CHAPTER

5

WORK, ENERGY AND POWER

CLASS 11TH

NCERT EXERCISE AND SOLUTIONS - PHYSICS

5.1 The sign of work done by a force on a body is important to understand. State carefully if the following quantities are positive or negative:

(a) work done by a man in lifting a bucket out of a well by means of a rope tied to the bucket.

(b) work done by gravitational force in the above case,

(c) work done by friction on a body sliding down an inclined plane,

(d) work done by an applied force on a body moving on a rough horizontal plane with uniform velocity,

(e) work done by the resistive force of air on a vibrating pendulum in bringing it to rest.

SOLUTION:

Work done, W = T.S = Fscos θ

(a) Work done 'positive', because force is acting in the direction of displacement i.e., $\theta = 0°$.

(b) Work done is negative, because force is acting against the displacement i.e., $\theta = 180°$.

(c) Work done is negative, because force of friction is acting against the displacement i.e., $\theta = 180°$

(d) Work done is positive, because body moves in the direction of applied force i.e., $\theta = 0°$

(e) Work done is negative, because the resistive force of air opposes the motion i.e., $\theta = 180°$.

5.2 A body of mass 2 kg initially at rest moves under the action of an applied horizontal force of 7 N on a table with coefficient of kinetic friction = 0.1. Compute the

(a) work done by the applied force in 10 s,

(b) work done by friction in 10 s,

(c) work done by the net force on the body in 10 s,

(d) change in kinetic energy of the body in 10 s and interpret your results.

SOLUTION:

Given – mass (m) = 2kg, horizontal force (F), coefficient of kinetic friction = 0.1

Need to find – work done (W)

We know that μ_k = frictional force (F)/normal reaction (N)

frictional force (F) = μ_k × normal reaction

$f = 0.1 \times 2\ kgwt$

$f = 0.1 \times 2 \times 9.8$ N = 1.96 N

net effective force = (7 – 1.96) N = 5.04 N

acceleration = 5.04/2 ms^{-2} = 2.52 ms^{-2}

distance, $s = \frac{1}{2} \times 2.52 \times 10 \times 10 = 126$ m

work done by applied force = 7 × 126 J = 882 J

(b) Work done by friction = –1.96 × 126 = –246.96 J

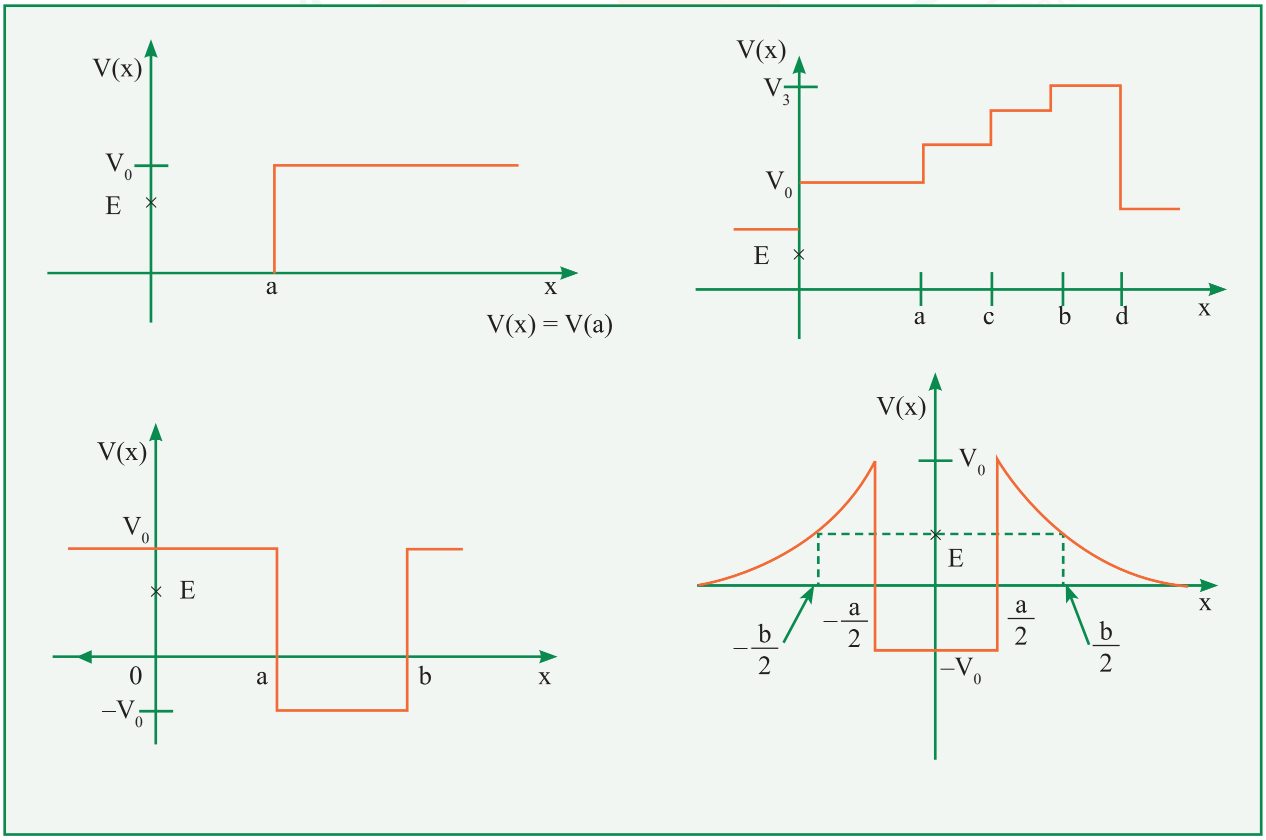
(c) Work done by net force = 5.04 × 126 = 635.04 J

(d) Change in the kinetic energy of the body = work done by the net force in 10 seconds = 635.04 J (This is in accordance with work-energy theorem).

5.3 Given in Fig. are examples of some potential energy functions in one dimension. The total energy of the particle is indicated by a cross on the ordinate axis. In each case, specify the regions, if any, in which the particle cannot be found for the given energy. Also, indicate the minimum total energy the particle must have in each case. Think of simple physical contexts for which these potential energy shapes are relevant.

SOLUTION:

We know that total energy $E = K.E. + P.E.$ or $K.E. = E - P.E.$ and kinetic energy can never be negative. The object cannot exist in the region, where its K.E. would become negative.



(a) In the region between $x = 0$ and $x = $ a, potential energy is zero. So, kinetic energy is positive. In the region $x > a$, the potential energy has a value greater than E. So, kinetic energy will be negative in this region. Thus, the particle cannot be present in the region $x > a$.

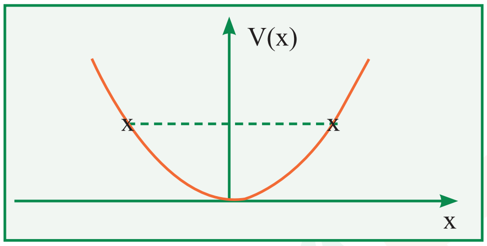
The minimum total energy that the particle can have in this case is zero.

(b) Here P.E. > E, the total energy of the object and as such the kinetic energy of the object would be negative. Thus, objects cannot be present in any region of the graph.

(c) Here x = 0 to $x = a$ and $x > b$, the P.E. is more than E, so K.E. is negative. The particle cannot exist in these portions.

(d) The object cannot exist in the region between $x = -b/2$ to $x = -a/2$ and $x = -a/2$ to $x = -b/2$. Because in this region P.E. > E.

5.4 The potential energy function for a particle executing linear simple harmonic motion is given by V(x) = $kx^2/2$, where k is the force constant of the oscillator. For k = 0.5 N m^{-1}, the graph of $V(x)$ versus x is shown in Fig. Show that a particle of total energy 1 J moving under this potential must 'turn back' when it reaches. $x = \pm 2$ m



SOLUTION:

Given – force constant (k) = 0.5 Nm^{-1}, total energy of particle (E) = 1 J.

Here, The particle can go up to a maximum distance x_m where its total energy is transformed into elastic potential energy.

$$\frac{1}{2}kx_m^2 = E$$

$$\Rightarrow x_m = \sqrt{\frac{2E}{K}} = \sqrt{\frac{2\times 1}{0.5}} = \sqrt{4} = \pm 2m$$

Hence proved.

5.5 Answer the following:

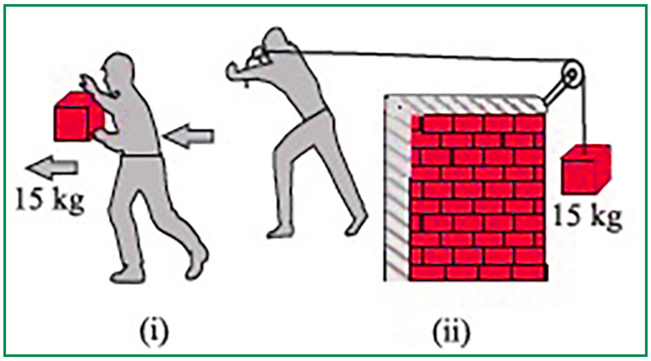
(a) The casing of a rocket in flight burns up due to friction. At whose expense is the heat energy required for burning obtained? The rocket or the atmosphere?

(b) Comets move around the sun in highly elliptical orbits. The gravitational force on the comet due to the sun is not normal to the comet's velocity in general. Yet the work done by the gravitational force over every complete orbit of the comet is zero. Why ?

(c) An artificial satellite orbiting the earth in very thin atmosphere loses its energy gradually due to dissipation against atmospheric resistance, however small. Why then does its speed increase progressively as it comes closer and closer to the earth?

(d) In Fig. (i) the man walks 2 m carrying a mass of 15 kg on his hands. In Fig. (ii), he walks the

same distance pulling the rope behind him. The rope goes over a pulley, and a mass of 15 kg hangs at its other end. In which case is the work done greater?



SOLUTION:

(a) Heat energy required for burning of casing of a rocket comes from the rocket itself. As a result of work done against friction the kinetic energy of rocket continuously decreases - and this work against friction reappears as heat energy.

(b) This is because gravitational force is a conservative force. Work done by the gravitational ' force of the sun over a closed path in every complete orbit of the comet is zero.

(c) As an artificial satellite gradually loses its energy due to dissipation against atmospheric resistance, its potential decreases rapidly. As a result, kinetic energy of satellite slightly increases i.e., its speed increases progressively.

(d) In Fig. (i), force is applied on the mass, by the man in vertically upward direction but distance is moved along the horizontal.

$$\theta = 90°. W = FS \cos 90° = \text{zero}$$

In Fig. (ii), force is applied along the horizontal and the distance moved is also along the horizontal. Therefore, $\theta = 0°$

$$W = FS\cos\theta = mg \times s\cos 0°$$

$$W = 15 \times 9.8 \times 2 \times 1 = 294 \text{ joule}$$

Thus, work done in (ii) case is greater.

5.6 Underline the correct alternative :

(a) When a conservative force does positive work on a body, the potential energy of the body increases/decreases/remains unaltered.

(b) Work done by a body against friction always results in a loss of its kinetic/potential energy.

(c) The rate of change of total momentum of a many-particle system is proportional to the external force/sum of the internal forces on the system.

(d) In an inelastic collision of two bodies, the quantities which do not change after the collision are the total kinetic energy/total linear momentum/total energy of the system of two bodies.

SOLUTION:

(a) Potential energy of the body decreases because the body in this case goes closer to the centre of the force.

(b) Kinetic energy, because friction does its work against the motion.

(c) Internal forces cannot change the total or net momentum of a system. Hence the rate of change of total momentum of many particles system is proportional to the external force on the system.

(d) In an inelastic collision of two bodies, the quantities which do not change after the collision are the total kinetic energy/total linear momentum/ total energy of the system of two bodies.

5.7 State if each of the following statements is true or false. Give reasons for your answer.

(a) In an elastic collision of two bodies, the momentum and energy of each body is conserved.

(b) Total energy of a system is always conserved, no matter what internal and external forces on the body are present.

(c) Work done in the motion of a body over a closed loop is zero for every force in nature.

(d) In an inelastic collision, the final kinetic energy is always less than the initial kinetic energy of the system.

SOLUTION:

(a) False, the total momentum and total energy of the system are conserved.

(b) False, the external force on the system may increase or decrease the total energy of the system.

(c) False, the work done during the motion of a body over a closed loop is zero only when body is moving under the action of a conservative force (such as gravitational or electrostatic force). Friction is not a conservative force hence work done by force of friction (or work done on the body against friction) is not zero over a closed loop.

(d) True, usually in an inelastic collision the final kinetic energy is always less than the initial kinetic energy of the system.

5.8 Answer carefully, with reasons:

(a) In an elastic collision of two billiard balls, is the total kinetic energy conserved during the short time of collision of the balls (i.e. when they are in contact)?

(b) Is the total linear momentum conserved during the short time of an elastic collision of two balls ?

(c) What are the answers to (a) and (b) for an inelastic collision?

(d) If the potential energy of two billiard balls depends only on the separation distance between their centres, is the collision elastic or inelastic? (Note, we are talking here of potential energy corresponding to the force during collision, not gravitational potential energy).

SOLUTION:

(a) In this case total kinetic energy is not conserved because when the bodies are in contact dining elastic collision even, the kinetic energy is converted into potential energy.

(b) Yes, because total momentum conserves as per law of conservation of momentum.

(c) The answers remain unchanged.

(d) It is a case of elastic collision because in this case the forces will be of conservative nature.

5.9 A body is initially at rest. It undergoes one-dimensional motion with constant acceleration. The power delivered to it at time *t* is proportional to

(i) $t^{1/2}$

(ii) t

(iii) $t^{3/2}$

(iv) t^2

SOLUTION:

(ii) From $v = u + at$

$v = 0 + at = at$

As power, $p = F \times v$

$p = (ma) \times at = ma^2 t$

Since m and a are constants, therefore, p is proportional to t.

5.10 A body is moving unidirectionally under the influence of a source of constant power. Its displacement in time *t* is proportional to

(i) $t^{1/2}$

(ii) t

(iii) $t^{3/2}$

(iv) t^2

SOLUTION:

(iii) $t^{3/2}$

$[p] = [F]\,[v] = [MLT^{-2}]\,[LT^{-1}]$

$\Rightarrow [p] = [ML^2T^{-3}]$

or L^2T^{-3} = constant $\Rightarrow \frac{L^2}{T^2}$ = constant

$\therefore L^2 \propto T^3 \Rightarrow L \propto T^{3/2}$

5.11 A body constrained to move along the z-axis of a coordinate system is subject to a constant force F given by

$$\mathbf{F} = -\tilde{y} + 2\hat{j} + 3\hat{k}\ \mathbf{N}$$

where $\hat{\imath}, \hat{\jmath}, \hat{K}$ are unit vectors along the *x*-, *y* - and *z*-axis of the system respectively. What is the work done by this force in moving the body a distance of 4 m along the -axis?

SOLUTION:

Given – $\mathbf{F} = -\tilde{y} + 2\hat{j} + 3\hat{k}\ \mathbf{N}$

Need to find – work done(w)

Since the body is displaced 4 m along z -axis only,

$$\therefore \vec{S} = 0\hat{i} + 0\hat{j} + 4\hat{k}$$

Also

$$\vec{F} = -\hat{i} + 2\hat{j} + 3\hat{k}$$

Work done,

$$W = \vec{F} \cdot \vec{S}$$
$$W = (-\hat{i} + 2\hat{j} + 3\hat{k}) \cdot (\hat{i} + 0\hat{j} + 4\hat{k})$$
$$W = 12(\hat{k} \cdot \hat{k}) \text{ Joule} = 12 \text{ Joule.}$$

Hence work done is 12J.

5.12 An electron and a proton are detected in a cosmic ray experiment, the first with kinetic energy 10 keV, and the second with 100 keV. Which is faster, the electron or the proton? Obtain the ratio of their speeds. (Electron mass = 9.11 × 10^{-31}kg, proton mass = 1.67 × 10^{-27} kg, 1eV = 1.60 × 10^{-19} J).

SOLUTION:

Given – Electron mass = 9.11 × 10^{-31}kg, proton mass = 1.67 × 10^{-27} kg, 1eV = 1.60 × 10^{-19} J

Need to find – which is faster- electron or the proton

$K_e = 10\ keV$ and $K_p = 100\ keV$

$m_e = 9.11 \times 10^{-31}\ kg$ and $m_p = 1.67 \times 10^{-27}\ kg$

$$K = \frac{1}{2}mv^2 \text{ or } v = \sqrt{\frac{2K}{m}}$$

Hence,

$$\frac{v_e}{v_p} = \sqrt{\frac{K_e}{K_p} \times \frac{m_p}{m_e}} = \sqrt{\frac{10keV}{100keV} \times \frac{1.67 \times 10^{-27} kg}{9.11 \times 10^{-31} kg}}$$
$$= 13.54$$
$$\Rightarrow v_e = 13.54\ v_p$$

Thus, electron is travelling faster.

5.13 A rain drop of radius 2 mm falls from a height of 500 m above the ground. It falls with decreasing acceleration (due to viscous resistance of the air) until at half its original height, it attains its maximum (terminal) speed and moves with uniform speed thereafter. What is the work done by the gravitational force on the drop in the first and second half of its journey? What is the work done by the resistive force in the entire journey if its speed on reaching the ground is 10 mxs^{-2} ?

SOLUTION:

Given – radius (r) =2 mm = 2×10^{-3} m, Distance moved in each half of the journey (S) =500/2 = 250 m.
Density of water (p) = 10^3 kg/m^3

Need to find – work done(w)

Using-

Mass of rain drop volume of drop density

$m = 4/3\pi r^2 \times \rho = 4/3 \times 22/7\,(2 \times 10^{-3})^3 \times 10^3 = 3.35 \times 10^{-5}$ kg

$\therefore$ W = mg × s

$W = 3.35 \times 10^{-5} \times 9.8 \times 250$

W = 0.082 J

Whether the drop moves with decreasing acceleration or with uniform speed, work done by the gravitational force on the drop remains the same.

If there were no resistive forces, energy of drop on reaching the ground.

$E_1 = mgh = 3.35 \times 10^{-5} \times 9.8 \times 500 = 0.164$ J

Actual energy, $E_2 = 1/2\, mv^2 = 1/2 \times 3.35 \times 10^{-5}\,(10)^2 = 1.675 \times 10^{-3}$ J

Work done by the resistive forces,

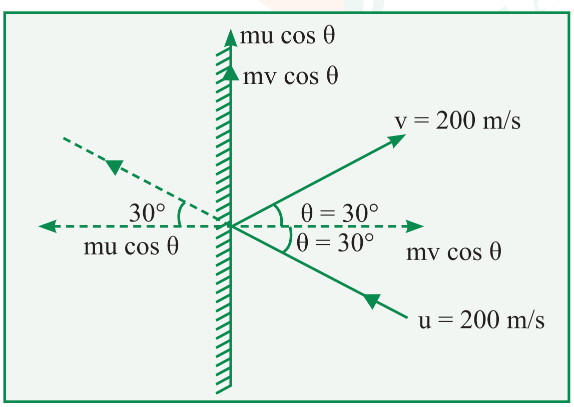
$W = E_1 - E_2$

$W = 0.164 - 1.675 \times 10^{-3}$ W = 0.1623 joule.

Hence the energy due to the resistive force is 0.162 J.

5.14 A molecule in a gas container hits a horizontal wall with speed 200 ms^{-1} and angle 30° with the normal, and rebounds with the same speed. Is momentum conserved in the collision? Is the collision elastic or inelastic?

SOLUTION:



Given – speed (u)= 200m/s

Let us consider the mass of the molecule be m and that of wall be M . The wall remains at rest due to its large mass. Resolving momentum of the molecule along x-axis and y-axis, we get

The x-component of momentum of molecule $= mu\cos\theta = -m200\cos 30^\circ = -100\sqrt{3}m$

y-component of the molecule = $musin\ \theta = m \times 200 \times \sin 30^\circ = 100\ m$

Before collision: x-component of total momentum (wall + molecule)

$$= 0 + (-100\sqrt{3}m) = -100\sqrt{3}m$$

y - component of momentum (wall + molecule) = 0 + 100 m = 100 m

After collision: -component of the momentum (wall + molecule)

$$= 0 + m200\cos 30^\circ = 100\sqrt{3}m$$

and -component = 0 + m 100 sin30° =100 m

We find that momentum of the (molecule + wall) system is conserved. The wall has a recoil momentum such that momentum of the wall + momentum of outgoing molecule equals the momentum of the incoming molecule.

Initial kinetic energy $\left(\frac{1}{2}mu^2\right)$ is the same as final K.E. $\left(\frac{1}{2}mv^2\right)$ of the molecule as

$u = v = 200\ m/s$ i.e., thus, the collision is elastic collision.

5.15 A pump on the ground floor of a building can pump up water to fill a tank of volume 30m³ in 15 min. If the tank is 40 m above the ground, and the efficiency of the pump is 30%, how much electric power is consumed by the pump?

SOLUTION:

Given – volume of water (V) =$30m^3$; time (t) =15 min = 15 × 60 = 900 s,

h (height) = 40m; efficiency (n) = 30%

Need to find – electrical power (p_i)

As the density of water = $p = 10^3\ kg\ m^{-3}$

Mass of water pumped, m = volume (V) x density (p)

$$m = 30 \times 10^3 \text{ kg}$$

Actual power consumed or output power p_0 = W/t = mgh/t

$\Rightarrow p_0 = (30 \times 10^3 \times 9.8 \times 40)/900 = 13070$ watt

If p_i is input power (required), then as

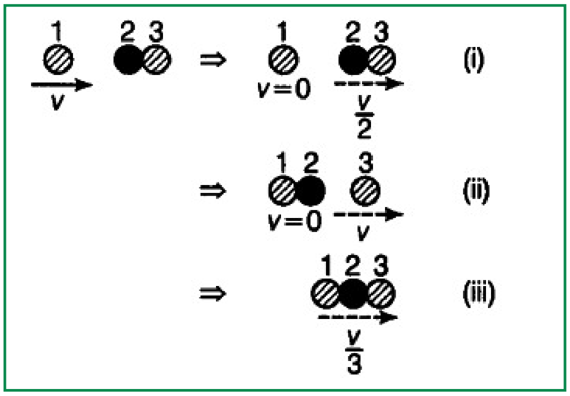
$$\eta = \frac{p_0}{p_i}$$

$$p_i = p_0 / \eta$$

p_i = 13070/ (30/100) = 43567 W

Hence electrical power consumed is 43.56W.

5.16 Two identical ball bearings in contact with each other and resting on a frictionless table are hit head-on by another ball bearing of the same mass moving initially with a speed V. If the collision is elastic, which of the following (Fig.) is a possible result after collision?



SOLUTION:

Let m be the mass of each ball bearing.

Before the collision, total K.E. of the system = $1/2mv^2 + 0 = 1/2mv^2$

After collision, K.E. of the system is: -

Case I, $E_1 = 1/2(2m)(v/2)^2 = 1/4mv^2$

Case II, $E_2 = 1/2mv^2$

Case III, $E_3 = 1/2(3\ m)(v/3)^2 = 1/6mv^2$

Thus, case II is the only possibility since K.E. is conserved in this case.

5.17 The bob A of a pendulum released from 30° to the vertical hits another bob B of the same mass at rest on a table as shown in Fig. . How high does the bob A rise after the collision? Neglect the size of the bobs and assume the collision to be elastic.

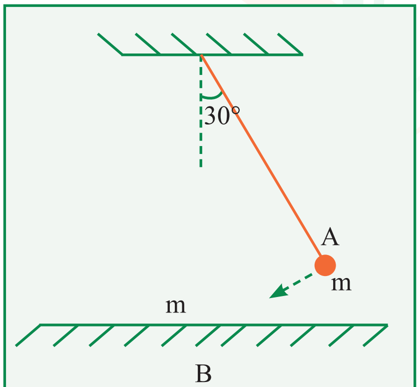
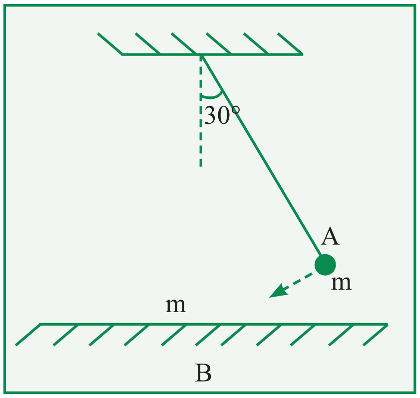


Fig.

SOLUTION:

Since collision is elastic therefore A would come to rest and B would begin to move with the velocity of A.



The bob transfers its entire momentum to the ball on the table. The bob does not rise at all.

5.18 The bob of a pendulum is released from a horizontal position. If the length of the pendulum is 1.5 m, what is the speed with which the bob arrives at the lowermost point, given that it dissipated 5% of its initial energy against air resistance ?

SOLUTION:

Given – length of the pendulum (l)=1.5m

Need to find – speed (v)

On releasing the bob of pendulum from horizontal position, it falls vertically downward by a distance equal to length of pendulum i.e., h = I = 1.5 m.

As 5% of loss in P.E. is dissipated against air resistance, the balance 95% energy is transformed into K.E. Hence,

$$\frac{1}{2}mv^2 = \frac{95}{100} \times mgh$$

$$\Rightarrow v = \sqrt{2 \times \frac{95}{100} \times gh} = \sqrt{\frac{2 \times 95 \times 9.8 \times 1.5}{100}} = 5.3 ms^{-1}$$

Hence velocity at the lowermost point is 5.3m/s.

5.19 A trolley of mass 300 kg carrying a sandbag of 25 kg is moving uniformly with a speed of 27 km/h on a frictionless track. After a while, sand starts leaking out of a hole on the floor of the trolley at the rate of 0.05 kg s^{-1}. What is the speed of the trolley after the entire sand bag is empty ?

SOLUTION:

The trolley has a mass of 300 kg and carries a sandbag of 25 kg. The initial speed of the trolley is given as 27 km/h.

Since we typically work in SI units, we need to convert the speed from km/h to m/s.

$$-27 \text{ km/h} = \frac{27 \times 1000 \text{ m}}{3600 \text{ s}} = 7.5 \text{ m/s}$$

The initial momentum of the system (trolley + sandbag) can be calculated using the formula:

Initial Momentum = Total Mass × Initial Speed

Total mass $= 300 \text{ kg} + 25 \text{ kg} = 325 \text{ kg}$

Initial momentum $= 325 \text{ kg} \times 7.5 \text{ m/s} = 2437.5 \text{ kg m/s}$

As the sand leaks out at a rate of 0.05 kg/s, the mass of the trolley decreases, but the sand falling inside the trolley does not exert any external force on the system. Therefore, the momentum of the system will be conserved.

When the entire sandbag is empty, the mass of the trolley will be:

Final mass = 300 kg (trolley) +0 kg (empty sandbag) = 300 kg

Since momentum is conserved, the initial momentum must equal the final momentum:

Initial Momentum = Final Momentum

$2437.5 \text{ kg m/s} = \text{Final Mass} \times \text{Final Speed}$

Let v_f be the final speed of the trolley:

$2437.5 \text{ kg m/s} = 300 \text{ kg} \times v_f$

Now, we can solve for v_f :

$$v_f = \frac{2437.5 \text{ kg m/s}}{300 \text{ kg}} = 8.125 \text{ m/s}.$$

To convert the final speed back to km/h:

$$v_f = 8.125 \text{ m/s} \times \frac{3600 \text{ s}}{1000 \text{ m}} = 29.25 \text{ km/h}$$

Conclusion

The speed of the trolley after the entire sandbag is empty is 29.25 km/h.

5.20 A body of mass 0.5 kg travels in a straight line with velocity $v = ax^{3/2}$ where a = 5 $m^{-1/2}$ s^{-1}. What is the work done by the net force during its displacement from $x = 0$ to x =2m ?

SOLUTION:

Given – mass (m) = 0.5kg, $u = ax^{3/2}$, a = 5 $m^{-1/2}$

Need to find – work done(W)

Initial velocity at $x = 0$, $v_1 = a \times 0 = 0$

Final velocity at $x = 2$, $v_2 = a(2)^{3/2} = 5 \times (2)^{3/2}$

Using work energy theorem-

Work done = increase in

$$W = 1/2m\left(v_2^2 - v_1^2\right) = 1/2 \times 0.5\left[\left(5 \times (2)^{3/2}\right)^2 - 0\right] = 50J$$

Hence work done is 50J.

5.21 The blades of a windmill sweep out a circle of area A.

(a) If the wind flows at a velocity perpendicular to the circle, what is the mass of the air passing through it in time?

(b) What is the kinetic energy of the air?

(c) Assume that the windmill converts 25% of the wind's energy into electrical energy, and that A = 30 m², v = 36 and the density of air is 1.2 kg m⁻³. What is the electrical power produced?

SOLUTION:

(a) Volume of wind flowing per second = Av

Mass of wind flowing per second = Avo

Mass of air passing in second = Avpt

(b) Kinetic energy of air $= \frac{1}{2}mv^2 = \frac{1}{2}(Av\rho t)v^2 = \frac{1}{2}Av^3\rho t$

(c) Electrical energy produced $= \frac{25}{100} \times \frac{1}{2}Av^3\rho t = \frac{Av^3\rho t}{8}$

Electrical power $= \frac{Av^3\rho t}{8t} = \frac{Av^3\rho}{8}$

Now,

$$A = 30\text{m}^2, v = 36kmh^{-1} = 36 \times \frac{5}{18}ms^{-1} = 10ms^{-1}$$

and $\rho = 1.2kgms^{-1}$

Electrical power $= \frac{30 \times 10 \times 10 \times 10 \times 1.2}{8}\text{W} = 4500\text{W} = 4.5KW.$

5.22 A person trying to lose weight (dieter) lifts a 10 kg mass, one thousand times, to a height of 0.5 m each time. Assume that the potential energy lost each time she lowers the mass is dissipated.

(a) How much work does she do against the gravitational force?

(b) Fat supplies 3.8×10^7 of energy per kilogram which is converted to mechanical energy with a 20% efficiency rate. How much fat will the dieter use up?

SOLUTION:

Given – mass (m)=10 kg, height (h)=0.5 m, number of times(n)=1000

Need to find – work done(W)

(a) work done against gravitational force.

$W = n(mgh)$

$W = 1000 \times (10 \times 9.8 \times 0.5) = 49000$ J

(b) Mechanical energy supplied by 1 kg of fat (E) = $3.8 \times 10^7 \times 20/100$

E = 0.76×10^7 J/kg

$\therefore$ Fat used up by the dieter = 1 kg/(0.76×10^7) × 49000 = 6.45×10^{-3} kg

5.23 A family uses 8 kW of power. (a) Direct solar energy is incident on the horizontal surface at an average rate of 200 W per square meter. If 20% of this energy can be converted to useful electrical energy, how large an area is needed to supply 8 kW ? (b) Compare this area to that of the roof of a typical house.

SOLUTION:

Given – Power used by family (p) = 8KW = 8000 W

Need to find – solar energy (E), area needed(A)

(a) As only 20% of solar energy can be converted to useful electrical energy, hence, power 8000 W to be supplied by solar energy = 8000 W/20 = 40000 W

As solar energy is incident at a rate of $200 Wm^{-2}$, hence the area needed

$$A = 4000W/200Wm^{-2} = 200\ m^2$$

(b) The area needed is comparable to the roof area of a large sized house.