ATOM

12.1 Choose the correct alternative from the clues given at the end of the each statement:

(a) The size of the atom in Thomson's model is .... the atomic size in Rutherford's model. (much greater than/no different from/much less than.)

(b) In the ground state of .... electrons are in stable equilibrium, while in .... electrons always experience a net force. (Thomson's model/ Rutherford's model.)

(c) A classical atom based on .... is doomed to collapse. (Thomson's model/ Rutherford's model.)

(d) An atom has a nearly continuous mass distribution in a ....but has a highly non-uniform mass distribution in ....(Thomson's model/ Rutherford's model.)

(e) The positively charged part of the atom possesses most of the mass in .... (Rutherford's model/both the models.)

SOLUTION:

(a) The size of the atom in Thomson's model is no different from the atomic size in Rutherford's model.

(b) In the ground state of Thomson's model electrons are in stable equilibrium, while in Rutherford's model electrons always experience a net force.

(c) A classical atom based on Rutherford's model is doomed to collapse.

(d) An atom has a nearly continuous mass distribution in a Thomson's model but has a highly non-uniform mass distribution in Rutherford's model.

(e) The positively charged part of the atom possesses most of the mass in both the models.

12.2 Suppose you are given a chance to repeat the alpha-particle scattering experiment using a thin sheet of solid hydrogen in place of the gold foil. (Hydrogen is a solid at temperatures below 14 K.) What results do you expect?

SOLUTION:

In the alpha-particle scattering experiment, replacing the gold foil with a thin sheet of solid hydrogen would result in smaller scattering angles. This happens because hydrogen has a lower mass (6.64×10^{-27} kg) than the incident alpha particles. Since the mass of the alpha particles (6.64×10^{-27} kg) is greater than that of the hydrogen nuclei, they would not rebound. Therefore, alpha particles would not be deflected backward if solid hydrogen were used in the experiment.

12.3 A difference of 2.3 eV separates two energy levels in an atom. What is the frequency of radiation emitted when the atom make a transition from the upper level to the lower level?

SOLUTION:

Separation of two energy levels in an atom is:

$$E = 2.3 \text{ eV}$$
$$E = 2.3 \times 1.6 \times 10^{-19}$$
$$E = 3.68 \times 10^{-19} \text{ J}$$

To find the frequency (v) of radiation we use the formula,

$$E = hv \qquad (\because h = \text{Planck's constant} = 6.626 \times 10^{-34}\ Js)$$
$$v = \frac{E}{h}$$
$$v = \frac{3.68 \times 10^{-19}}{6.62 \times 10^{-34}}$$
$$v = 5.55 \times 10^{14}\ Hz$$

Therefore, the frequency of the radiation is 5.55×10^{14} Hz.

12.4 The ground state energy of hydrogen atom is –13.6 eV. What are the kinetic and potential energies of the electron in this state?

SOLUTION:

Ground state total energy of hydrogen atom (E) = −13.6 eV.

Kinetic energy (K.E.) = −E = −(−13.6) = 13.6 eV. And

Potential energy (P.E.) = −2E = −2(−13.6) = −27.2 eV.

12.5 A hydrogen atom initially in the ground level absorbs a photon, which excites it to the n = 4 level. Determine the wavelength and frequency of photon.

SOLUTION:

For ground state (n = 1), the energy is given by,

$$E_n = \frac{-13.6}{n_1^2} eV$$

Put n = 1 we get,

$$E_1 = \frac{-13.6}{1^2} eV$$
$$E_1 = -13.6 eV$$

For excited state (n = 4), the energy is given by,

$$E_n = \frac{-13.6}{n_1^2} eV$$

Put n = 1 we get,

$$E_4 = \frac{-13.6}{4^2} = \frac{-13.6}{16}$$

$$E_4 = -\frac{13.6}{16} eV$$

The energy absorbed by photon is given by,

$$E = E_4 - E_1$$

$$E = \frac{-13.6}{16} - (-13.6)$$

$$E = \frac{13.6 \times 15}{16} eV = \frac{(13.6 \times 15) \times (1.6 \times 10^{-19})}{16} J$$

$$E = 2.04 \times 10^{-18} \text{J}$$

For a photon the frequency is given by,

$$E = hv \qquad (\because h = \text{Planck's constant} = 6.626 \times 10^{-34} Js)$$

$$v = \frac{E}{h}$$

$$v = \frac{2.04 \times 10^{-18}}{6.62 \times 10^{-34}}$$

$$v = 3.1 \times 10^{15} Hz$$

And the wavelength is given by,

$$\lambda = \frac{c}{\upsilon}$$

$$\lambda = \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{2.04 \times 10^{-18}}$$

$$\lambda = 9.7 \times 10^{-8} \text{m} = 97 nm$$

Therefore, the frequency and wavelength are 3.1×10^{15} *Hz* and 97 nm respectively.

12.6 (a) Using the Bohr's model calculate the speed of the electron in a hydrogen atom in the *n* = 1, 2, and 3 levels. (b) Calculate the orbital period in each of these levels.

SOLUTION:

We know that the speed (v) of electron in different energy levels is:

$$v_n = \frac{e^2}{n 4\pi \epsilon_0 (h/2\pi)} = \frac{e^2}{2n \epsilon_0 h}$$

(a) For ground state i.e. n = 1, the speed is given by,

$$v_1 = \frac{e^2}{2 \epsilon_0 h}$$

Where, e = 1.610-19 C, h = 6.6210-34 Js, and $\in_0 = 8.85\times10^{-12}\ N^{-1}C^2m^{-2}$.

Now,

$$v_1 = \frac{\left(1.6\times10^{-19}\right)^2}{2\times8.85\times10^{-12}\times6.62\times10^{-34}}$$

$$v_1 = 0.0218\times10^8 = 2.18\times10^6\ m/s$$

For energy state i.e. n = 2, the speed is given by,

$$v_2 = \frac{e^2}{4\in_0 h}$$

Where, e = 1.610-19 C, h = 6.6210-34 Js, and $\in_0$ = 8.85 × 10^{-12} N^{-1} C^2 m^{-2}.

Now,

$$v_2 = \frac{\left(1.6\times10^{-19}\right)^2}{4\times8.85\times10^{-12}\times6.62\times10^{-34}}$$

$$v_2 = 0.0109\times10^8 = 1.09\times10^6\ m/s$$

For energy state i.e. n = 3, the speed is given by,

$$v_3 = \frac{e^2}{4\in_0 h}$$

Where, e = 1.610-19 C, h = 6.6210-34 Js, and $\in_0$ = 8.85 × 10^{-12} N^{-1} C^2 m^{-2}.

Now,

$$v_3 = \frac{\left(1.6\times10^{-19}\right)^2}{6\times8.85\times10^{-12}\times6.62\times10^{-34}}$$

$$v_3 = 0.0727\times10^7 = 7.27\times10^5\ m/s$$

Therefore, the speed of the electron in a hydrogen atom in n = 1, n = 2 and n = 3 is 2.18 × 10^6 m/s, 1.09 × 10^6 m/s, 7.21 ×10^5 m/s respectively.

(b) Orbital period for energy states is given by,

$$T_n = \frac{2\pi r_n}{v_n} \qquad \left(\because r_n = \frac{n^2h^2\in_0}{\pi me^2}\right)$$

Now,

$$T_n = \frac{2\pi n^2h^2\in_0}{\pi m v_n e^2} = \frac{2n^2h^2\in_0}{m v_n e^2}$$

For ground state n = 1, the orbital period is,

$$T_1 = \frac{2(1)^2h^2\in_0}{m v_1 e^2}$$

Where, e = 1.610-19 C, h = 6.6210-34 Js, and $\in_0$ = 8.85 × 10^{-12} N^{-1} C^2 m^{-2}, m = 9.1 ×10^{-31} kg and v_1 = 2.18 ×10^6 m/s.

Put these values in above equation we get,

$$T_1 = \frac{2\times(1)^2\times\left(6.62\times10^{-34}\right)^2\times8.85\times10^{-12}}{2.18\times10^6\times9.1\times10^{-31}\times\left(1.6\times10^{-19}\right)^2}$$

$$T_1 = 15.27\times10^{-17} = 1.527\times10^{-16}\ s$$

For energy state n = 2, the orbital period is,

$$T_2 = \frac{2(2)^2 h^2 \epsilon_0}{m v_2 e^2}$$

Where, e = 1.610-19 C, h = 6.6210-34 Js, and ϵ_0 = 8.85 × 10^{-12} N^{-1} C^2 m^{-2}, $m = 9.1 \times 10^{-31}$ *kg*, and $v_2 = 1.09 \times 10^6$ m / s.

Put these values in above equation we get,

$$T_2 = \frac{2 \times (2)^2 \times \left(6.62 \times 10^{-34}\right)^2 \times 8.85 \times 10^{-12}}{2.18 \times 10^6 \times 9.1 \times 10^{-31} \times \left(1.6 \times 10^{-19}\right)^2}$$

$$T_2 = 1.22 \times 10^{-15} \text{s}$$

For energy state n = 2, the orbital period is,

$$T_3 = \frac{2(3)^2 h^2 \epsilon_0}{m v_3 e^2}$$

Where, e = 1.610-19 C, h = 6.6210-34 Js, and ϵ_0 = 8.85 × 10^{-12} N^{-1} C^2 m^{-2}, $m = 9.1 \times 10^{-31}$ *kg*, and $v_3 = 9.27 \times 10^5$ m/s.

Put these values in above equation we get,

$$T_3 = \frac{2 \times (3)^2 \times \left(6.62 \times 10^{-34}\right)^2 \times 8.85 \times 10^{-12}}{2.18 \times 10^6 \times 9.1 \times 10^{-31} \times \left(1.6 \times 10^{-19}\right)^2}$$

$$T_3 = 4.12 \times 10^{-15} \text{s}$$

Therefore, the orbital period in each of these levels is 1.52×10^{-16} s, 1.22×10^{-15} s, and 4.12×10^{-15} s respectively.

12.7 The radius of the innermost electron orbit of a hydrogen atom is 1.525.3 × 10^{-11} m. What are the radii of the *n* = 2 and *n* = 3 orbits?

SOLUTION:

Given – Radius (r_1) = 5.3×10^{-11} m for n =1.

Need to find – Radius for n = 2 and n = 3 orbits.

We know that the radius of nth orbit is related to the innermost orbit is given by,

$$r_n = n^2 r_1$$

Radius of the n = 2 orbit is given by,

$$r_2 = (n)^2 r_1$$

$$r_2 = (2)^2 \times 5.3 \times 10^{-11} = 4 \times 5.3 \times 10^{-11}$$

$$r_2 = 2.12 \times 10^{-10} m$$

Radius of the n = 3 orbit is given by,

$$r_3 = (n)^2 r_1$$

$$r_3 = (3)^2 \times 5.3 \times 10^{-11} = 9 \times 5.3 \times 10^{-11}$$

$$r_3 = 4.77 \times 10^{-10} m$$

Therefore, the radii of an electron for *n* = 2 and *n* = 3 orbits are 2.12×10^{-10} m and 4.77×10^{-10} m respectively.

12.8 A 12.5 eV electron beam is used to bombard gaseous hydrogen at room temperature. What series of wavelengths will be emitted?

SOLUTION:

Given – Energy = 12.5 eV.

Need to find – Series of wavelengths emitted.

We know that the energy of the gaseous hydrogen in its ground state at room temperature = −13.6 eV.

When gaseous hydrogen is bombarded with an electron beam, the energy of the gaseous hydrogen is,

$$E = (13.6 \text{ eV}) + (12.5 \text{ eV}) = -1.1 \text{ eV}$$

Formula for orbital energy is given by,

$$E = \frac{-13.6}{(n)^2} eV$$

For n = 3, $$E = \frac{-13.6}{9} = -1.5eV$$

The energy is roughly equivalent to that of gaseous hydrogen, suggesting that the electron has transitioned from the n = 1 level to the n = 3 level.

During de-excitation, the electron may transition directly from n = 3 to n = 1, producing a spectral line in the Lyman series of the hydrogen spectrum.

Wave number for Lyman series is given by,

$$\frac{1}{\lambda} = R_y\left(\frac{1}{1^2} - \frac{1}{n^2}\right) \qquad (\because R_y = \text{Rydberg constant} = 1.097 \times 10^7 \text{ m}^{-1})$$

For n = 3, the wavelength is:

$$\frac{1}{\lambda} = 1.097\times10^7\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = 1.097\times10^7\left(1 - \frac{1}{9}\right)$$

$$\frac{1}{\lambda} = 1.097\times10^7 \times \frac{8}{9}$$

$$\lambda = \frac{9}{8\times1.097\times10^7} = 102.55nm$$

If the electron jumps from n = 3 to n = 2, then the wavelength of the radiation is:

$$\frac{1}{\lambda} = 1.097\times10^7\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 1.097\times10^7\left(\frac{1}{4} - \frac{1}{9}\right)$$

$$\frac{1}{\lambda} = 1.097\times10^7 \times \frac{5}{36}$$

$$\lambda = \frac{36}{5\times1.097\times10^7} = 656.33nm$$

This radiation corresponds to the Balmer series of the hydrogen spectrum.

If the electron jumps from n = 2 to n = 1, then the wavelength of the radiation is:

$$\frac{1}{\lambda} = 1.097\times10^7\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = 1.097\times10^7\left(1 - \frac{1}{4}\right)$$

$$\frac{1}{\lambda} = 1.097 \times 10^7 \times \frac{3}{4}$$

$$\lambda = \frac{4}{3 \times 1.097 \times 10^7} = 121.54nm$$

This radiation corresponds to the Lyman series of the hydrogen spectrum.

Therefore, in Lyman series, two wavelengths i.e., 102.5 nm and 121.5 nm are emitted and in the Balmer series, one wavelength i.e., 656.33 nm is emitted.

12.9 In accordance with the Bohr's model, find the quantum number that characterises the earth's revolution around the sun in an orbit of radius 1.5 × 10^{11} m with orbital speed 3 × 10^4 m/s. (Mass of earth = 6.0 × 10^{24} kg.)

SOLUTION:

Given – Orbital radius (r) = 1.5 × 10^{11} m, Orbital speed (v) = 3 × 10^4 m/s,

Mass of earth (m) = 6.0 × 10^{24} kg.

Need to find – Quantum number (n).

From Bohr's model, angular momentum is quantized and is given by,

$$mvr = \frac{nh}{2\pi}$$

$$n = \frac{mvr2\pi}{h}$$

$$n = \frac{2\pi \times 6 \times 10^{24} \times 3 \times 10^4 \times 1.5 \times 10^{11}}{6.62 \times 10^{-34}}$$

$$n = 25.61 \times 10^{73} = 2.6 \times 10^{74}$$

Therefore, the quanta number that characterizes the Earth' revolution is 2.6 × 10^{74}.