CHAPTER 3

MOTION IN A PLANE

CLASS 11TH

NCERT EXERCISE AND SOLUTIONS - PHYSICS

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3.1 State, for each of the following physical quantities, if it is a scalar or a vector: volume, mass, speed, acceleration, density, number of moles, velocity, angular frequency, displacement, angular velocity.

SOLUTION:

Scalars: Volume, mass, speed, density, number of moles, angular frequency.

Vectors: Acceleration, velocity, displacement, angular velocity.

3.2 Pick out the two scalar quantities in the following list: force, angular momentum, work, current, linear momentum, electric field, average velocity, magnetic moment, relative velocity.

SOLUTION:

Work and current are the scalar quantities.

3.3 Pick out the only vector quantity in the following list: Temperature, pressure, impulse, time, power, total path length, energy, gravitational potential, coefficient of friction, charge.

SOLUTION:

Impulse.

3.4 State with reasons, whether the following algebraic operations with scalar and vector physical quantities are meaningful :

(a) adding any two scalars,
(b) adding a scalar to a vector of the same dimensions,
(c) multiplying any vector by any scalar,
(d) multiplying any two scalars,
(e) adding any two vectors,
(f) adding a component of a vector to the same vector.

SOLUTION:

(a) No, because only the scalars of same dimensions can be added.

(b) No, because a scalar cannot be added to a vector.

(c) Yes, multiplying a vector with a scalar gives the scalar (number) times the vector quantity which makes sense, and one gets a bigger vector.

(d) Yes, two scalars multiplied yield a meaningful result, for example multiplication of rise in temperature of water and its mass gives the amount of heat absorbed by that mass of water.

(e) No, because the two vectors of same dimensions can be added.

(f) Yes, because both are vectors of the same dimensions.

3.5 Read each statement below carefully and state with reasons, if it is true or false:

(a) The magnitude of a vector is always a scalar,

(b) each component of a vector is always a scalar,

(c) the total path length is always equal to the magnitude of the displacement vector of a particle.

(d) the average speed of a particle (defined as total path length divided by the time taken to cover the path) is either greater or equal to the magnitude of average velocity of the particle over the same interval of time,

(e) Three vectors not lying in a plane can never add up to give a null vector.

SOLUTION:

(a) True, magnitude of the velocity of a body moving in a straight line may be equal to the speed of the body.

(b) False, each component of a vector is always a vector, not scalar.

(c) False, total path length can also be more than the magnitude of displacement vector of a particle.

(d) True, because the total path length is either greater than or equal to the magnitude of the displacement vector.

(e) True, this is because the resultant of two vectors will not lie in the plane of third vector and hence cannot cancel its effect to give null vector.

3.6 Establish the following vector inequalities geometrically or otherwise :

(a) $|\mathbf{a} + \mathbf{b}| \leq |\mathbf{a}| + |\mathbf{b}|$

(b) $|\mathbf{a} + \mathbf{b}| \geq ||\mathbf{a}| - |\mathbf{b}||$

(c) $|\mathbf{a} - \mathbf{b}| \leq |\mathbf{a}| + |\mathbf{b}|$

(d) $|\mathbf{a} - \mathbf{b}| \geq ||\mathbf{a}| - |\mathbf{b}||$

When does the equality sign above apply?

SOLUTION:

Consider two vectors $\vec{A}$ and $\vec{B}$ be represented by the sides $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ of a parallelogram OPSQ. According to parallelogram law of vector addition; ($\vec{A} + \vec{B}$) will be represented by $\overrightarrow{OS}$ as shown in Fig. Thus,

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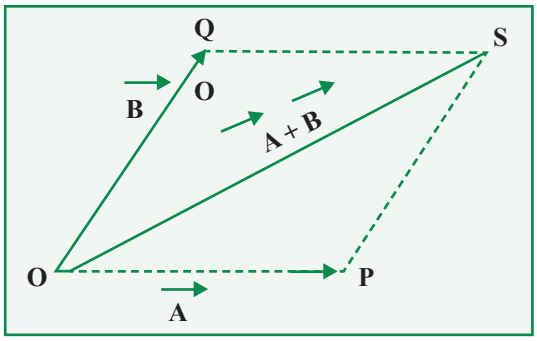

and

$$OP = |\vec{A}|, OQ = PS = |\vec{B}| \; OS = |\vec{A} + \vec{B}|$$

(a) To prove $|\vec{A} + \vec{B}| \le |\vec{A}| + |\vec{B}|$

We know that the length of one side of a triangle is always less than the sum of the lengths of the other two sides. Hence from OPS, we have

$OS < OP + PS$ or $OS < OP + OQ$ or $|\vec{A} + \vec{B}| < |\vec{A}| + |\vec{B}|$

If the two vectors $\vec{A}$ and $\vec{B}$ are acting along the same straight line and in the same direction then $|\vec{A} + \vec{B}| = |\vec{A}| + |\vec{B}|$

Combining the conditions mentioned in (i) and (ii) we have

$|\vec{A} + \vec{B}| \le |\vec{A}| + |\vec{B}|$

(b) To prove $|\vec{A} + \vec{B}| \ge ||\vec{A}| - |\vec{B}||$

From OPS, we have $OS + PS > OP$ or $OS > |OP - PS|$ or $OS > |OP - OQ|$

... (iii) ($\because$ PS = OQ)

The modulus of (OP – PS) has been taken because the L.H.S. is always positive but the R.H.S. may be negative if OP < PS. Thus from (iii) we have.

$$|\vec{A} + \vec{B}| > ||\vec{A}| - |\vec{B}||$$

If the two vectors $\vec{A}$ and $\vec{B}$ are acting along a straight line in opposite directions, then

$$|\vec{A} + \vec{B}| = ||\vec{A}| - |\vec{B}||$$

Combining the conditions mentioned in (iv) and (v) we get.

$|\vec{A} + \vec{B}| \ge ||\vec{A}| - |\vec{B}||$

(c) To prove $|\vec{A} - \vec{B}| \le |\vec{A}| + |\vec{B}|$

In fig. $\overrightarrow{OL}$ and $\overrightarrow{OM}$ represents vectors $\vec{A}$ and $\vec{B}$ respectively. Here $\overrightarrow{ON}$ represents $\vec{A} - \vec{B}$ Consider the ,

$$ON < MN + OM$$
$$\text{or } |\vec{A} - \vec{B}| < |\vec{A}| + |-\vec{B}|$$

Or $|\vec{A} - \vec{B}| < |\vec{A}| + |\vec{B}|$

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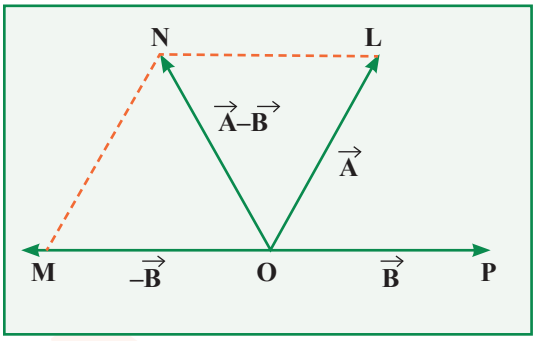
When $\vec{A}$ and $\vec{B}$ are along the same straight line, but point in the opposite direction, then

$$|\vec{A}-\vec{B}|=|\vec{A}|+|\vec{B}|$$

Combining equation (vi) and (vii), we get

$$|\vec{A}-\vec{B}|\leq|\vec{A}|+|\vec{B}|.$$



(d) To prove $|\vec{A}-\vec{B}|\geq||\vec{A}|-|\vec{B}||$

Let us consider the ΔOMN,

$ON+OM>MN$ or $ON>|MN-OM|$ Since $MN=OL \therefore ON>|OL-OM|$

or

$|\vec{A}-\vec{B}|>||\vec{A}|-|\vec{B}||$

When $\vec{A}$ and $\vec{B}$ are along the same straight line and point in the same direction, then

$|\vec{A}-\vec{B}|=|\vec{A}|-|\vec{B}|$

Combining equations (viii) and (ix), we get

$|\vec{A}-\vec{B}|\geq||\vec{A}|-|\vec{B}||$

3.7 Given a+b+c+d = 0, which of the following statements are correct :

(a) a, b, c, and d must each be a null vector,

(b) The magnitude of (a+c) equals the magnitude of (b+d),

(c) The magnitude of a can never be greater than the sum of the magnitudes of b, c and d,

(d) b+c must lie in the plane of a and d if a and d are not collinear, and in the line of a and d, if they are collinear?

SOLUTION:

(a) This statement is not correct. Each need not be a null vector. Even when $\vec{a}=-\vec{b}$ and $\vec{c}=-\vec{d}$ they can form a null vector.

(b) This statement is correct. When $|\vec{a}+\vec{c}|=|\vec{b}+\vec{d}|$, the addition may be a null vector, if $\vec{a}+\vec{c}=-(\vec{b}+\vec{d})$ and are collinear.

(c) This statement is correct. Let $|\vec{a}|>|\vec{b}+\vec{c}+\vec{d}|$. If it is true the vector sum cannot be zero. Even if $\vec{b},\vec{c},\vec{d}$ form a triangle, the vector sum $\vec{b}+\vec{c}+\vec{d}=0$ but then $\vec{a}+\vec{b}+\vec{c}+\vec{d}$ is not zero.

(d) This statement is correct. If $\vec{b}+\vec{c}$ do not lie in the plane of $\vec{a}+\vec{d}$ the vector sum $(\vec{a}+\vec{b})+(\vec{c}+\vec{d})$ is not zero because the addends will have different magnitude and different direction.

3.8 Three girls skating on a circular ice ground of radius 200 m start from a point P on the edge of the ground and reach a point Q diametrically opposite to P following different paths as shown in Fig. 3.19. What is the magnitude of the displacement vector for each? For which girl is this equal to the actual length of path skate?

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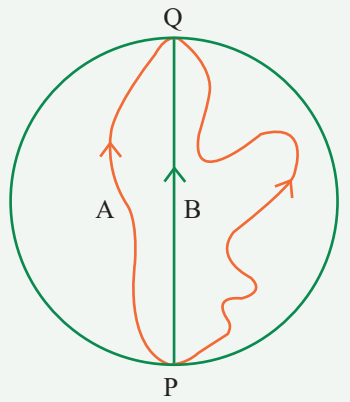

Figure 3.19

SOLUTION:

Given – radius = 200m

Need to find – Magnitude of displacement vector

Displacement for each girl $\overrightarrow{PQ}$

Magnitude of the displacement for each girl = PQ = diameter of circular ice ground

PQ = 2×200=400 m.

Therefore, magnitude of displacement vector is 400m

3.9 A cyclist starts from the centre O of a circular park of radius 1 km , reaches the edge P of the park, then cycles along the circumference, and returns to the centre along QO as shown in Fig. 3.20. If the round trip takes 10 min , what is the (a) net displacement, (b) average velocity, and (c) average speed of the cyclist?

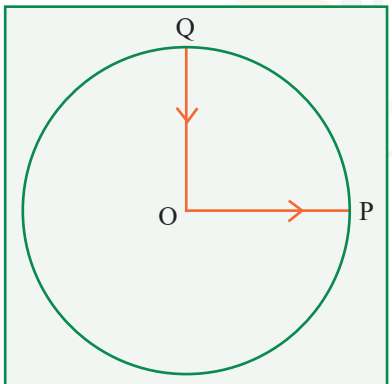

Figure 3.20

SOLUTION:

Given – radius (r) = 1km, time taken by round trip (t) = 10min

Need to find – net displacement, average velocity and average speed (s)

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(a) Since both the initial and final positions are the same therefore the net displacement is zero.

(b) Average velocity is the ratio of net displacement and total time taken. Since the net displacement is zero therefore the average velocity is also zero.

(c) Average speed (s) = $\dfrac{\text{distance covered}}{\text{time taken}}$

$$s = \frac{OP + \text{ Actual distance } PQ + QO}{10 \text{ minute}}$$

$$s = \frac{1\text{km} + \frac{1}{4} \times 2\pi \times 1\text{km} + 1\text{km}}{10/60\text{h}}$$

$$s = 6\left(2 + \frac{22}{14}\right)\text{kmh}^{-1} = 6 \times \frac{50}{14}\text{kmh}^{-1}$$

$s = 21.43 \text{ km}^{\text{h}-1}$

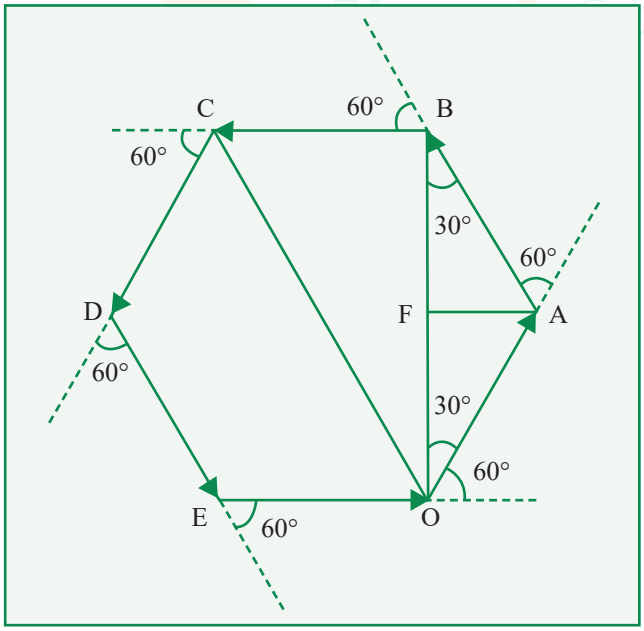
Hence average speed is

3.10 On an open ground, a motorist follows a track that turns to his left by an angle of 60° after every 500 m. Starting from a given turn, specify the displacement of the motorist at the third, sixth and eighth turn. Compare the magnitude of the displacement with the total path length covered by the motorist in each case.

SOLUTION:

Given – Angle = 60, Distance after which motorist turn (d)= 500m

Need to find – Magnitude of displacement (s) and total path length (L)



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(i) The path followed by the motorist will be a closed hexagonal path.

Suppose the motorist starts his journey from the, point 0. He takes the turn at point C .

Displacement $= \overrightarrow{OC}$

Here

$$OC = \sqrt{(OB)^2 + (BC)^2} = \sqrt{(OF + FB)^2 + (BC)^2}$$

$$OC = \sqrt{\left(500\cos 30^\circ + 500\cos 30^\circ\right)^2 + (500)^2}$$

$$OC = \sqrt{\left(2 \times 500 \times \frac{\sqrt{3}}{2}\right)^2 + (500)^2}$$

$$OC = 500\sqrt{4} = 1000\text{m} = 1\text{km}$$

Total path length (L) = 500 m+500 m+500 m=1500 m = 1.5 km

$$\text{Ratio} = \frac{\text{Magnitude of displacement}}{\text{Total path length}} = \frac{1}{1.5} = \frac{2}{3} = 0.67$$

(ii) The motorist will take the sixth turn at .

Displacement is zero. So, the displacement vector is a null vector.

Path length is 3000 m, i.e., 3 km .

Ratio of magnitude of displacement and path length is zero.

(iii) The motorist will take the turn at .

Magnitude of displacement (s) $= 2 \times 500\cos 30^\circ = 500\sqrt{3}\text{m} = \frac{\sqrt{3}}{2}\text{km} = 0.866\text{km}$

Path length (L) = 8×500m 4 km

Ratio of magnitude of displacement and path length is $\frac{\sqrt{3}/2}{4}$ i.e., $\frac{\sqrt{3}}{8} = 0.22$

3.11 A passenger arriving in a new town wishes to go from the station to a hotel located 10 km away on a straight road from the station. A dishonest cabman takes him along a circuitous path 23 km long and reaches the hotel in 28 min . What is

(a) the average speed of the taxi,

(b) the magnitude of average velocity? Are the two equal?

SOLUTION:

Given – actual path length travelled (L) = 23 km; Displacement (d) =10 km, time taken (t) = 28min

Need to find – Average speed of the taxi (s) and magnitude of average velocity (v)

Time taken,

$$t = 28\text{min} = \frac{28}{60}\text{h}$$

(a) $\text{Average speed of taxi(s)} = \frac{\text{actual path length(L)}}{\text{time taken(t)}}$

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$$s = \frac{23}{28/60} \text{km/h} = 49.3 \text{km/h}$$

(b) Magnitude of average velocity(v) $= \dfrac{\text{displacement(d)}}{\text{time taken(t)}}$

$$v = \frac{10}{28/60} \text{km/h} = 21.4 \text{km/h}$$

The average speed is not equal to the magnitude of average velocity. The two are equal for the motion of taxi along a straight path in one direction.

3.12 The ceiling of a long hall is 25 m high. What is the maximum horizontal distance that a ball thrown with a speed of 40 m s^{-1} can go without hitting the ceiling of the hall?

SOLUTION:

Given – Maximum height (h_{max}) = 25 m, Velocity of projection, (v) = 40 ms^{-1}

Need to find – Maximum horizontal distance (R)

Using the formula: $h_{max} = \dfrac{v^2 \sin^2\theta}{2g}$

$$\sin^2\theta = \frac{25 \times 2 \times 9.8}{40 \times 40} = 0.30625$$

$$\sin\theta = 0.5534$$

$$\theta = \sin^{-1}(0.5534) = 33.6^\circ$$

Using- $R = \dfrac{v^2 \sin 2\theta}{g}$

$$R = \frac{40 \times 40 \sin 67.2^\circ}{9.8}$$

$$R = \frac{1600}{9.8} \times 0.9219\text{m} = 150.53\text{m}$$

Hence Maximum horizontal distance (R)= 150.53m

3.13 A cricketer can throw a ball to a maximum horizontal distance of 100 m. How high above the ground can the cricketer throw the same ball?

SOLUTION:

Given – horizontal distance (R_{max}) = 100m

Need to find – Height up to which ball can be thrown (s)

Using the formula-

$$R_{max} = \frac{v^2}{g} \Rightarrow 100 = \frac{v^2}{g}$$

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Using equation of motion-

$v^2 - u^2 = 2as$

$\{v = 0, a = -g, s = R_{max} = 100\text{m}\}$

$(0)^2 - u^2 = 2(-g) \times s$

$s = \frac{1}{2}\frac{u^2}{g}$

$u = v$

$s = \frac{1}{2}\frac{v^2}{g} = \frac{1}{2} \times 100$

s = 50 m

Hence, Height up to which ball can be thrown (s) = 50 m

3.14 A stone tied to the end of a string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 s, what is the magnitude and direction of acceleration of the stone?

SOLUTION:

Given – radius (r) = 80 cm = 0.8 m, linear velocity (v) = $\frac{14}{25}$ rev /s, angular velocity $(\omega) = 2\pi v = 2 \times \frac{22}{7} \times \frac{14}{25} \text{rad/s} = \frac{88}{25} \text{rad} \cdot \text{s}^{-1}$

Need to find – Magnitude of acceleration(a)

Using The centripetal acceleration,

$a = \omega^2 r$

$a = \left(\frac{88}{25}\right)^2 \times 0.80 = 9.90\text{ms}^{-2}$

The direction of centripetal acceleration is 9.90 ms^{-2} along the string directed towards the centre of circular path.

3.15 An aircraft executes a horizontal loop of radius 1.00 km with a steady speed of 900 . Compare its centripetal acceleration with the acceleration due to gravity.

SOLUTION:

Given – radius (r) = 1 km=10^3 m, speed (v) = 900 km h^{-1} = $900 \times \frac{5}{18}$ = 250 ms^{-1}

Need to find – compare centripetal acceleration (a_c)with acceleration due to gravity (g)

Using the formula: Centripetal acceleration $= a_c = \frac{v^2}{r}$

$a_c = \frac{(250)^2}{10^3} = 62.5\text{ms}^{-2}$

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Now, Ratio - $\frac{a_c}{g} = \frac{62.5}{9.8} = 6.38.$

3.16 Read each statement below carefully and state, with reasons, if it is true or false:

(a) The net acceleration of a particle in circular motion is always along the radius of the circle towards the centre.

(b) The velocity vector of a particle at a point is always along the tangent to the path of the particle at that point

(c) The acceleration vector of a particle in uniform circular motion averaged over one cycle is a null vector

SOLUTION:

(a) False, the net acceleration of a particle in circular motion is along the radius of the circle towards the centre only in uniform circular motion.

(b) True, because while leaving the circular path, the particle moves tangentially to the circular path.

(c) True, the direction of acceleration vector in a uniform circular motion is directed towards the centre of circular path. It is constantly changing with time. The resultant of all these vectors will be a zero vector.

3.17 The position of a particle is given by :-

$$r = 3.0t\hat{i} - 2.0t^2\hat{J} + 4.0\hat{k}\text{m}$$

where is in seconds and the coefficients have the proper units for to be in metres.

(a) Find the and of the particle?

(b) What is the magnitude and direction of velocity of the particle at ?

SOLUTION:

Given – $\vec{r}(t) = \left(3.0t\hat{i} - 2.0t^2\hat{J} + 4.0\hat{k}\right)\text{m}$.

Need to find – velocity(v) and acceleration(a)

(a) $\vec{v}(t) = \frac{\overrightarrow{dr}}{dt} = (3.0\hat{i} - 4.0t\hat{J})\text{m/s}$

$\vec{a}(t) = \frac{\overrightarrow{dv}}{dt} = (-4.0\hat{J})\text{m/s}^2$

(b) Magnitude of velocity at ,

$$v_{(t=2s)} = \sqrt{(3.0)^2 + (-4.0\times 2)^2} = \sqrt{9+64} = \sqrt{73}$$

$$v_{(t=2s)} = 8.54\text{ms}^{-1}$$

This velocity will subtend an angle from -axis, where $\beta = \frac{(-4.0\times 2)}{(3.0)} = -2.667$

$\beta = \tan^{-1}(-2.6667)$

$\beta = -69.44° = 69.44°$ from negative x -axis.

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3.18 A particle starts from the origin at t = 0 s with a velocity of 10.0ĵ m/s and moves in the x-y plane with a constant acceleration of (8.0î+2.0ĵ)ms^{-2}.

(a) At what time is the x-coordinate of the particle 16 m? What is the y-coordinate of the particle at that time?

(b) What is the speed of the particle at the time?

SOLUTION:

Given – $\vec{r}_{(t=0s)} = 0, \vec{v}_{(0)} = 10.0\hat{J}\text{m/s}$ and $\vec{a}(t) = (8.0\hat{i} + 2.0\hat{J})\text{ms}^{-2}$

Need to find – Time at which x coordinate of particle is 16m (t)

(a) $x_0 = 0, u_x = 0$, $a_x = 8.0$ m s^{-2} and x = 16 m

Using relation

$$s = x - x_0 = u_x t + \frac{1}{2}a_x t^2$$

$$16 - 0 = 0 + \frac{1}{2} \times 8.0 \times t^2 \Rightarrow t = 2\text{s}$$

$$y = y_0 + u_y t + \frac{1}{2}a_y t^2 = 0 + 10.0 \times 2 + \frac{1}{2} \times 2.0 \times (2)^2$$

$$y = 20 + 4 = 24\text{m}$$

(b) Velocity of particle at t = 2 s along x-axis and along y -axis

$v_x = u_x + a_x t = 0 + 8.0 \times 2 = 16.0$ m/s

$v_y = u_y + a_y t = 10.0 + 2.0 \times 2 = 14.0$ m/s

Speed of particle at t = 2 s

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(16.0)^2 + (14.0)^2} = 21.26\text{ms}^{-1}$$

3.19 19î and ĵ are unit vectors along x - and y - axis respectively. What is the magnitude and direction of the vectors î + ĵ and î – ĵ What are the components of a vector A = 2î+ 3ĵ along the directions of î+ ĵ and î – ĵ? [You may use graphical method]

Solution:

(i) $\hat{i} + \hat{j} = \sqrt{(1)^2 + (1)^2 + 2 \times 1 \times 1 \times \cos 90^\circ} = \sqrt{2} = 1.414$ units

$$\tan\theta = \frac{1}{1} = 1, \quad \therefore \theta = 45^\circ$$

So the vector î+ ĵ makes an angle of 45° with x-axis.

(ii) $\hat{i} + \hat{j} = \sqrt{(1)^2 + (1)^2 + 2 \times 1 \times 1 \times \cos 90^\circ} = \sqrt{2} = 1.414$ units

$|\hat{i} - \hat{j}| = \sqrt{2} = 1.414$ units

The vector î – ĵ makes an angle of –45° with x-axis.

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(iii) Let us now determine the component of $\vec{A} = 2\hat{i} + 3\hat{j}$ in the direction of $\hat{i} + \hat{j}$

Let

$\vec{B} = \hat{i} + \hat{j}$

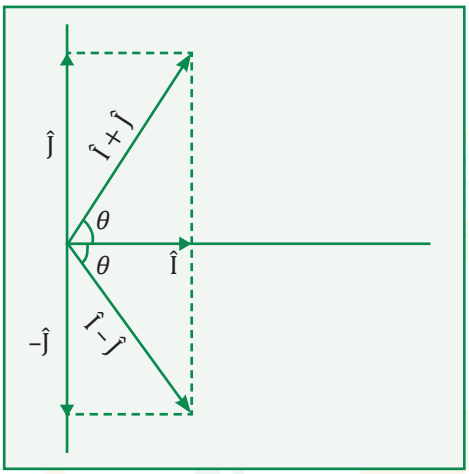
$\vec{A} \cdot \vec{B} = AB\cos\theta = (A\cos\theta)B$

So the component of $\vec{A}$ in the direction of $\vec{B} = \dfrac{\vec{A} \cdot \vec{B}}{B}$

$$= \frac{(2\hat{i} + 3\hat{J}) \cdot (\hat{i} + \hat{J})}{\sqrt{(1)^2 + (1)^2}} = \frac{2\hat{i} \cdot \hat{i} + 2\hat{i} \cdot \hat{J} + 3\hat{J} \cdot \hat{i} + 3\hat{J} \cdot \hat{J}}{\sqrt{2}} = \frac{5}{\sqrt{2}} \text{ units}$$

(iv) Component of in the direction of $\hat{i} - \hat{J} = \dfrac{(2\hat{i} + 3\hat{J}) \cdot (\hat{i} - \hat{J})}{\sqrt{2}} = -\dfrac{1}{\sqrt{2}}$ units.

B



3.20 For any arbitrary motion in space, which of the following relations are true :

(a) $v_{average} = (1/2)\,(v(t_1) + v(t_2))$

(b) $v\ averase = [r(t_2) - r(t_1)]/(t_2 - t_1)$

(c) $v(t) = v(0) + at$

(d) $r(t) = r(0) + v(0)t + (1/2)\,a\,t^2$

(e) $a_{average} = [v(t_2) - v(t_1)]/(t_2 - t_1)$

(The "average" stands for average of the quantity over the time interval t_1 to t_2)

SOLUTION:

(b) and (e) are true; others are false because relations (a), (c) and (d) hold only for uniform acceleration.

3.21 Read each statement below carefully and state, with reasons and examples, if it is true or false :

A scalar quantity is one that

(a) is conserved in a process

(b) can never take negative values

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(c) must be dimensionless

(d) does not vary from one point to another in space

(e) has the same value for observers with different orientations of axes.

SOLUTION:

(a) False, because kinetic energy is a scalar but does not remain conserved in an inelastic collision.

(b) False, because potential energy in a gravitational field may have negative values.

(c) False, because mass, length, time, speed, work etc., all have dimensions.

(d) False, because speed, energy etc., vary from point to point in space.

(e) True, because a scalar quantity will have the same value for observers with different orientations of axes since a scalar has no direction of its own.

3.22 An aircraft is flying at a height of 3400 m above the ground. If the angle subtended at a ground observation point by the aircraft positions 10.0 s a part is , what is the speed of the aircraft?

SOLUTION:

Given – height (h)= 3400m, OC = 3400 m and ∠AOC = ∠COB = 15°.

Need to find – speed of the aircraft (v)

In Fig, O is the observation point at the ground. A and B are the positions of aircraft for which ∠AOB = 30°.

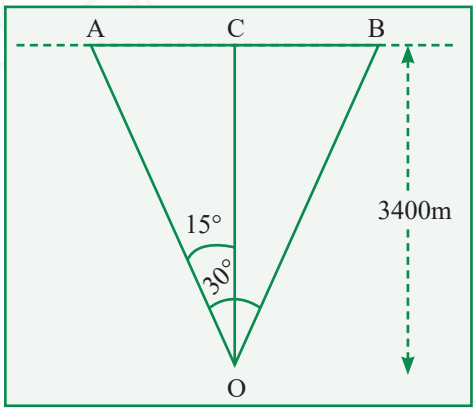
Draw a perpendicular OC on AB. Here In ΔAOC, AC = OC

tan15° = 3400×0.2679 = 910.86 m.

AB = AC + CB = AC + AC = 2AC = 2 × 910.86 m

Speed of the aircraft, $v = \dfrac{\text{distance } AB}{\text{time}}$

$$v = \frac{2 \times 910.86}{10} = 182.17\text{ms}^{-1} = 182.2\text{ms}^{-1}$$



https://t.me/veda11and12