WAVE OPTICS

10.1 Monochromatic light of wavelength 589 nm is incident from air on a water surface. What are the wavelength, frequency and speed of (a) reflected, and (b) refracted light? Refractive index of water is 1.33.

SOLUTION:

Given – Wavelength (λ) = 589 nm = 589×10^{-9} m, Refractive index of water (μ) = 1.33, Speed of light in air/vacuum (c) = 3×10^8 m/s.

Need to find – Wavelength (λ), Frequency (ν) and speed (v) of reflected and refracted ray.

(a) In case of reflected ray medium is same. Hence, the wavelength, speed and frequency of the reflected ray is same as incident ray.

So, Wavelength of reflected ray = Wavelength of incident ray = λ = 589 nm = 5×8910^{-9} m.

Speed of reflected ray = Speed of incident ray = 3×10^8 m/s.

Frequency of reflected ray is given by,

$$v = \frac{c}{\lambda}$$

$$v = \frac{(3\times10^8)}{(589\times10^{-9})} = 5.09\times10^{14}\,Hz$$

Therefore, the speed, frequency, and wavelength of the reflected light are 3×10^8 m/s, 5.09×10^{14} Hz and 589 nm respectively.

(b) In case of refracted ray medium is different. Hence, only the wavelength and speed changes but frequency of the reflected ray is same as incident ray.

So, Frequency of refracted ray = Frequency of incident ray = 5.09×10^{14} Hz.

Speed of refracted ray in water is given by,

$$\mu = \frac{c}{v} \Rightarrow v = \frac{c}{\mu}$$

$$v = \frac{3\times10^8}{1.33} = 2.26\times10^8\,m/s$$

Wavelength of refracted ray in water is given by,

$$\lambda = \frac{v}{v}$$

$$\lambda = \frac{2.26\times10^{8}}{5.09\times10^{14}}$$

$$\lambda = 444.007\times10^{-9}\,\text{m} = 444.01nm$$

Therefore, the speed, frequency, and wavelength of refracted light are 2.26×10^8 m/s, 444.01 nm, and respectively.

10.2 What is the shape of the wavefront in each of the following cases:

(a) Light diverging from a point source.

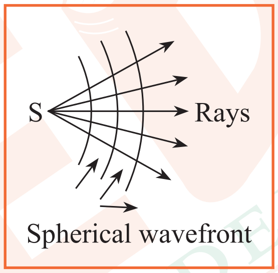
(b) Light emerging out of a convex lens when a point source is placed at its focus.

(c) The portion of the wavefront of light from a distant star intercepted by the Earth.

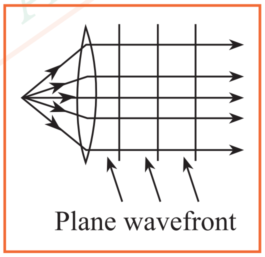
SOLUTION:

Need to find – Shape of wavefront.

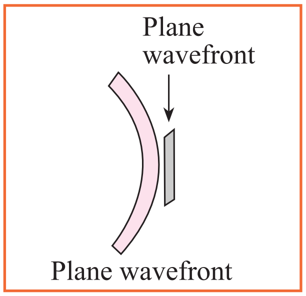
(a) The wavefront of light diverging from a point source is spherical in shape. This is illustrated in the accompanying figure, where the spherical wavefronts emanate outward from the point source.



(b) When light passes through a convex lens with a point source at its focus, the wavefront becomes flat and parallel, as shown in the figure.



(c) The part of the wavefront from a distant star that reaches the Earth is nearly flat or plane.



10.3 (a) The refractive index of glass is 1.5. What is the speed of light in glass? (Speed of light in vacuum is 3.0×10^8 m/s)

(b) Is the speed of light in glass independent of the colour of light? If not, which of the two colours red and violet travels slower in a glass prism?

SOLUTION:

(a) Refractive index of glass (μ) = 1.5, Speed of light (c) = 3.0×10^8 m/s.

Speed of light in glass is given by,

$$\mu = \frac{c}{v} \Rightarrow v = \frac{c}{\mu}$$

$$v = \frac{3\times10^8}{1.5} = 2\times10^8 \text{ m/s}$$

Therefore, the speed of light in glass is 3.0×10^8 m/s.

(b) The speed of light in glass is not independent of the colour of light. The refractive index for violet light is higher than that for red light, so violet light travels slower than red light in glass, including in a glass prism.

10.4 In a Young's double-slit experiment, the slits are separated by 0.28 mm and the screen is placed 1.4 m away. The distance between the central bright fringe and the fourth bright fringe is measured to be 1.2 cm. Determine the wavelength of light used in the experiment.

SOLUTION:

Given – Separation between the slits (d) = 0.28 mm = 0.28×10^{-3} m, Distance between the slit and scree (D) = 1.4 m, Distance between central fringe and fourth fringe (y) = 1.2 cm = 1.2×10^{-2} m, n = 4.

Need to find – Wavelength of light (λ).

For constructive interference, we get,

$$y = n\lambda\frac{D}{d}$$

$$\lambda = \frac{yd}{nD}$$

$$\lambda = \frac{(1.2\times10^{-2})\times(0.28\times10^{-3})}{(4\times1.4)}$$

$$\lambda = 6\times10^{-7} = 600nm$$

Therefore, the wavelength of light is 600 nm.

10.5 In Young's double-slit experiment using monochromatic light of wavelength λ, the intensity of light at a point on the screen where path difference is λ, is K units. What is the intensity of light at a point where path difference is λ/3?

SOLUTION:

Given – Wavelength of light = λ, Path difference (Δx) = λ, Path difference (Δx') = $\frac{\lambda}{3}$, Intensity of light (I) = K.

Let I_1 and I_2 be the intensity of the two light waves. Their resultant intensities are:

$$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

For monochromatic light $I_1 = I_2 = I$.

Phase difference (ϕ) = $\frac{2\pi}{\lambda} \times$ Path difference (Δx)

$$\phi = \frac{2\pi}{\lambda} \times \lambda = 2\pi$$

Now the resultant intensity is:

$$I_R = I + I + 2\sqrt{I.I}\cos 2\pi \qquad (\because \cos 2\pi = 1)$$

$$I_R = 4I \qquad (\because IR = K)$$

$$I = \frac{K}{4} \qquad \ldots\ldots\ldots\ (i)$$

Now, when the path difference (Δx) = $\frac{\lambda}{3}$ and

So, Phase difference (ϕ) = $\frac{2\pi}{\lambda} \times$ Path difference (Δx)

$$\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{3} = \frac{2\pi}{3}$$

Hence, the resultant intensity is given by,

$$I_R = I + I + 2\sqrt{I.I}\cos\frac{2\pi}{3} \qquad \left(\because \cos\frac{2\pi}{3} = -1\right)$$

$$I_R = I + I - 2I(\frac{1}{2}) = 2I - I = I$$

$$I_R = I$$

From equation (i), $I_R = \frac{K}{4}$

Therefore, the resultant intensity is $\frac{K}{4}$.

10.6 A beam of light consisting of two wavelengths, 650 nm and 520 nm, is used to obtain interference fringes in a Young's double-slit experiment.

(a) Find the distance of the third bright fringe on the screen from the central maximum for wavelength 650 nm.

(b) What is the least distance from the central maximum where the bright fringes due to both the wavelengths coincide?

SOLUTION:

Given – First wavelength (λ_1) = 650 nm, Second wavelength (λ_2) = 520 nm.

Let, Distance of the slits from the screen =D, Distance between the two slits = d.

(a) Distance of the n^{th} bright fringe on the screen from the central maximum is given by,

$$x = n\lambda_1\left(\frac{D}{d}\right)$$

For third bright fringe (n = 3),

$$\therefore x = 3\times 650\frac{D}{d} = 1950\left(\frac{D}{d}\right)nm$$

(b) Let the n^{th} bright fringe due to wavelength λ_2 and $(n - 1)^{th}$ bright fringe due to wavelength λ_1 coincide on the screen. We can equate the conditions for bright fringes as:

$$n\lambda_2 = (n-1)\lambda_1$$

$$520n = 650n - 650$$

$$130n = 650$$

$$n = 5$$

Hence, the least distance from the central maximum can be obtained by the relation:

$$x = n\lambda_2\frac{D}{d}$$

$$x = 5\times 520\frac{\text{D}}{\text{d}}$$

$$x = 2600\frac{D}{d}nm$$

Therefore, the least distance from the central maximum where the bright fringes coincide is $2600\frac{D}{d}nm$.