CHAPTER 13

# NUCLEI

CLASS 12TH

## NCERT EXERCISE AND SOLUTIONS - PHYSICS



**13.1 Obtain the binding energy (in MeV) of a nitrogen nucleus ($_7N^{14}$), given m (7N14) = 14.00307 u**

**SOLUTION:**

Given – Atomic mass of nitrogen M ($_7N^{14}$) = 14.00307 u.

Need to find – Binding energy (in MeV).

We know that the nitrogen ($_7N^{14}$) nucleus contains 7 protons and 7 neutrons.

Mass defect of nucleus is given by,

$$\Delta m = (7m_p + 7m_n) - M$$

Where, Mass of proton ($m_p$) = 1.007825 u, Mass of neutron ($m_n$) = 1.008665 u, mass of nucleus (M) = 14.00307 u.

Now,

$$\Delta m = 7 \times 1.007825 + 7 \times 1.008665 - 14.00307$$

$$\Delta m = 7.054775 + 7.06055 - 14.00307$$

$$\Delta m = 0.11236u$$

The binding energy in MeV is given by,

$$E_b = (\Delta mc^2) \times (931.5 MeV / c^2) \qquad (\because c = 3 \times 10^8 \text{ m/s})$$

$$E_b = (0.11236 \times c^2) \times 931.5 \left(\frac{MeV}{c^2}\right)$$

$$E_b = 104.66334 MeV$$

Therefore, the binding energy of nitrogen ($_7N^{14}$) nucleus is 104.66334 MeV.

**13.2 Obtain the binding energy of the nuclei $_{26}Fe^{56}$ and $_{83}Bi^{209}$ in units of MeV from the following data: m ($_{26}Fe^{56}$) = 55.934939 u, *m* ($_{83}Bi^{209}$) = 208.980388 u.**

**SOLUTION:**

Given – Atomic mass of Iron M ($_{26}Fe^{56}$) = 55.934939 u, atomic mass of Bismuth M’ ($_{83}Bi^{209}$) = 208.980388 u.

Need to find – Binding energy in MeV.

We know that the iron ($_{26}Fe^{56}$) nucleus contains 26 protons and 30 neutrons.

Mass defect of nucleus is given by,

$$\Delta m = (26m_p + 30m_n) - M$$

Where, Mass of proton ($m_p$) = 1.007825 u, Mass of neutron ($m_n$) = 1.008665 u, mass of nucleus (M) = 55.934939 u.

Now,
$$\Delta m = 26 \times 1.007825 + 30 \times 1.008665 - 55.934939$$
$$\Delta m = 26.20345 + 30.25995 - 55.934939$$
$$\Delta m = 0.528461u$$

The binding energy in MeV is given by,

$$E_b = (\Delta mc^2) \times (931.5 MeV / c^2) \qquad (\because c = 3 \times 10^8 \text{ m/s})$$
$$E_b = (0.528461 \times c^2) \times 931.5 \left(\frac{MeV}{c^2}\right)$$
$$E_b = 492.26 MeV$$

Average binding energy per nucleon is given by,

$$\frac{E_B}{A} = \frac{492.26}{56} = 8.79 MeV$$

Similarly,

We know that the bismuth ($_{83}Bi^{209}$) nucleus contains 83 protons and 126 neutrons.

Mass defect of nucleus is given by,

$$\Delta m' = (83m_p + 126m_n) - M'$$

Where, Mass of proton ($m_p$) = 1.007825 u, Mass of neutron ($m_n$) = 1.008665 u, mass of nucleus (M) = 208.980388 u.

Now,
$$\Delta m' = 83 \times 1.007825 + 126 \times 1.008665 - 208.980388$$
$$\Delta m' = 83.649475 + 127.091790 - 208.980388$$
$$\Delta m' = 1.760877u$$

The binding energy in MeV is given by,

$$E_b = (\Delta mc^2) \times (931.5 MeV / c^2) \qquad (\because c = 3 \times 10^8 \text{ m/s})$$
$$E_b = (1.760877 \times c^2) \times 931.5 \left(\frac{MeV}{c^2}\right)$$
$$E_b = 1640.26 MeV$$

Average binding energy per nucleon is given by,

$$\frac{E_B}{A} = \frac{1640.26}{209} = 7.848 MeV$$

Therefore, the binding energy of the nuclei $_{26}Fe^{56}$ and $_{83}Bi^{209}$ in units of MeV is 492.26 MeV and 1640.26 MeV, respectively.

**13.3 A given coin has a mass of 3.0 g. Calculate the nuclear energy that would be required to separate all the neutrons and protons from each other. For simplicity assume that the coin is entirely made of $_{29}Cu^{63}$ atoms (of mass 62.92960 u).**

**SOLUTION:**

Given – Mass of the coin (m) = 3 g, atomic mass of Copper M ($_{29}Cu^{63}$) = 62.92960 u.

Need to find – Nuclear energy to separate neutrons and protons.

Number of atoms in 3 g of Cu is given by,

$$n = \frac{m}{M} = \frac{N}{N_A} \Rightarrow N = N_A \times \frac{m}{M}$$

$$N = \frac{\left(6.023\times10^{23}\times3\right)}{63} = 2.86\times10^{22}$$

We know that the copper ($_{29}Cu^{63}$) nucleus contains 29 protons and 34 neutrons.

Mass defect of nucleus is given by,

$$\Delta m = (29m_p + 34m_n) - M$$

Where, Mass of proton (mp) = 1.007825 u, Mass of neutron ($m_n$) = 1.008665 u, mass of nucleus (M) = 62.92960 u.

Now,

$$\Delta m = 29\times1.00783 + 34\times1.00867 - 62.92960$$

$$\Delta m = 29.22707 + 34.29478 - 62.92960$$

$$\Delta m = 0.59225 \text{ u}$$

The binding energy in MeV is given by,

$$E_b = (\Delta mc^2)\times(931.5MeV/c^2) \qquad (\because c = 3\times10^8 \text{ m/s})$$

$$E_b = (0.59225\times c^2)\times931.5\left(\frac{MeV}{c^2}\right)$$

$$E_b = 551.68MeV$$

Nuclear energy for 3 g of Cu is given by,

$$E_{b(net)} = N \times E_b$$

$$E_b\ net) = N \times E_b$$

$$E_{b(net)} = (2.86\times10^{22})\times(551.68)$$

$$E_{b(net)} = 1.58\times10^{25}MeV$$

Therefore, the binding energy required to separate neutrons and protons is $1.58\times10^{25}MeV$.

**13.4 Obtain approximately the ratio of the nuclear radii of the gold isotope $_{79}Au_{197}$ and the silver isotope $_{47}Ag^{107}$.**

**SOLUTION:**

Let Nuclear radius of the gold isotope $_{79}Au^{197} = R_{Au}$ and Mass number $A_{Au}$ = 197,

Nuclear radius of the silver isotope $_{47}Ag^{107} = R_{Ag}$ and Mass number $A_{Ag} = 107$.

The ratio of radii of the two nuclei is given by,

$$\frac{R_{Au}}{R_{Ag}} = \left(\frac{A_{Au}}{A_{Ag}}\right)^{\frac{1}{3}}$$

$$\frac{R_{Au}}{R_{Ag}} = \left(\frac{197}{107}\right)^{\frac{1}{3}} = 1.2256$$

Therefore, the ratio of the nuclear radii of the gold and silver isotopes is about 1.23.

**13.5 The Q value of a nuclear reaction $A + b \rightarrow C + d$ is defined by $Q = [m_A + m_b - m_C - m_d]c^2$ where the masses refer to the respective nuclei. Determine from the given data the Q-value of the following reactions and state whether the reactions are exothermic or endothermic.**

**(i)** ${}_1^1\text{H} + {}_1^3\text{H} \rightarrow {}_1^2\text{H} + {}_1^2\text{H}$

**(ii)** ${}_6^{12}\text{C} + {}_6^{12}\text{C} \rightarrow {}_{10}^{20}\text{Ne} + {}_2^4\text{He}$

**Atomic masses are given to be**

**m ($_1H^2$) = 2.014102 u**

**m ($_1H^3$) = 3.016049 u**

**m ($_6C^{12}$) = 12.000000 u**

**m ($_{10}Ne^{20}$) = 19.992439 u**

## SOLUTION:

(i) Given reaction – ${}_1^1\text{H} + {}_1^3\text{H} \rightarrow {}_1^2\text{H} + {}_1^2\text{H}$

Atomic masses – m ($_1H^1$) = 1.007825 u, m ($_1H^2$) = 2.014102 u, and m ($_1H^3$) = 3.016049 u.

From question, The Q-value is given by,

$$Q = \left[m\left({}_1^1\text{H}\right) + m\left({}_1^3\text{H}\right) - 2m\left({}_1^2\text{H}\right)\right]c^2$$

$$Q = \left[(1.007825\ u) + (3.016049\ u) - 2(2.014102\ u)\right]c^2$$

$$Q = \left(-0.00433c^2\right)u$$

We know that 1 u = 931.5 MeV/$c^2$

So, $$Q = (-0.00433)c^2 \times 931.5 MeV / c^2 = -4.0334 MeV$$

Therefore, the Q-value is – 4.0334 MeV and negative sign shows the reaction is endothermic.

(ii) Given reaction – ${}_6^{12}\text{C} + {}_6^{12}\text{C} \rightarrow {}_{10}^{20}\text{Ne} + {}_2^4\text{He}$

Atomic masses – $m$ ($_6C^{12}$) = 12.000000 u, $m$ ($_{10}Ne^{20}$) = 19.992439 u, and $m$ ($_2He^4$) = 4.002603 u

From question, The Q-value is given by,

$$Q = \left[2m\left({}_6^{12}\text{C}\right) - m\left({}_{10}^{20}Ne\right) - m\left({}_2^4He\right)\right]c^2$$

$$Q = [2 \times 12.0 - 19.992439 - 4.002603]c^2$$

$$Q = \left(0.004958c^2\right)u$$

We know that 1 u = 931.5 MeV/c$^2$

So, $$Q = \left(0.004958c^2\right) \times 931.5 MeV / c^2 = 4.618377 MeV$$

Therefore, the Q-value is 4.618377 MeV, and it shows the reaction is exothermic.

**13.6 Suppose, we think of fission of a $_{26}Fe^{56}$ nucleus into two equal fragments, $_{13}Al^{28}$. Is the fission energetically possible? Argue by working out Q of the process. Given m ($_{26}Fe^{56}$) = 55.93494 u and *m* ($_{13}Al^{28}$) = 27.98191 u.**

**SOLUTION:**

Fission of $_{26}Fe^{56}$ is given as:

$$_{13}Fe^{56} \rightarrow 2\,_{13}Al^{28}$$

Atomic masses – *m* ($_{26}Fe^{56}$) = 55.93494 u and *m* ($_{13}Al^{28}$) = 27.98191 u.

The Q-value of the nuclear reaction is given by,

$$Q = \left[ m\left(^{56}_{26}Fe\right) - 2m\left(^{28}_{13}Al\right)\right]c^2$$

$$Q = [55.93494 - 2 \times 27.98191]c^2$$

$$Q = \left(-0.02888c^2\right)u$$

We know that 1u = 931.5 MeV/c$^2$

So, $$Q = (-0.02888c^2) \times 931.5 MeV / c^2 = -26.902 MeV$$

Therefore, the Q-value of the fission is – 26.902 MeV and negative in nature i.e. fission is not possible energetically because for energetically possible fission reaction Q-value must be positive.

**13.7 The fission properties of $_{94}Pu^{239}$ are very similar to those of $_{92}U^{235}$. The average energy released per fission is 180 MeV. How much energy, in MeV, is released if all the atoms in 1 kg of pure $_{94}Pu^{239}$ undergo fission?**

**SOLUTION:**

Given – Average energy released per fusion (Eav) = 180 MeV, Mass of pure $_{94}Pu^{239}$ for fission = 1 kg.

Number of atoms in 1 kg = 1000g of Pu is given by,

$$n = \frac{m}{M} = \frac{N}{N_A} \Rightarrow N = N_A \times \frac{m}{M}$$

$$N = \frac{6.023 \times 10^{23}}{239} \times 1000 = 2.52 \times 10^{24} \text{ atoms}$$

Total energy released during the fission of 1 kg of $_{94}Pu^{239}$ is given by,

$$E_{net} = E_{av} \times 2.52 \times 10^{24}$$

$$E_{net} = 180 \times 2.52 \times 10^{24}$$
$$E_{net} = 4.536 \times 10^{26}\,\text{MeV}$$

Therefore, the energy released when 1 kg of ${}_{94}Pu^{239}$ undergo fission is $4.536 \times 10^{26}$ MeV.

**13.8 How long can an electric lamp of 100W be kept glowing by fusion of 2.0 kg of deuterium? Take the fusion reaction as ${}_1^2H + {}_1^2H \rightarrow {}_2^3He + n + 3.27 MeV$.**

**SOLUTION:**

Given – Power (P) = 100 W, Amount of deuterium (m) = 2 kg.

Fusion reaction – ${}_1^2H + {}_1^2H \rightarrow {}_2^3He + n + 3.27MeV$

Number of atoms in 2 kg = 2000g of deuterium is given by,

$$n = \frac{m}{M} = \frac{N}{N_A} \Rightarrow N = N_A \times \frac{m}{M}$$

$$N = \frac{6.023 \times 10^{23}}{2} \times 2000 = 6.023 \times 10^{26} \text{ atoms}$$

From above reaction – Energy releases is 3.27 MeV when two atoms of deuterium fuse.

Total energy per nucleus released in fusion is given by,

$$E = \frac{3.27}{2} \times 6.023 \times 10^{26} = 9.847 \times 10^{26}\,\text{MeV}$$

In joule it is:

$$E = \frac{3.27}{2} \times 6.023 \times 10^{26} \times 1.6 \times 10^{-19} \times 10^{6}$$

$$E = 1.576 \times 10^{14}\,\text{J}$$

Energy consumed by lamp per second is:

$$E = P \times t = P = 100J$$

The total time for which the lamp will glow Is:

$$t = \frac{E}{P} = \frac{1.576 \times 10^{14}}{100}\,\text{s}$$

$$t = \frac{1.576 \times 10^{14}}{100 \times 60 \times 60 \times 24 \times 365} \approx 4.9 \times 10^{4} \text{ year}$$

Therefore, the time for which lamp will glow is about $4.9 \times 10^4$ years.

**13.9 Calculate the height of the potential barrier for a head on collision of two deuterons. (Hint: The height of the potential barrier is given by the Coulomb repulsion between the two deuterons when they just touch each other. Assume that they can be taken as hard spheres of radius 2.0 fm.)**

**SOLUTION:**

Given – Radius of spheres (R) = 2 fm = $2 \times 10^{15}$ m.

When two deuterons collide head-on, the distance between their centre (d) is given by, Distance (d) = Radius of first deuteron + Radius of second deuteron

Distance (d) = 2R = 2(2 fm) = 4 × $10^{15}$ m.

Charge on deuteron nucleus = Charge on electron = $1.610^{-19}$ C.

Potential energy of two deuteron system is given by,

$$V = \frac{e^2}{4\pi \epsilon_0 d} \qquad \left(\because \frac{1}{4\pi \epsilon_0} = 9\times10^9\, Nm^2C^{-2}\right)$$

$$V = \frac{9\times10^9 \times \left(1.6\times10^{-19}\right)^2}{4\times10^{-15}} J$$

$$V = \frac{9\times10^9 \times \left(1.6\times10^{-19}\right)^2}{4\times10^{-15} \times \left(1.6\times10^{-19}\right) eV}$$

$$V = 360 keV$$

Therefore, the height of potential barrier is 360 KeV.

**13.10 From the relation $R = R_0A^{1/3}$, where $R_0$ is a constant and A is the mass number of a nucleus, show that the nuclear matter density is nearly constant (i.e. independent of A).**

**SOLUTION:**

Given – $R = R_0A^{1/3}$

Need to find – Show that nuclear matter density is nearly constant.

We know that the nuclear matter density is:

$$\rho = \frac{\text{Mass of the nucleus}}{\text{Volume of the nucleus}}$$

$$\rho = \frac{mA}{\frac{4}{3}\pi R^3}$$

$$\rho = \frac{3mA}{4\pi\left(R_0A^{\frac{1}{3}}\right)^3}$$

$$\rho = \frac{3mA}{4\pi R_0^3 A}$$

$$\rho = \frac{3m}{4\pi R_0^3}$$

Therefore, the nuclear matter density is independent of atomic mass A.