \frac{-3+2}{54} \Rightarrow \frac{1}{v} = -\frac{1}{54}$$

$$v = -54\text{cm}$$



Therefore, the distance of image from the mirror is -54 cm i.e. image is real in nature and for sharp image screen should be placed at that point.

Size of the image is given by,

$$m = \frac{h_i}{h_o} = -\frac{v}{u} \Rightarrow \frac{h_i}{h_o} = -\frac{v}{u}$$

$$\frac{h_i}{+2.5} = -\frac{(-54)}{(-27)} \Rightarrow h_i = -5\text{ cm}$$

Therefore, the size of the image is -5 cm i.e. image is inverted and magnified.

If the candle is moved closer to the concave mirror, the real image will shift farther away from the mirror. Consequently, the screen must be moved farther from the mirror to locate the sharp image.



- 9.2 A 4.5 cm needle is placed 12 cm away from a convex mirror of focal length 15 cm. Give the location of the image and the magnification. Describe what happens as the needle is moved farther from the mirror.

SOLUTION:

Given – Size of needle (h_0) = 4.5 cm, Object distance (u) = -12 cm, Focal length (f) = +15 cm.

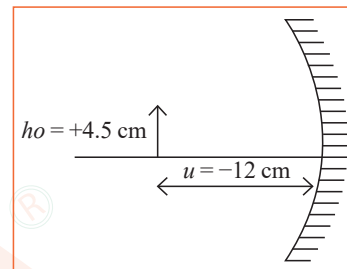
Need to find – Location of image and magnification.

By using mirror formula,

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \Rightarrow \frac{1}{(+15)} = \frac{1}{v} + \frac{1}{(-12)} \text{ or } \frac{1}{v} = \frac{1}{15} + \frac{1}{12}$$

$$\frac{1}{v} = \frac{4+5}{60} \Rightarrow \frac{1}{v} = \frac{9}{60}$$

$$v = +6.66 \text{ cm}$$



Therefore, the distance of image from the mirror is +6.66 cm i.e. image is virtual in nature.

Magnification is given by,

$$m = -\frac{v}{u}$$

$$m = -\frac{(60/9)}{(-12)} = \frac{5}{9} \Rightarrow m = +0.55 \text{ cm}$$

Size of the image is given by,

$$m = \frac{h_i}{h_o} \Rightarrow h_i = m \times h_o$$

$$h_i = \frac{5}{9} \times (4.5) = 2.5 \text{ cm}$$

Therefore, the size of the image is +2.5 cm i.e. image is erect, virtual and small.

When the needle is moved farther from the mirror, the image also shifts closer to the focus and decreases in size. As the object distance (u) increases, the image distance (v) approaches the focus (f) but never moves beyond it.

- 9.3 A tank is filled with water to a height of 12.5 cm. The apparent depth of a needle lying at the bottom of the tank is measured by a microscope to be 9.4 cm. What is the refractive index of water? If water is replaced by a liquid of refractive index 1.63 up to the same height, by what distance would the microscope have to be moved to focus on the needle again?

SOLUTION:

Given – Height till water filled (h) = 12.5 cm, Apparent depth of needle = 9.4 cm,

Refractive index of liquid (μ_l) = 1.63.

Formula for apparent depth is:

$$\text{Apparent depth} = \frac{\text{Real depth}}{\mu_w}$$



$$9.4\text{cm} = \frac{12.5\text{cm}}{\mu_w}$$

$$\mu_w = \frac{12.5}{9.4} = 1.33$$

Therefore, the refractive index of water is 1.33.

Now, if water is replaced by a liquid (refractive index = 1.63) then,

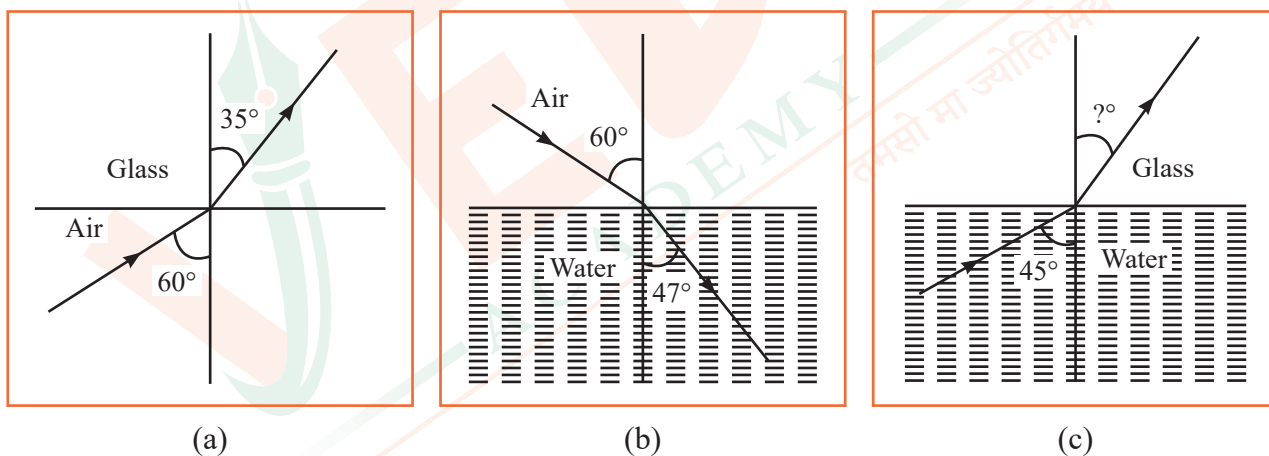
$$\text{Apparent depth} = \frac{\text{Real depth}}{\mu_l}$$

$$\text{Apparent depth} = \frac{12.5}{1.63} = 7.67\text{cm}$$

The microscope must be shifted from its initial position to refocus on the needle, which is now located at a depth of 7.67 cm.

$$\text{Shift distance} = (9.4 - 7.67) = 1.73\text{cm}$$

- 9.4 Figures (a) and (b) show refraction of a ray in air incident at 60° with the normal to a glass-air and water-air interface, respectively. Predict the angle of refraction in glass when the angle of incidence in water is 45° with the normal to a water-glass interface [Fig. (c)].



SOLUTION:

Given – Angle of incident (i) = 60° (for fig. a and b) and 45° (for fig. c).

(a) By applying Snell's law, refractive index of the glass w.r.t. air is:

$${}^a\mu_g = \frac{\sin 60^\circ}{\sin 35^\circ} = \frac{0.8660}{0.5736}$$

$${}^a\mu_g = 1.51$$

(b) By applying Snell's law, refractive index of the water w.r.t. air is:

$${}^a\mu_{tw} = \frac{\sin 60^\circ}{\sin 47^\circ} = \frac{0.8660}{0.7314}$$

$${}^a\mu_{tw} = 1.184$$



(c) By applying Snell's law, angle of refraction (water to glass) is:

$${}^w\mu_g = \frac{\sin 45^\circ}{\sin r}$$

$$\frac{1.51}{1.18} = \frac{\sin 45^\circ}{1.18} = \frac{0.7071}{1.18}$$

$$\sin r = \frac{1.18 \times 0.7071}{1.51} = 0.55$$

$$r \cong 38.68^\circ$$

Therefore, the refractive index of glass and water w.r.t. air is 1.51 and 1.18 respectively and for fig. (c) the angle of refraction is 38.68° .

9.5 A small bulb is placed at the bottom of a tank containing water to a depth of 80cm. What is the area of the surface of water through which light from the bulb can emerge out? Refractive index of water is 1.33. (Consider the bulb to be a point source.)

SOLUTION:

Given – depth of tank = 80 cm, Refractive index of water = 1.33.

From figure there are three cases:

- (i) Light rays' incident below critical angle shows refraction.
- (ii) Light rays' incident at critical angle grazes the water surface.
- (iii) Light rays' incident at more than critical angle shows TIR.

Formula for critical angle is:

$$\sin C = \frac{1}{{}^a\mu_w}$$

$$\sin C = \frac{1}{1.33} = \frac{3}{4}$$

From figure,

$$\tan C = \frac{R}{OP}$$

$$R = \tan C \times OP = \tan C(0.80)$$

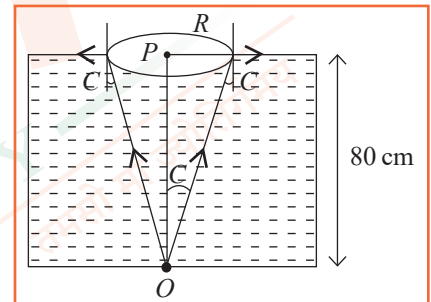
$$\text{Area of the surface (A)} = \pi R^2 = \pi \times \tan^2 C(0.64)$$

$$A = \pi(0.64) \times \frac{\sin^2 C}{\cos^2 C}$$

$$A = \pi(0.64) \times \frac{9}{16} \times \frac{16}{7} \quad \left(\because \sin C = \frac{3}{4} \text{ and } \cos C = \frac{\sqrt{7}}{4} \right)$$

$$A = \frac{22}{7} \times 0.64 \times \frac{9}{7}$$

$$A = 2.6 \text{ m}^2$$



- 9.6 A prism is made of glass of unknown refractive index. A parallel beam of light is incident on a face of the prism. The angle of minimum deviation is measured to be 40° . What is the refractive index of the material of the prism? The refracting angle of the prism is 60° . If the prism is placed in water (refractive index 1.33), predict the new angle of minimum deviation of a parallel beam of light.

SOLUTION:

Given – Angle of minimum deviation (δ_m) = 40° , Angle of refraction (r) = 60° , Refractive index of water (${}^a\mu_w$) = 1.33.

Formula for refractive index of glass w.r.t. air is:

$${}^a\mu_g = \frac{\sin \frac{A + \delta_m}{2}}{\sin \frac{A}{2}}$$

$${}^a\mu_g = \frac{\sin \frac{60^\circ + 40^\circ}{2}}{\sin \frac{60^\circ}{2}}$$

$${}^a\mu_g = \frac{\sin 50^\circ}{\sin 30^\circ} = \frac{0.766}{0.50} = 1.532$$

Therefore, refractive index of glass w.r.t. air is 1.532.

Now, the prism emerged in water, new angle of minimum deviation is:

$${}^w\mu_g = \frac{\sin \frac{A + \delta'_m}{2}}{\sin \frac{A}{2}}$$

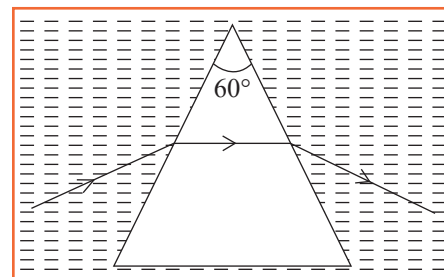
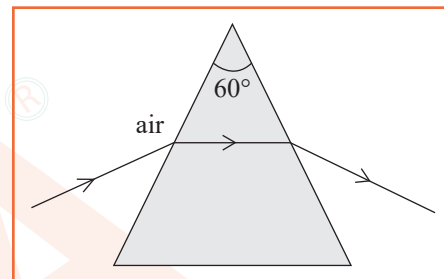
$$\frac{{}^a\mu_g}{{}^a\mu_w} = \frac{\sin \frac{60^\circ + \delta'_m}{2}}{\sin \frac{60^\circ}{2}}$$

$$\sin \left(30^\circ + \frac{\delta'_m}{2} \right) = \frac{1}{2} \left[\frac{1.532}{1.33} \right] = 0.5759$$

$$30^\circ + \frac{\delta'_m}{2} = 35^\circ 10'$$

$$\delta'_m = 2(35^\circ 10' - 30^\circ) = 10^\circ 20'$$

Therefore, the new angle of minimum deviation is $10^\circ 20'$.



9.7 Double-convex lenses are to be manufactured from a glass of refractive index 1.55, with both faces of the same radius of curvature. What is the radius of curvature required if the focal length is to be 20 cm?

SOLUTION:

Given – Refractive index of glass (${}^a\mu_g$) = 1.55, focal length (f) = + 20 cm.

Both faces of double-convex lens are same, let $R_1 = R_2 = R$ (same in magnitude but opposite sign).

Lens makers formula is given as:

$$\frac{1}{f} = ({}^a\mu_g - 1) \left[\frac{1}{R_1} - \frac{1}{R_2} \right]$$

$$\frac{1}{f} = (1.55 - 1) \left[\frac{1}{R} - \frac{1}{-R} \right] = 0.55 \left[\frac{2}{R} \right]$$

$$\frac{1}{20} = \frac{1.10}{R}$$

$$R = 20 \times 1.1 = 22 \text{ cm}$$

Therefore, the radius or curvature required is 22 cm for each face.

9.8 A beam of light converges at a point P. Now a lens is placed in the path of the convergent beam 12 cm from P. At what point does the beam converge if the lens is (a) a convex lens of focal length 20cm, and (b) a concave lens of focal length 16cm?

SOLUTION:

Given – Focal length of convex lens (f) = 20 cm, Focal length of concave lens (f') = 16 cm.

(a) Convex lens placed in the path of converging beam of light.

From Lens formula,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

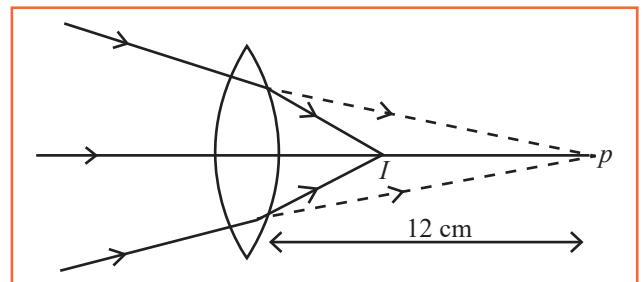
Where, f = + 20 cm, u = + 12 cm, v = ?

So,

$$\frac{1}{(+20)} = \frac{1}{v} - \frac{1}{(+12)}$$

$$\frac{1}{v} = \frac{1}{20} + \frac{1}{12} = \frac{3+5}{60}$$

$$\frac{1}{v} = \frac{8}{60} \Rightarrow v = \frac{60}{8} = +7.5 \text{ cm}$$



Therefore, the image I is formed by the further convergence of beams at +7.5 cm from the lens.

(b) Concave lens placed in the path of converging beam of light.

From Lens formula,

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$$\frac{1}{f'} = \frac{1}{v} - \frac{1}{u}$$

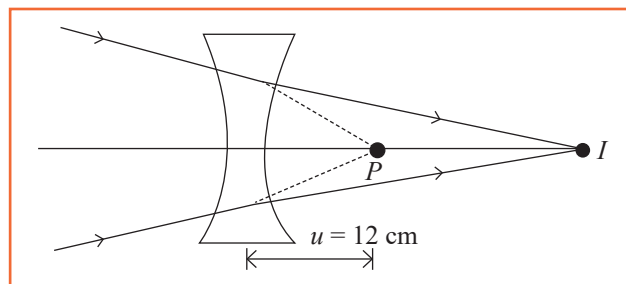
Where, $f = 16 \text{ cm}$, $u = +12 \text{ cm}$, $v = ?$

So,

$$\frac{1}{(-16)} = \frac{1}{v} - \frac{1}{(+12)}$$

$$\frac{1}{v} = \frac{1}{12} - \frac{1}{16} = \frac{4-3}{48}$$

$$\frac{1}{v} = \frac{1}{48} \Rightarrow v = +48 \text{ cm}$$



Therefore, the image I is formed by the further divergence of beams at +48 cm from the lens.

- 9.9** An object of size 3.0 cm is placed 14 cm in front of a concave lens of focal length 21 cm. Describe the image produced by the lens. What happens if the object is moved further away from the lens?

SOLUTION:

Given – Size of object (h_0) = +3 cm, Distance of object from the lens (u) = -14 cm, Focal length of concave lens (f) = -21 cm.

From Lens formula,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

Where, $f = 21 \text{ cm}$, $u = 14 \text{ cm}$, $v = ?$

So,

$$\frac{1}{(-21)} = \frac{1}{v} - \frac{1}{(-14)}$$

$$\frac{1}{v} = \left(\frac{1}{21} - \frac{1}{14} \right) = \frac{-2-3}{42}$$

$$\frac{1}{v} = \frac{-5}{42} \Rightarrow v = -\frac{42}{5} = -8.4 \text{ cm}$$

From magnification formula,

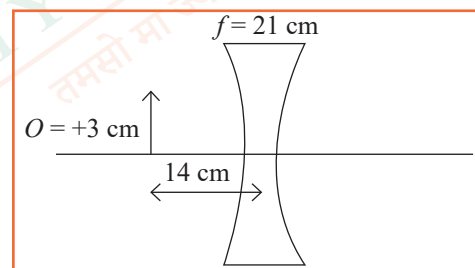
$$m = \frac{h_i}{h_0} = +\frac{v}{u}$$

$$\frac{h_i}{h_0} = +\frac{v}{u} \Rightarrow \frac{h_i}{+3} = \frac{-8.4}{-14}$$

$$h_i = +1.8 \text{ cm}$$

Therefore, the image is situated at 8.4 cm from the lens on the same side of the object and the image is virtual, erect and smaller than the object.

When the object moves away from the lens, the virtual image moves toward the focus but never beyond it. The image also gets smaller as it moves towards the focus.



9.10 What is the focal length of a convex lens of focal length 30cm in contact with a concave lens of focal length 20cm? Is the system a converging or a diverging lens? Ignore thickness of the lenses.

SOLUTION:

Given – Focal length of convex lens (f) = + 30 cm, Focal length of concave lens (f') = -20 cm,

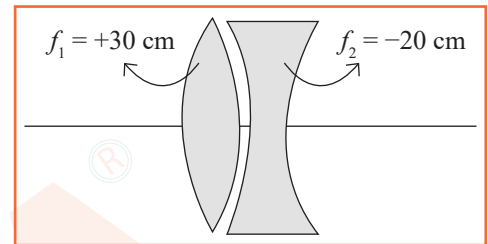
Need to find – Focal length and nature or combined system.

The equivalent focal length of the combined system is given by,

$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\frac{1}{f_{eq}} = \frac{1}{30} + \frac{1}{-20} = \frac{2-3}{60}$$

$$f_{eq} = -60cm$$



Therefore, the focal length of the system is -60 cm, i.e., the system is behaving as a diverging lens.

9.11 A compound microscope consists of an objective lens of focal length 2.0 cm and an eyepiece of focal length 6.25 cm separated by a distance of 15 cm. How far from the objective should an object be placed in order to obtain the final image at (a) the least distance of distinct vision (25 cm), and (b) at infinity? What is the magnifying power of the microscope in each case?

SOLUTION:

Given – Focal length of object lens (f_o) = 2 cm, Focal length of eyepiece (f_e) = 6.25 cm, Separation (d) = 15 cm.

(a) Need to obtain the final image at the least distance of distinct vision (D) = 25 cm.

Let, the distance of object from objective lens = u_o and

The distance of object for the eyepiece = u_e .

From Lens formula for eyepiece,

$$\frac{1}{f_e} = \frac{1}{v_e} - \frac{1}{u_e}$$

Where, $f_e = 6.25$ cm, $v_e = 25$ cm, $u_e = ?$

So,

$$\frac{1}{+6.25} = \frac{1}{-25} - \frac{1}{u_e}$$

$$\frac{1}{u_e} = \frac{1}{(-25)} - \frac{1}{(+6.25)}$$

$$\frac{1}{u_e} = \frac{-4}{25} - \frac{1}{25} = \frac{-1}{5}$$

$$u_e = -5cm$$

The distance of image formed by objective lens is:

$$v_o = (d - u_e) = (15 - 5) = 10 \text{ cm}$$

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Now, From Lens formula for objective,

$$\frac{1}{f_0} = \frac{1}{v_0} - \frac{1}{u_0}$$

Where, $f_0 = 2 \text{ cm}$, $v_0 = +10 \text{ cm}$, $u_0 = ?$

$$\text{So, } \frac{1}{2} = \frac{1}{+10} - \frac{1}{u_0}$$

$$\frac{1}{u_0} = \frac{1}{10} - \frac{1}{2} = \frac{1-5}{10}$$

$$u_0 = -2.5 \text{ cm}$$

Magnification formula is:

$$m = -\frac{v_o}{u_o} \left[1 + \frac{D}{f_e} \right]$$

$$m = -\frac{10}{-2.5} \left[1 + \frac{25}{6.25} \right] = 4[5]$$

$$m = 20$$

Therefore, the object should be placed at 2.5 cm from the objective lens and the magnification is 20.

(b) Need to obtain the final image at infinity.

Let again, the distance of object from objective lens = u_0 and

The distance of object for the eyepiece = $u_e = f_e = 6.25 \text{ cm}$ because we need to get final image at infinity

i.e. $v_e = \infty$

Image distance of objective lens (v_0) = $(d - f_e)$

$$v_0 = (15 - 6.25)$$

$$v_0 = 8.75 \text{ cm}$$

From Lens formula for objective,

$$\frac{1}{f_0} = \frac{1}{v_0} - \frac{1}{u_0}$$

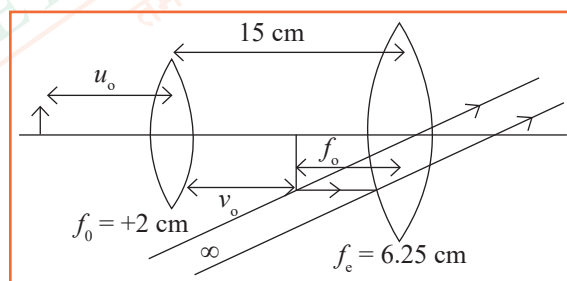
Where, $f_0 = 2 \text{ cm}$, $v_0 = 8.75 \text{ cm}$, $u_0 = ?$

$$\frac{1}{2} = \frac{1}{8.75} - \frac{1}{u_0}$$

$$\frac{1}{u_0} = \frac{1}{8.75} - \frac{1}{2} = \frac{2-8.75}{17.5}$$

$$\frac{1}{u_0} = -\frac{6.75}{17.5} \Rightarrow u_0 = -\frac{17.5}{6.75} \text{ cm}$$

$$u_0 = -2.59 \text{ cm}$$



Magnification formula is:



$$m = -\frac{v_o}{u_o} \left[\frac{D}{f_e} \right]$$

$$m = -\frac{8.75}{-2.59} \left[\frac{25}{6.25} \right] = 3.37[4]$$

$$m = 13.5$$

Therefore, the object should be placed at 2.59 cm from the objective lens and the magnification is 13.5.

9.12 A person with a normal near point (25cm) using a compound microscope with objective of focal length 8.0 mm and an eyepiece of focal length 2.5cm can bring an object placed at 9.0mm from the objective in sharp focus. What is the separation between the two lenses? Calculate the magnifying power of the microscope.

SOLUTION:

Given – Normal near point = 25 cm, Focal length of objective (f_o) = 8 mm, Focal length of eyepiece (f_e) = 2.5 cm, Distance of object from objective lens (u_o) = 9 mm.

Need to find – Separation between the two lenses (d) and magnifying power (m) of microscope.

We know that,

$$\text{Separation (d)} = |v_o + u_e|$$

From Lens formula for objective,

$$\frac{1}{f_o} = \frac{1}{v_o} - \frac{1}{u_o}$$

Where, $f_o = 8 \text{ mm}$, $v_o = ?$, $u_o = -9 \text{ mm}$

So,

$$\frac{1}{(+8)} = \frac{1}{v_o} - \frac{1}{(-9)}$$

$$\frac{1}{v_o} = \frac{1}{8} - \frac{1}{9} = \frac{1}{72}$$

$$v_o = 72 \text{ mm} = 7.2 \text{ cm}$$

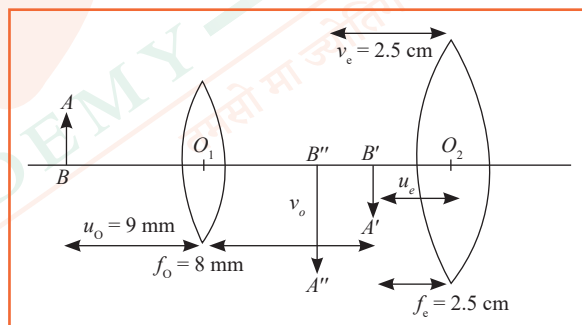
Now, from Lens formula for eyepiece,

$$\frac{1}{f_e} = \frac{1}{v_e} - \frac{1}{u_e}$$

Where, $f_e = 2.5 \text{ cm}$, $v_e = D = -25 \text{ cm}$, $u_e = ?$

$$\frac{1}{2.5} = \frac{1}{-25} - \frac{1}{u_e}$$

$$\frac{1}{u_e} = -\frac{1}{25} - \frac{2}{5} = -\frac{11}{25}$$



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$$u_e = -\frac{25}{11} = -2.27 \text{ cm}$$

Separation between lenses (d) = $|v_o + u_e| = (7.2 + 2.2) = 9.4 \text{ cm}$

Magnification formula is:

$$m = -\frac{v_o}{u_o} \left[1 + \frac{D}{f_e} \right]$$

$$m = -\frac{7.2}{-0.9} \left[1 + \frac{25}{2.5} \right] = 8[11]$$

$$m = 88$$

Therefore, the separation between the lenses is 9.4 cm and the magnification is 88.

- 9.13** A small telescope has an objective lens of focal length 144cm and an eyepiece of focal length 6.0cm. What is the magnifying power of the telescope? What is the separation between the objective and the eyepiece?

SOLUTION:

Given – Focal length of objective (f_o) = 144 cm, Focal length of eyepiece (f_e) = 6 cm,
Need to find – Magnifying power (m) of microscope and separation between the lenses (d).
We know that,

$$\text{Magnifying power} = \frac{f_o}{f_e} = \frac{144}{6} = 24$$

and the separation between the objective and eyepiece is given by,

$$d = f_o + f_e = 144 + 6 = 150 \text{ cm}$$

Therefore, the magnifying power of microscope is 24 and separation between the lenses is 150 cm.

- 9.14** (a) A giant refracting telescope at an observatory has an objective lens of focal length 15 m. If an eyepiece of focal length 1.0 cm is used, what is the angular magnification of the telescope?
(b) If this telescope is used to view the moon, what is the diameter of the image of the moon formed by the objective lens? The diameter of the moon is $3.48 \times 10^6 \text{ m}$, and the radius of lunar orbit is $3.8 \times 10^8 \text{ m}$.

SOLUTION:

(a) Focal length of objective (f_o) = 15 m = 1500 cm, Focal length of eyepiece (f_e) = 1 cm.
Angular magnification of the telescope is given by,

$$m = \frac{f_o}{f_e} = -\frac{1500 \text{ cm}}{1.0 \text{ cm}} = -1500$$

(b) Diameter of the moon (d_m) = $3.48 \times 10^6 \text{ m}$, Radius of lunar orbit (r_o) = $3.8 \times 10^8 \text{ m}$.

The objective lens forms the image of the moon at its focus because



the moon is at a nearly infinite distance compared to the lens's focal length.

$$\text{Radius of moon } (r_m) = \frac{3.48 \times 10^6}{2} = 1.74 \times 10^6 \text{ m}$$

Here,

Distance of lunar object = Radius of orbit (r_o) = $3.8 \times 10^8 \text{ m}$,

Height of object = Radius of the moon (r_m) = $1.74 \times 10^6 \text{ m}$, and

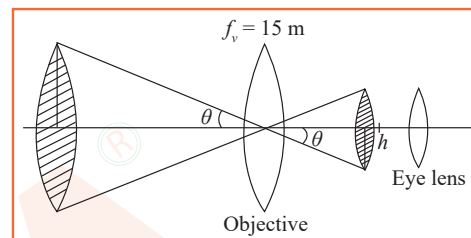
Distance of image = Focal length of objective lens (f_o) = 15 cm.

Radius of image (r_i) of the moon is given by,

$$\tan \theta = -\frac{r_m}{r_o} = \frac{r_i}{f_o}$$

$$r_i = \frac{r_m \times f_o}{r_o} = \frac{(1.74 \times 10^6) \times 15}{(3.8 \times 10^8)}$$

$$r_i = 6.87 \times 10^{-2} \text{ m} = 6.87 \text{ cm}$$



and the diameter of the image of the moon is given by,

$$d_i = 2r_i = 2(6.87 \times 10^{-2}) \text{ m} = 13.74 \text{ cm}$$

Therefore, the diameter of the image is 13.74 cm.

9.15 Use the mirror equation to deduce that:

- an object placed between f and $2f$ of a concave mirror produces a real image beyond $2f$.
- a convex mirror always produces a virtual image independent of the location of the object.
- the virtual image produced by a convex mirror is always diminished in size and is located between the focus and the pole.
- an object placed between the pole and focus of a concave mirror produces a virtual and enlarged image.

[Note: This exercise helps you deduce algebraically properties of images that one obtains from explicit ray diagrams.]

SOLUTION:

(a) Object position b/w f and $2f$, deduce real image formed beyond $2f$.

For concave mirror: Focal length (f) and distance of object from mirror (u) both are negative.

$$\therefore \frac{1}{2f} > \frac{1}{u} > \frac{1}{f} \quad \text{or} \quad \frac{-1}{2f} < \frac{-1}{u} < \frac{-1}{f}$$

$$\frac{1}{f} - \frac{1}{2f} < \frac{1}{f} - \frac{1}{u} < \frac{1}{f} - \frac{1}{f} \quad \left(\because \frac{1}{f} - \frac{1}{u} = \frac{1}{v} \right)$$

$$\frac{1}{2f} < \frac{1}{v} < 0$$

i.e., v should be less than zero to form an image on the left of the mirror.

Also, $|2f| > |v|$ so, the real image is formed beyond $2f$.

(b) Convex mirror: Focal length (f) = +ve, Object distance (u) = -ve.



From mirror formula,

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

i.e., $\frac{1}{v} > 0 \Rightarrow v > 0$

Therefore, distance of image is always positive (virtual in nature) whatever the value of u .

(c) Convex mirror: Focal length (f) = +ve, Object distance (u) = -ve.

From mirror formula,

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

$$\frac{1}{v} > \frac{1}{f}$$

i.e., $v < f$ (both are +ve)

Magnification is given by,

$$m = -\frac{v}{u}$$

$$m = -\frac{(+v)}{(-u)} = +\frac{v}{u}$$

$$m < 1 \text{ and } +ve$$

Therefore, the image is formed between pole and the focus, and the magnification is less than one but positive which shows the image is virtual and diminished.

(d) Concave mirror: Focal length (f) = -ve, Object distance (u) = -ve.

Object placed between f and p then,

$$\frac{1}{f} > \frac{1}{u}$$

From mirror formula,

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\frac{1}{v} < 0 \text{ or } v > 0 \text{ (positive)}$$

i.e., image formed is virtual in nature.

Magnification is given by,

$$m = \frac{-v}{u}$$



$$\frac{1}{v} < \frac{1}{u} \Rightarrow v > |u|$$

So, $m > 1$
i.e., Image is enlarged.

- 9.16** A small pin fixed on a tabletop is viewed from above from a distance of 50 cm. By what distance would the pin appear to be raised if it is viewed from the same point through a 15cm thick glass slab held parallel to the table? Refractive index of glass = 1.5. Does the answer depend on the location of the slab?

SOLUTION:

Given – Refractive index of glass = 1.5.

Need to find – Shift in the image by the thick glass slab.

We know that the shift only depends upon thickness of glass slab and refractive index of glass.

i.e., Shift = Real thickness – Apparent of thickness

$$\text{Shift} = t_g \left[1 - \frac{1}{\mu_g} \right]$$

$$\text{Shift} = 15 \left[\frac{0.5}{1.5} \right] = 5\text{cm}$$

The answer does not depend on the location of the slab.

- 9.17** (a) Figure shows a cross-section of a ‘light pipe’ made of a glass fibre of refractive index 1.68. The outer covering of the pipe is made of a material of refractive index 1.44. What is the range of the angles of the incident rays with the axis of the pipe for which total reflections inside the pipe take place, as shown in the figure.
(b) What is the answer if there is no outer covering of the pipe?

SOLUTION:

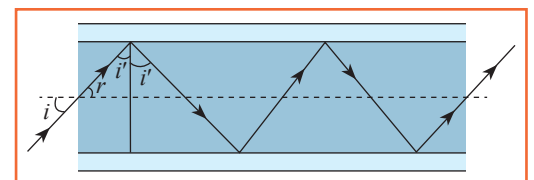
Given – Refractive index of glass fibre = 1.68, Refractive index of outer covering = 1.44.

(a) Firstly, discuss the condition of TIR,

The critical angle is given by,

$$\sin C = \frac{1}{\mu_2} = \frac{\mu_2}{\mu_1}$$

$$\sin C = \frac{1.44}{1.68} = 0.8571$$



The critical angle (C) = 59°.

Condition for total internal reflection from core to cladding is:

$$i_2 > 59^\circ \text{ or } r \leq \frac{\pi}{2} - 59^\circ \text{ or } r \leq 31^\circ$$

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Now, for refraction at first surface air to core is given by Snell's law:

$$\frac{\sin i_1}{\sin r} = {}^a\mu_1$$

$$\sin i_1 = {}^a\mu_1 \sin r = 1.68 \sin 31^\circ \text{ or } i_1 \approx 60^\circ$$

Therefore, all incident rays which makes angle of incidence between 0° to 60° shows TIR.

(b) When there is no outer covering critical angle from core to surface.

The critical angle is:

$$\sin C = \frac{1}{{}^1\mu_a} = \frac{\mu_a}{\mu_1}$$

$$\sin C = \frac{1}{1.68} \Rightarrow C = \sin^{-1}\left(\frac{1}{1.68}\right)$$

The critical angle (C) = 36.5° .

Condition for total internal reflection from core to surface is:

$$i_2 > 36.5^\circ \text{ or } r < \frac{\pi}{2} - 36.5^\circ \text{ or } r < 53.5^\circ$$

Now, the range of incident angle at first surface air to core is given by Snell's law:

$${}^a\mu_1 = \frac{\sin i_1}{\sin r}$$

$$\sin i_1 = 1.68 \times \sin 53.5^\circ = 1.68 \times 0.8039$$

$$\sin i_1 = 1.35 \text{ or } i \approx 90^\circ.$$

Therefore, all incident rays on the first surface, ranging from 0° to 90° , will undergo total internal reflection within the core.

9.18 The image of a small electric bulb fixed on the wall of a room is to be obtained on the opposite wall 3m away by means of a large convex lens. What is the maximum possible focal length of the lens required for the purpose?

SOLUTION:

Given – Distance between object and image is 3 m.

Let the object distance (u) = x m and image distance (v) = (3 – x) m.

From Lens formula we get,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{f} = \frac{1}{(3-x)} - \frac{1}{-x}$$

$$\frac{1}{f} = \frac{x+3-x}{x(3-x)}$$

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$$3f = 3x - x^2$$

$$x^2 - 3x + 3f = 0$$

$$x = \frac{+3 \pm \sqrt{9 - 4 \times (3f)}}{2}$$

$$x = \frac{+3 \pm \sqrt{9 - 12f}}{2}$$

Condition for image to be obtained on the screen, i.e., m real image. $9 - 12f \geq 0$ or $9 \geq 12f$ or $f \leq 0.75$ m. so, maximum focal length is 0.75 m.

Alternate method:

Distance between the object and the image (d) = 3 m

Maximum focal length of the convex lens = f_{\max}

For real images, the maximum focal length is given as:

$$f_{\max} = \frac{d}{4} = \frac{3}{4} = 0.75\text{m}$$

Therefore, for the required purpose, the maximum possible focal length of the convex lens is 0.75 m.

9.19 A screen is placed 90 cm from an object. The image of the object on the screen is formed by a convex lens at two different locations separated by 20 cm. Determine the focal length of the lens.

SOLUTION:

Given – Distance between object and screen = 90 cm, Separation b/w two locations (d) = 20 cm.

Need to find – Focal length (f) of the lens.

From figure,

$$u_1 + v_1 = 90 \quad \dots\dots\dots (i)$$

$$v_1 - u_1 = 20 \quad \dots\dots\dots (ii)$$

After solving equation (i) and (ii) we get,

$$u_1 = 35\text{cm}$$

$$v_1 = 55\text{cm}$$

From Lens formula we get,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

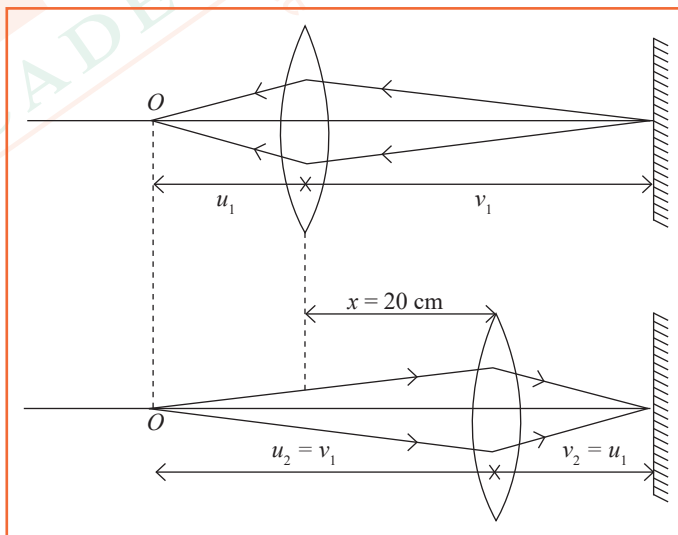
$$\frac{1}{f} = \frac{1}{55} - \frac{1}{-35} = \frac{1}{55} + \frac{1}{35}$$

$$\frac{1}{f} = \frac{35 + 55}{55 \times 35}$$

$$f = \frac{55 \times 35}{90}$$

$$f = +21.38\text{cm}$$

Therefore, the focal length of the lens is 21.38 cm.



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- 9.20 (a) Determine the 'effective focal length' of the combination of the two lenses in Exercise 9.10, if they are placed 8.0cm apart with their principal axes coincident. Does the answer depend on which side of the combination a beam of parallel light is incident? Is the notion of effective focal length of this system useful at all? (b) An object 1.5 cm in size is placed on the side of the convex lens in the arrangement (a) above. The distance between the object and the convex lens is 40cm. Determine the magnification produced by the two-lens system, and the size of the image.

SOLUTION:

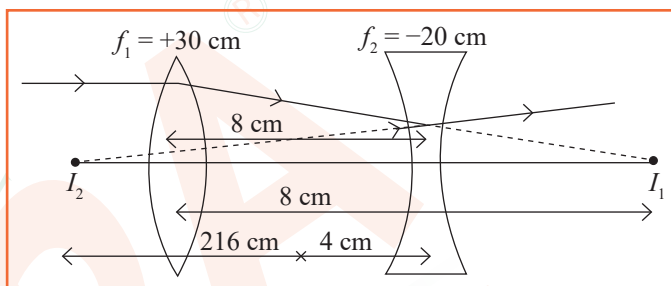
(a) **Case 1:** Let a parallel beam of light incident first on convex lens then after refraction passing through Concave lens.

Refraction at convex lens given by lens formula is:

$$\frac{1}{f_1} = \frac{1}{v_1} - \frac{1}{u_1}$$

$$\frac{1}{30} = \frac{1}{v_1} - \frac{1}{\infty}$$

$$v_1 = f_1 = 30\text{cm}$$



So, the virtual object for concave lens is at,

$$u_2 = (v_1 - d) = (30 - 8) = +22 \text{ cm}$$

Similarly, refraction at concave lens given by lens formula is:

$$\frac{1}{f_2} = \frac{1}{v_2} - \frac{1}{u_2}$$

$$\frac{1}{-20} = \frac{1}{v_2} - \frac{1}{+22} \Rightarrow \frac{1}{v_2} = \frac{1}{22} - \frac{1}{20}$$

$$v_2 = -220\text{cm}$$

The parallel beam of light seems to diverge from a point located 216 cm away from the centre of the two-lens system.

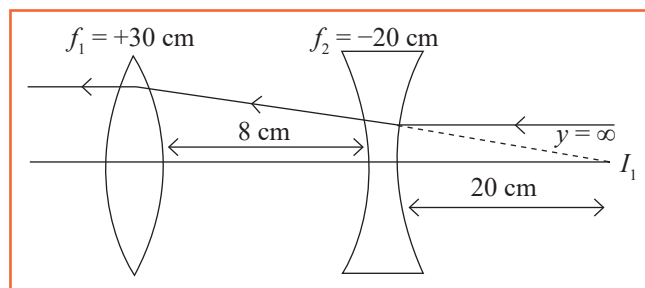
Case 2: Let a parallel beam of light incident first on concave lens.

Refraction at concave lens given by lens formula is:

$$\frac{1}{f_1} = \frac{1}{v_1} - \frac{1}{u_1}$$

$$\frac{1}{-20} = \frac{1}{v_1} - \frac{1}{\infty}$$

$$v_1 = f_1 = -20\text{cm}$$



The image I₁ will act as real object for convex lens at 28 cm.

$$\frac{1}{f_2} = \frac{1}{v_2} - \frac{1}{u_2}$$



$$\frac{1}{30} = \frac{1}{v_2} - \frac{1}{-28} \Rightarrow \frac{1}{v_2} = \frac{1}{30} - \frac{1}{28}$$

$$v_2 = -420 \text{ cm}$$

Thus, the parallel incident beam appears to diverge from a point $420 - 4 = 416 \text{ cm}$ to the left of the centre of the two-lens system. Therefore, the answer depends on which side of the lens system the parallel beam is incident. As a result, the effective focal length differs in the two situations.

(b) Now an object of 1.5 cm size is kept 40 cm in front of convex lens in the same system of lenses.

Distance between the object and convex lens is 40 cm .

Refraction through first lens is given by,

$$\frac{1}{f_1} = \frac{1}{v_1} - \frac{1}{u_1}$$

$$\frac{1}{30} = \frac{1}{v_1} - \frac{1}{-40}$$

$$\frac{1}{v_1} = \frac{1}{30} - \frac{1}{40} = \frac{1}{120}$$

$$v_1 = 120 \text{ cm}$$

Magnification is given by,

$$m_1 = \frac{h_i}{h_o} = \frac{v}{u}$$

$$m_1 = \frac{h_i}{1.5} = \frac{120}{-40}$$

$$h_i = (1.5) \times (-3) = -4.5 \text{ cm}$$

i.e.,
$$m_1 = \frac{-4.5}{1.5} = -3$$

Similarly, Refraction through second lens is given by,

$$\frac{1}{f_2} = \frac{1}{v_2} - \frac{1}{u_2}$$

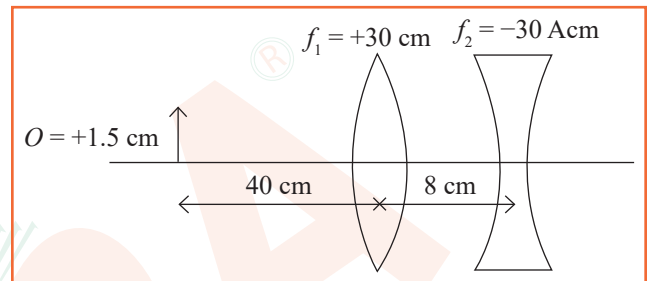
$$\frac{1}{-20} = \frac{1}{v_2} - \frac{1}{+112}$$

$$\frac{1}{v_2} = \frac{1}{112} - \frac{1}{20} = -\frac{92}{112 \times 20}$$

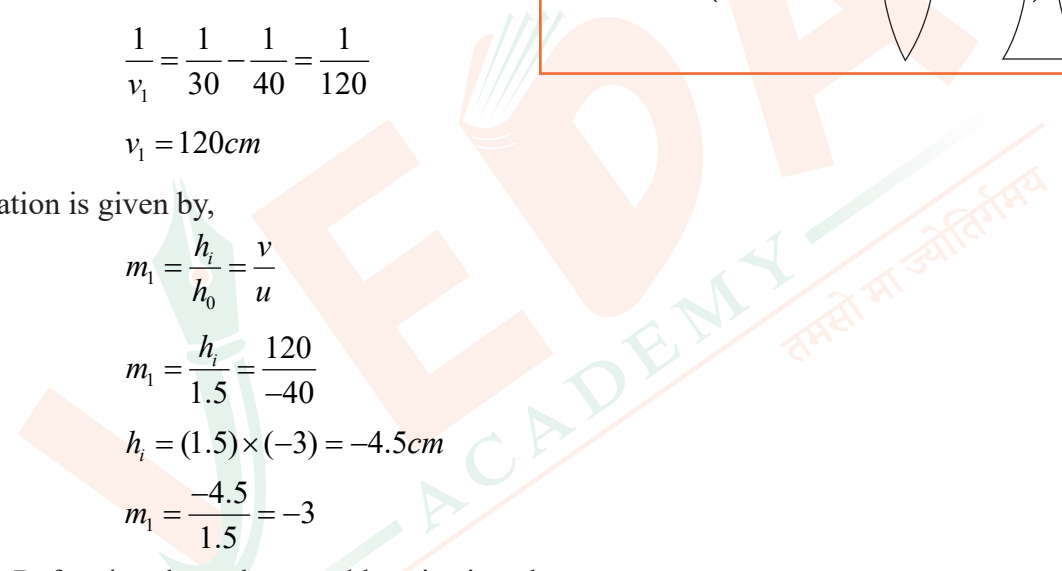
$$v_2 = -24.34 \text{ cm}$$

Magnification is given by,

$$m_2 = \frac{h_i}{h_o} = \frac{v_2}{u_2}$$



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$$m_2 = \frac{-24.32}{+112}$$

i.e., $m_2 = -0.217$

Total magnification by two lenses is given by,

$$m = m_1 \times m_2$$

$$m = (-3) \times (-0.217)$$

$$m = 0.652$$

Finally, size of the obtained image is:

$$m = \frac{h_i}{h_o}$$

$$0.652 = \frac{h_i}{1.5}$$

$$h_i = 1.5 \times 0.652 = 0.98 \text{ cm}$$

Therefore, the magnification by combined system is 0.652 and the size of the final image is 0.98 cm.

- 9.21** At what angle should a ray of light be incident on the face of a prism of refracting angle 60° so that it just suffers total internal reflection at the other face? The refractive index of the material of the prism is 1.524.

SOLUTION:

Given – Angle of refraction (r) = 60° , Refractive index of the prism (μ) = 1.524.

Need to find – Angle of incident (i).

We know that for total internal reflection to occur at the second surface of the prism, the beam must be incident at an angle equal to or greater than the critical angle.

For critical angle we know that,

$$\sin C = \frac{1}{\mu_g}$$

$$C = \sin^{-1} \left(\frac{1}{\mu_g} \right) = \sin^{-1} \left(\frac{1}{1.524} \right)$$

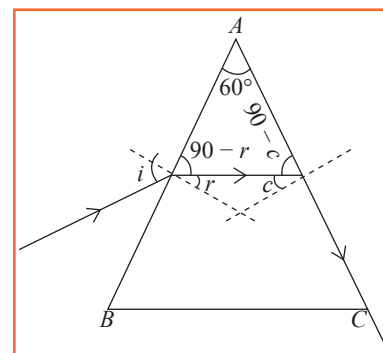
$$C = 41^\circ$$

From figure we can calculate 'r',

$$60^\circ + (90^\circ - r) + (90^\circ - C) = 180^\circ$$

$$r = 19^\circ$$

Angle of incident (i) can be calculated with the help of Snell's law is given as:



$${}^a\mu_g = \frac{\sin i}{\sin r}$$

$$1.524 = \frac{\sin i}{\sin 19^\circ}$$

$$\sin i = 1.524(\sin 19^\circ)$$

$$i = \sin i = 1.524 \times 0.3256$$

$$i \cong 29.75^\circ$$

Therefore, the angle of incidence for which the ray of light shows TIR is 29.75° .

9.22 A card sheet divided into squares each of size 1 mm^2 is being viewed at a distance of 9 cm through a magnifying glass (a converging lens of focal length 9 cm) held close to the eye.

(a) What is the magnification produced by the lens? How much is the area of each square in the virtual image?

(b) What is the angular magnification (magnifying power) of the lens?

(c) Is the magnification in (a) equal to the magnifying power in (b)? Explain.

SOLUTION:

Given – Area of square (A) = 1 mm^2 , Focal length (f) = 9 cm , distance (d) = 9 cm .

(a) Magnification by the lens is given by,

$$m = \frac{h_i}{h_o} = \frac{v}{u}$$

From figure, $u = -9 \text{ cm}$, $f = +10 \text{ cm}$, $v = ?$

To find v we use lens formula given as:

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{10} = \frac{1}{v} - \frac{1}{-9}$$

$$\frac{1}{v} = \frac{1}{10} - \frac{1}{9} = -\frac{1}{90}$$

$$v = -90 \text{ cm}$$

Put this value of v in magnification formula we get,

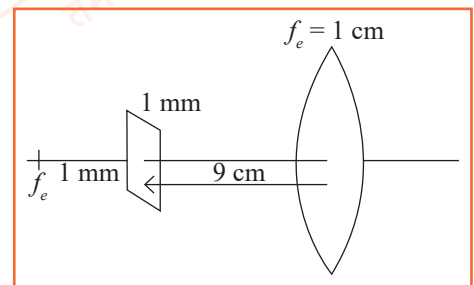
$$m = \frac{-90}{-9} = 10$$

Given area is 1 mm^2 , so for this we consider the size of object is 1 mm , then the size of image is:

$$m = \frac{h_i}{h_o}$$

$$10 = \frac{h_i}{1}$$

$$h_i = 10 \text{ mm}$$



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Area of the image (A') = $(10 \text{ mm})^2 = 100 \text{ mm}^2$

Therefore, the magnification produced by lens is 10 and the area of virtual image is 100 mm^2 or 10 cm^2 .

(b) Angular magnification is given by,

$$m = \frac{D}{u} = \frac{25}{9} = 2.78$$

$$m = 2.78$$

(c) No. The linear magnification (m) = $\frac{v}{u} = 10$ and Angular magnification (m) = $\frac{D}{u} = 2.78$ have difference values.

Linear magnification and angular magnification have similar magnitude when $D = v$ i.e., image is at least distance of distinct vision ($D = 25 \text{ cm}$).

9.23 (a) At what distance should the lens be held from the card sheet in Exercise 9.22 in order to view the squares distinctly with the maximum possible magnifying power?

(b) What is the magnification in this case?

(c) Is the magnification equal to the magnifying power in this case? Explain.

SOLUTION:

(a) For maximum magnifying power, the image should be formed at the least distance of distinct vision, which is 25 cm.

To find the position of object we use lens formula given as:

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{10} = \frac{1}{-25} - \frac{1}{u}$$

$$-\frac{1}{u} = \frac{1}{10} + \frac{1}{25} = \frac{7}{50}$$

$$u = -\frac{50}{7} = -7.14 \text{ cm}$$

Therefore, the position object from lens is 7.14 cm left from the lens.

(b) Magnification is given by,

$$m = \frac{h_i}{h_o} = \frac{v}{u}$$

$$m = \frac{-25}{-\frac{50}{7}} = \frac{7}{2} = 3.5$$

$$m = 3.5$$

Therefore, the linear magnification is 3.5.



(c) Magnifying power is given as:

$$m = \frac{D}{u_{\min}} \text{ or } m = \left(1 + \frac{D}{f_c}\right)$$

$$m = \frac{25}{\frac{50}{7}} \text{ or } m = \left[1 + \frac{25}{10}\right]$$

$$m = 3.5$$

It can be observed that when the image is at the least distance of distinct vision, the angular magnification and linear magnification have approximately the same values.

9.24 What should be the distance between the object in Exercise 9.23 and the magnifying glass if the virtual image of each square in the figure is to have an area of 6.25 mm^2 . Would you be able to see the squares distinctly with your eyes very close to the magnifier?

[Note: Exercises 9.22 to 9.24 will help you clearly understand the difference between magnification in absolute size and the angular magnification (or magnifying power) of an instrument.]

SOLUTION:

Given: Area of virtual image of each square (A) = 6.25 mm^2 .

Side of image is (a) = $\sqrt{6.25} = 2.5 \text{ mm}$ which is also known as linear magnification.

Focal length (f) = 10 cm.

Position of object and image is given by magnification and lens formula is:

$$\begin{aligned} \text{Magnification (m)} &= \frac{h_i}{h_o} = \frac{v}{u} \\ \frac{2.5 \text{ mm}}{1 \text{ mm}} &= \frac{v}{u} \\ v &= 2.5u \end{aligned}$$

From lens formula,

$$\begin{aligned} \frac{1}{f} &= \frac{1}{v} - \frac{1}{u} \\ \frac{1}{10} &= \frac{1}{2.5u} - \frac{1}{u} \\ \frac{1}{10} &= \frac{-3}{5u} \Rightarrow u = -6 \end{aligned}$$

$$u = -6 \text{ cm and}$$

$$v = 2.5(-6) = -15 \text{ cm}$$

Therefore, the required virtual image is -15 cm which is closer than normal near point. Thus, the eye cannot observe the image distinctly.

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9.25 Answer the following questions:

- (a) The angle subtended at the eye by an object is equal to the angle subtended at the eye by the virtual image produced by a magnifying glass. In what sense then does a magnifying glass provide angular magnification?
- (b) In viewing through a magnifying glass, one usually positions one's eyes very close to the lens. Does angular magnification change if the eye is moved back?
- (c) Magnifying power of a simple microscope is inversely proportional to the focal length of the lens. What then stops us from using a convex lens of smaller and smaller focal length and achieving greater and greater magnifying power?
- (d) Why must both the objective and the eyepiece of a compound microscope have short focal lengths?
- (e) When viewing through a compound microscope, our eyes should be positioned not on the eyepiece but a short distance away from it for best viewing. Why? How much should be that short distance between the eye and eyepiece.

SOLUTION:

- (a) In a magnifying glass, the object is positioned closer than 25 cm, creating an image at 25 cm. This closer placement increases the object's angular size compared to its size at 25 cm. As a result, even though the virtual image and the object subtend the same angle at the eye, angular magnification is still achieved.
- (b) As the eye moves backward, away from the lens, the angular magnification slightly decreases. This happens because both the angle subtended by the image at the eye (α) and the angle subtended by the object at the eye (α) decrease, with the reduction in the angle subtended by the object being relatively smaller.
- (c) Reducing the focal length of a lens requires it to be thicker and have a smaller radius of curvature. However, in a thick lens, spherical and chromatic aberrations become more significant. Additionally, crafting lenses with very short focal lengths is challenging. As a result, a simple convex lens typically cannot achieve a magnifying power greater than 3 in practice.
- (d) The angular magnification of the compound microscope's eyepiece is:

$$m = \left[1 + \frac{D}{f_e} \right]$$

From the above formula, if f_e is small, then angular magnification of the eyepiece will be large.

The angular magnification of the compound microscope's objective lens is:

$$m = \frac{f_o}{u_o}$$

From figure magnification is large when $u_o > f_o$.

Therefore, f_e and f_o are both small for the given condition.

- (e) Placing the eye too close to the eyepiece limits the amount of light collected and reduces the field of view. By positioning the eye slightly farther away, where the pupil area is larger, the eye can capture all the light refracted by the objective, resulting in a clear and well-defined image.



9.26 An angular magnification (magnifying power) of 30X is desired using an objective of focal length 1.25cm and an eyepiece of focal length 5cm. How will you set up the compound microscope?

SOLUTION:

Given – Angular magnification (m) = 30x, Focal length of objective (f_o) = 1.25 cm, Focal length of eyepiece (f_e) = 5 cm.

To determine the distance between the objective and the eyepiece for a required magnification of 30x, assume the final image formed by the eyepiece is at the least distance of distinct vision.

Magnification is given by,

$$m_e = \left(1 + \frac{D}{f_e}\right) = \frac{D}{u_e}$$

$$m_e = 1 + \frac{25}{5} = \frac{25}{u_e}$$

$$m_e = 6 \text{ and } u_e = \frac{25}{6} \text{ cm}$$

Similarly, magnification by objective lens is given by,

$$m = m_o \times m_e$$

$$m_o = \frac{m}{m_e} = \frac{30}{6} = 5$$

and we know that, $m_o = \frac{v_o}{u_o} = -5$ (\because real image is formed by objective.)

$$v_o = -5u_o$$

Now, from lens formula for object is given by,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{1.25} = \frac{1}{-2.5u} - \frac{1}{u}$$

$$\frac{1+5}{5u_o} = -\frac{4}{5}$$

$$5u_o = -7.5 \Rightarrow u_o = -1.5 \text{ cm}$$

$$v_o = -5u_o = 7.5 \text{ cm}$$

So, the required distance between objective and eyepiece lens is:

$$L = v_o + |u_e| = 7.5 + \frac{25}{6} = 11.67 \text{ cm}$$

Therefore, the separation between the objective and eyepiece is 11.67 cm.

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9.27 A small telescope has an objective lens of focal length 140 cm and an eyepiece of focal length 5.0 cm. What is the magnifying power of the telescope for viewing distant objects when

- (a) the telescope is in normal adjustment (i.e., when the final image is at infinity)?
- (b) the final image is formed at the least distance of distinct vision (25 cm)?

SOLUTION:

Given – Focal length of objective (f_o) = 140 cm, Focal length of eyepiece (f_e) = 5 cm.

(a) In normal adjustment magnifying power is:

$$m = -\frac{f_o}{f_e}$$

$$m = \frac{140}{5} = 28$$

Therefore, the magnifying power is 28.

(b) For the image at least distance of distinct vision the magnifying power is:

$$m = \frac{f_o}{f_e} \left[1 + \frac{f_e}{D} \right]$$

$$m = 28 \left[\frac{30}{25} \right] = 33.6$$

Therefore, the magnifying power is 33.6

9.28 (a) For the telescope described in Exercise 9.27 (a), what is the separation between the objective lens and the eyepiece?

(b) If this telescope is used to view a 100 m tall tower 3 km away, what is the height of the image of the tower formed by the objective lens?

(c) What is the height of the final image of the tower if it is formed at 25 cm?

SOLUTION:

(a) In case of most relaxed eye,

$$\text{Separation (L)} = (f_o + f_e) = (140 + 5) = 145\text{cm}$$

In case of most strained eye,

$$\text{Separation (L)} = f_o + |u_e| \quad \dots\dots\dots (i)$$

To find object distance (u_e) for eye lens we use Lens formula,

$$\frac{1}{f_e} = \frac{1}{v_e} - \frac{1}{u_e}$$

$$\frac{1}{5} = \frac{1}{-25} - \frac{1}{u_e}$$

$$-\frac{1}{u_e} = \frac{1}{5} + \frac{1}{25} = \frac{6}{25}$$



$$u_e = -\frac{25}{6} = -4.16\text{cm}$$

Put this value of u_e in equation (i) we get,

$$L = 145 + 4.16 = 149.16\text{cm}$$

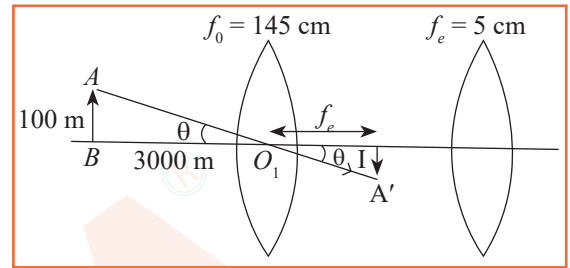
Therefore, the separation between objective and eyepiece is 145 cm and 149.16 cm respectively.

(b) From figure,

$$\tan \theta = \frac{A'B'}{B'O_1} = \frac{AB}{BO_1}$$

$$\text{height of image } A'B' = B'O_1 \times \frac{AB}{BO_1}$$

$$A'B' = 140 \times \frac{100}{3000} = 4.7\text{cm}$$



Therefore, the size of the image is 4.7 cm.

(c) From figure, need to find size of image ($A''B''$).

Let us assume that it formed at 25 cm.

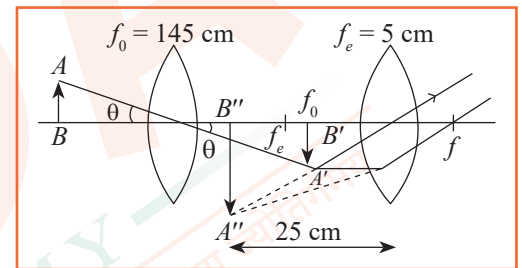
Magnification by the eyepiece is,

$$m_e = \left(1 + \frac{D}{f_e}\right) = \left(1 + \frac{25}{5}\right) = 6$$

Now,

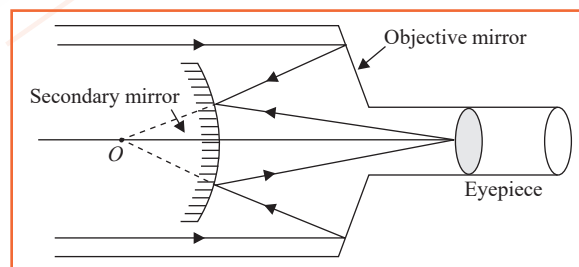
$$m_e = \frac{A''B''}{A'B'}$$

$$A''B'' = m_e \times A'B' = 6 \times 4.7 = 28.2\text{cm}$$



Therefore, the size of the image is 28.2 cm.

- 9.29** A Cassegrain telescope uses two mirrors as shown in Fig. Such a telescope is built with the mirrors 20 mm apart. If the radius of curvature of the large mirror is 220 mm and the small mirror is 140 mm, where will the final image of an object at infinity be?



SOLUTION:

Given – Separation between the mirrors (d) = 20 mm, Radius of curvature of large mirror (R_L) = 220 mm, Radius of curvature of small mirror (R_s) = 140 mm, Object at infinity.

From figure, image formed by concave mirror acts as a virtual object for convex mirror.



Here parallel rays coming from infinity will focus on focus point,

$$f_L = \frac{R_L}{2} = \frac{220}{2} = 110 \text{ mm}$$

Distance of virtual object for convex mirror = $(110 - 20) = 90 \text{ mm}$

For convex mirror we use mirror formula,

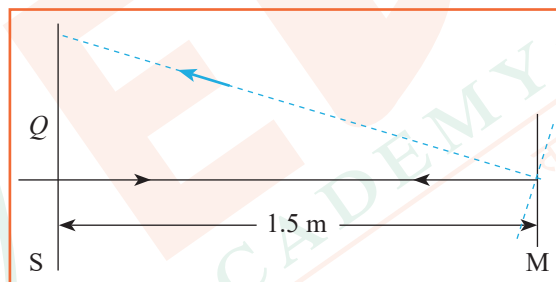
$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

Where, $u = -90 \text{ mm}$, $f = -70 \text{ mm}$ and $v = ?$

$$\begin{aligned} \text{So, } \frac{1}{-70} &= \frac{1}{v} + \frac{1}{-90} \\ \frac{1}{v} &= \frac{1}{90} - \frac{1}{70} = -\frac{2}{630} \\ v &= -315 \text{ mm} \end{aligned}$$

Therefore, the image is formed at 315 mm from convex mirror.

- 9.30** Light incident normally on a plane mirror attached to a galvanometer coil retraces backwards as shown in Fig. A current in the coil produces a deflection of 3.5° of the mirror. What is the displacement of the reflected spot of light on a screen placed 1.5 m away?



SOLUTION:

Given – Deflection of the mirror = 3.5° , Distance of screen = 1.5 m.

Deflection of the spot is given by,

$$\tan 7^\circ = \frac{d}{1.5}$$

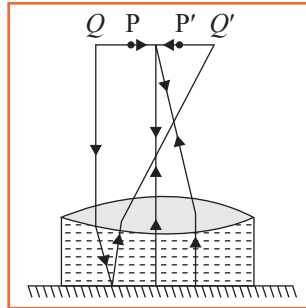
$$d = 1.5 \tan 7^\circ = 1.5[0.1228]$$

$$d = 0.1842 \text{ m} = 18.42 \text{ cm}.$$

Therefore, the deflection of the reflected spot is 18.42 cm.

- 9.31** Figure shows an equiconvex lens (of refractive index 1.50) in contact with a liquid layer on top of a plane mirror. A small needle with its tip on the principal axis is moved along the axis until its inverted image is found at the position of the needle. The distance of the needle from the lens is measured to be 45.0 cm. The liquid is removed, and the experiment is repeated. The new distance is measured to be 30.0 cm. What is the refractive index of the liquid?





SOLUTION:

Case 1: When there is air (no liquid) between the lens and plane mirror and the image is formed at 30 cm i.e. at the position of the object. For this the object must be placed at focus of Biconvex lens.

We know that focal length (f_0) = 30 cm.

The radius of curvature of convex lens is given by,

$$\frac{1}{f} = (\mu_g - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\frac{1}{30} = \left(\frac{3}{2} - 1 \right) \left(\frac{1}{R} - \frac{1}{-R} \right)$$

$$\frac{1}{30} = \frac{1}{2} \left(\frac{2}{R} \right)$$

$$R = 30 \text{ cm}$$

In this case the radius of curvature is 30 cm.

Case 2: When there is liquid is filled between the lens and plane mirror and the image is formed at 45 cm i.e. at the position of the object. For this the object must be placed at focus of equivalent lens of Biconvex of glass and Plano convex lens of liquid.

The equivalent focal length is given by,

$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2}$$

Where, Equivalent total length (f_{eq}) = 45 cm, Focal length of Biconvex lens (f_1) = 30 cm

Focal length of plano convex lens (f_2) = ?

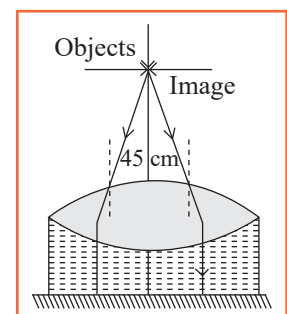
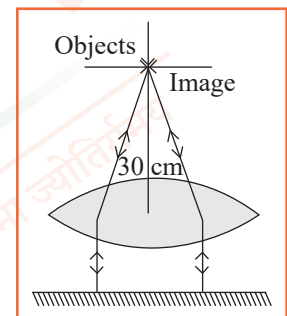
So,

$$\frac{1}{45} = \frac{1}{30} - \frac{1}{f_2}$$

$$\frac{1}{f_2} = \frac{1}{30} - \frac{1}{45} = \frac{1}{90}$$

$$f_2 = -90 \text{ cm (Negative)}$$

Now, the refractive index is given by,



$$\frac{1}{f_2} = [\mu - 1] \left[\frac{1}{-R} - \frac{1}{\infty} \right]$$

$$\frac{-1}{90} = [\mu - 1] \left[\frac{-1}{30} \right]$$

$$(\mu - 1) = \frac{1}{3}$$

$$\mu = 1 + \frac{1}{3} = \frac{4}{3}$$

Therefore, the refractive index of the liquid is 1.33.

