CHAPTER 6

SYSTEM OF PARTICLES AND ROTATIONAL MOTION

CLASS 11TH

NCERT EXERCISE AND SOLUTIONS - PHYSICS

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6.1 Give the location of the centre of mass of a (i) sphere, (ii) cylinder, (iii) ring, and (iv) cube, each of uniform mass density. Does the centre of mass of a body necessarily lie inside the body?

SOLUTION:

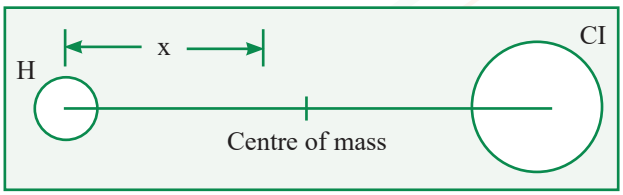
In all the four cases, as the mass density is uniform, centre of mass is located at their respective geometrical centres.

No, it is not necessary that the centre of mass of a body should lie on the body. For example, in case of a circular ring, centre of mass is at the centre of the ring, where there is no mass.

6.2 In the HCl molecule, the separation between the nuclei of the two atoms is about $1.27\text{Å}\left(1\text{Å}=10^{-10}\text{m}\right)$. Find the approximate location of the CM of the molecule, given that a chlorine atom is about 35.5 times as massive as a hydrogen atom and nearly all the mass of an atom is concentrated in its nucleus.

SOLUTION:

Let us choose the nucleus of the hydrogen atom as the origin for measuring distance. Mass of hydrogen atom, m_1 = 1 unit (say) Since cholorine atom is 35.5 times as massive as hydrogen atom, ∴ mass of cholorine atom, m_2 = 35.5 units



Now,

$x_1 = 0$ and $x_2 = 1.27\text{Å} = 1.27\times10^{-10}\,\text{m}$

Distance of centre of mass of HCl molecule from the origin is given by

$$X = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{1\times0 + 35.5\times1.27\times10^{-10}}{1+35.5}\,\text{m}$$

$$X = \frac{35.5\times1.27}{36.5}\times10^{-10}\,\text{m} = 1.235\times10^{-10}\,\text{m} = 1.235\text{Å}$$

6.3 A child sits stationary at one end of a long trolley moving uniformly with a speed on a smooth horizontal floor. If the child gets up and runs about on the trolley in any manner, what is the speed of the CM of the (trolley + child) system?

SOLUTION:

When the child gets up and runs about on the trolley, the speed of the centre of mass of the trolley and child remains unchanged irrespective of the manner of motion of child. It is because here child and trolley constitute one single system, and forces involved are purely internal forces. As there is no external force, there is no change in momentum of the system and velocity remains unchanged.

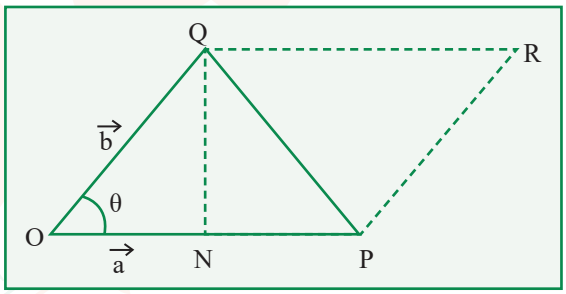
6.4 Show that the area of the triangle contained between the vectors and is one half of the magnitude of a × b.

SOLUTION:

Let $\vec{a}$ be represented $\overrightarrow{OP}$ and $\vec{b}$ be represented by $\overrightarrow{OQ}$. Let $\angle POQ = \theta$, Fig.

Complete the | gm OPRQ. Join PQ.

Draw QN ⊥ OP



In ΔOQN, $\sin\theta = \frac{QN}{OQ} = \frac{QN}{b}$

$QN = b\sin\theta$

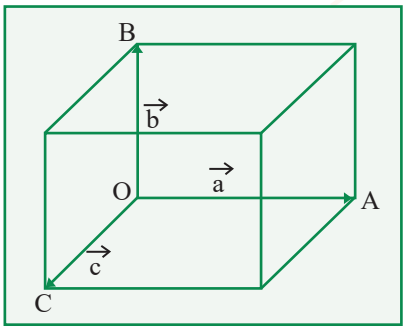
Now, by definition, $|\vec{a}\times\vec{b}| = ab\sin\theta = (OP)(QN)$

$= \frac{2(OP)(QN)}{2} = 2\times$ area of ΔOPQ

$\therefore$ area of $\Delta OPQ = \frac{1}{2}|\vec{a}\times\vec{b}|$ which was to be proved.

6.5 Show that a.(b×c) is equal in magnitude to the volume of the parallelepiped formed on the three vectors, a, b and c.

SOLUTION:



Let *a* parallelopiped be formed on the three vectors.

and

$\overrightarrow{OA} = \vec{a}, \overrightarrow{OB} = \vec{b}, \overrightarrow{OC} = \vec{c}$

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Now, $\vec{b}\times\vec{c} = bc\sin 90^\circ \hat{n} = bc\hat{n}$ where $\hat{n}$ is unit vector along $\overrightarrow{OA}$ perpendicular to the plane containing $\vec{b}$ and $\vec{c}$.

Now

$$\vec{a}\cdot(\vec{b}\times\vec{c}) = \vec{a}\cdot bc\hat{n}$$

$$\vec{a}\cdot(\vec{b}\times\vec{c}) = (a)(bc)\cos 0^\circ$$

$$\vec{a}\cdot(\vec{b}\times\vec{c}) = abc$$

which is equal in magnitude to the volume of the parallelopiped.

6.6 Find the components along the *x*, *y*, *z* axes of the angular momentum 1 of a particle, whose position vector is r with components *x*, *y*, *z* and momentum is with components p_x, p_y and p_z. Show that if the particle moves only in the *x* - *y* plane the angular momentum has only a -component.

SOLUTION:

We know that angular momentum $\vec{l}$ of a particle having position vector $\vec{r}$ and momentum $\vec{p}$ is given by

$$\vec{l} = \vec{r}\times\vec{p}$$

But

$\vec{r} = [x\hat{i} + y\hat{j} + z\hat{k}]$, where *x,y,z are the components of vector r.*

$$\vec{l} = \vec{r}\times\vec{p} = [x\hat{i} + y\hat{J} + z\hat{k}]\times\left[p_x\hat{i} + p_y\hat{J} + p_z\hat{k}\right]$$

or

$$\left(l_x\hat{i} + l_y\hat{J} + l_z\hat{k}\right) = \begin{vmatrix} \hat{i} & \hat{J} & \hat{k} \\ x & y & z \\ p_x & p_y & p_z \end{vmatrix}$$

$$= \left(yp_z - zp_y\right)\hat{i} + \left(zp_x - xp_z\right)\hat{J} + \left(xp_y - yp_x\right)\hat{k}$$

From this relation, we conclude that-

$l_x = yp_z - zp_y, l_y = zp_x - xp_z$ and $l_z = xp_y - yp_x$

If the given particle moves only in the *x* - *y* plane, then $z = 0$ and $p_z = 0$ and hence, $\vec{l} = \left(xp_y - yp_x\right)\hat{k}$, which is only the z-comonent of $\vec{i}$

It means that for a particle moving only in the *x* - *y* plane, the angular momentum has only the z component

6.7 Two particles, each of mass m and speed v, travel in opposite directions along parallel lines separated by a distance d. Show that the angular momentum vector of the two-particle system is the same whatever be the point about which the angular momentum is taken.

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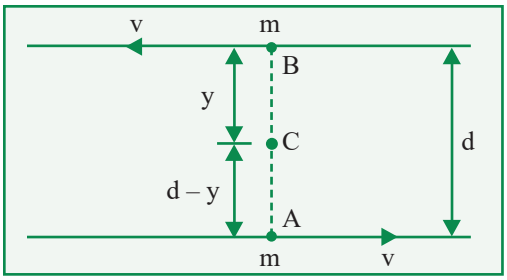
SOLUTION:



Angular momentum about A-

$L_A = mv \times 0 + mv \times d$

$L_A = mvd$

Angular momentum about B-

$L_B = mv \times d + mv \times 0$

$L_B = mvd$

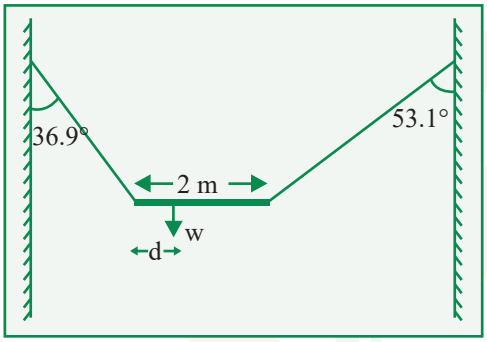
Angular momentum about *C*,

$L_C = mv \times y + mv \times (d - y) = mvd$

In all the three cases, the direction of angular momentum is the same.

$\therefore \vec{L}_A = \vec{L}_B = \vec{L}_C$

6.8 A non-uniform bar of weight W is suspended at rest by two strings of negligible weight as shown in Fig. The angles made by the strings with the vertical are 36.9° and 53.1° respectively. The bar is 2 m long. Calculate the distance d of the centre of gravity of the bar from its left end.



SOLUTION:

Given- $\theta_1 = 36.9°$, $\theta_2 = 53.1°$ (From figure)

Need to find – distance (d)

If T_1, T_2 are the tensions in the two strings, then for equilibrium along the horizontal,

$$T_1 \sin\theta_1 = T_2 \sin\theta_2$$

$$\frac{T_1}{T_2} = \frac{\sin\theta_2}{\sin\theta_1} = \frac{\sin 53.1^\circ}{\sin 36.9^\circ}$$

$$\frac{T_1}{T_2} = \frac{0.7404}{0.5477} = 1.3523$$

Let *d* be the distance of centre of gravity *C* of the bar from the left end.

For rotational equilibrium about *C*,

$T_1 \cos\theta_1 \times d = T_2 \cos\theta_2 (2 - d)$

$T_1 \cos 36.9° \times d = T_2 \cos 53.1° (2 - d)$

$T_1 \times 0.8366d = T_2 \times 0.6718\ (2 - d)$

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Put

$T_1 = 13523\ T_2$ and solve to get

$d = 0.745\ m$

6.9 A car weighs 1800 kg. The distance between its front and back axles is 1.8 m. Its centre of gravity is 1.05 m behind the front axle. Determine the force exerted by the level ground on each front wheel and each back wheel.

SOLUTION:

Given – mass of car(m) = 1800 kg, distance between front and back axles (d) = 1.8m

Need to find – force exerted by level ground (F)

Let F_1 and F_2 be the forces exerted by the level ground on front wheels and back wheels respectively.

Considering rotational equilibrium about the front wheels,

$F_2 \times 1.8 = mg \times 1.05$

$F_2 = 1.05/1.8 \times 1800 \times 9.8$ N = 10290 N

Force on each back wheel is =10290/2 N or 5145 N.

Considering rotational equilibrium about the back wheels.

$F_1 \times 1.8 = mg(1.8 - 1.05) = 0.75 \times 1800 \times 9.8$

$F_1 = 0.75 \times 1800 \times 9.8/1.8 = 7350$ N

Force on each front wheel is 7350/2 N or 3675 N .

6.10 Torques of equal magnitude are applied to a hollow cylinder and a solid sphere, both having the same mass and radius. The cylinder is free to rotate about its standard axis of symmetry, and the sphere is free to rotate about an axis passing through its centre. Which of the two will acquire a greater angular speed after a given time.

SOLUTION:

Let m and r be the respective masses of the hollow cylinder and the solid sphere.

The moment of inertia of the hollow cylinder about its standard axis, $I_I = mr^2$

The moment of inertia of the solid sphere about an axis passing through itscentre,

$$I_{II} = \frac{2}{5}mr^2$$

We have the relation:

$\tau = I\alpha$

Where, $\alpha =$ Angular acceleration $\tau =$ Torque

$I =$ Moment of inertia

For the hollow cylinder, $\tau_1 = I_1\alpha_1$

For the solid sphere, $\tau_{11} = I_{11}\alpha_{11}$

As an equal torque is applied to both the bodies, $\tau_1 = \tau_2$

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$$\therefore \frac{\alpha_{II}}{\alpha_1} = \frac{I_I}{I_{II}} = \frac{mr^2}{\frac{2}{5}mr^2} = \frac{2}{5}$$

$\alpha_{11} > \alpha_1$

Now, using the relation:

$\omega = \omega_0 + \alpha t$

Where,

$\omega_0 =$ Initial angular velocity

$t =$ Time of rotation

$\omega =$ Final angular velocity

For equal ω_0 and t, we have:

$\omega \propto \alpha \ldots$ (ii)

From equations (i) and (ii), we can write:

$\omega II > \omega I$

Hence, the angular velocity of the solid sphere will be greater than that of the hollow cylinder.

6.11 A solid cylinder of mass 20 kg rotates about its axis with angular speed 100 rads^{-1}. The radius of the cylinder is 0.25 m. What is the kinetic energy associated with the rotation of the cylinder? What is the magnitude of angular momentum of the cylinder about its axis?

SOLUTION:

Given – mass of the solid cylinder (M) =20 kg, Angular speed ω = 100 rads^{-1}; R = 0.25 m

Need to find – kinetic energy (E_Γ)

Moment of inertia of the cylinder about its axis

$$I = \frac{1}{2MR^2}$$

I = 1/2 × 20(0.25)^2 kg m^2 = 0.625 kg m^2

Rotational kinetic energy,

$$E_\Gamma = \frac{1}{2} Iw2$$

$$E_\Gamma = 1/2 \times 0.625 \times (100)^2 \text{ J} = 3125\text{J}$$

Angular momentum,

L = I w

L = 0.625 × 100 Js = 62.5 Js

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6.12 (a) A child stands at the centre of a turntable with his two arms outstretched. The turntable is set rotating with an angular speed of 40rev/min. How much is the angular speed of the child if he folds his hands back and thereby reduces his moment of inertia to 2/5 times the initial value? Assume that the turntable rotates without friction.

(b) Show that the child's new kinetic energy of rotation is more than the initial kinetic energy of rotation. How do you account for this increase in kinetic energy?

SOLUTION:

(a) Suppose, initial moment of inertia of the child is I_1 Then final moment of inertia,

$$I_2 = \frac{2}{5} I_1$$

$$v_1 = 40 \text{ rev min}^{-1}$$

By using the principle of conservation of angular momentum, we get

or

$$I_1\omega_1 = I_2\omega_2 \text{ or } I_1(2\pi v_1) = I_2(2\pi v_2)$$

$$v_2 = \frac{I_1 v_1}{I_2} = \frac{I_1 \times 40}{\frac{2}{5} \times I_1} = 100 rev \min^{-1}$$

(b)

$$\frac{\text{Final K.E. of rotation}}{\text{Initial K.E. of rotation}} = \frac{\frac{1}{2} I_2 \omega_2^2}{\frac{1}{2} I_1 \omega_1^2} = \frac{\frac{1}{2} I_2 (2\pi v_2)^2}{\frac{1}{2} I_1 (2\pi v_1)^2} = \frac{I_2 v_2^2}{I_1 v_1^2} = \frac{\frac{2}{5} I_1 \times (100)^2}{\frac{2}{5} I_1 \times (40)^2} = 2.5$$

Clearly, final (K.E.) rot becomes more because the child uses his internal energy when he folds his hands to increase the kinetic energy.

6.13 A rope of negligible mass is wound round a hollow cylinder of mass 3 kg and radius 40 cm. What is the angular acceleration of the cylinder if the rope is pulled with a force of 30 N? What is the linear acceleration of the rope? Assume that there is no slipping.

SOLUTION:

Given – mass of the cylinder (M) = 3kg, Radius (r) = 40 cm = 0.4 m

Need to find – α (angular acceleration)

Moment of inertia of the hollow cylinder about its axis.

$I = MR^2 = 3(0.4)^2 = 0.48$ kg m^2

Force applied $F = 30$ N

∴ Torque, $\tau = F \times R = 30 \times 0.4 = 12$ N-m.

If α is angular acceleration produced, then from $\tau = I\alpha$

$$\alpha = \frac{\tau}{I} = \frac{12}{0.48} = 25 rad s^{-2}$$

Linear acceleration, $a = R\alpha = 0.4 \times 25 = 10\ ms^{-2}$

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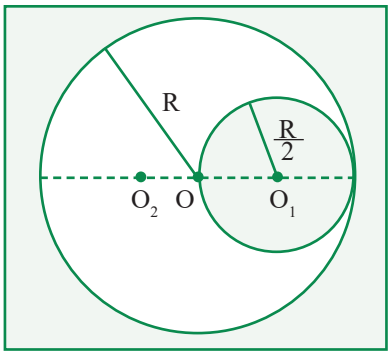
6.14 To maintain a rotor at a uniform angular speed of 200 rads^{-1}, an engine needs to transmit a torque of 180 N m. What is the power required by the engine? (Note: uniform angular velocity in the absence of friction implies zero torque. In practice, applied torque is needed to counter frictional torque). Assume that the engine is 100% efficient.



SOLUTION:

Given – angular speed (w) = 200 rads^{-1}; Torque, τ =180 N-m

Need to find – power required by the engine (P)

Since, Power, P = Torque (τ) × angular speed (w)

P =180 × 200 = 36000 watt = 36 KW

6.15 From a uniform disk of radius R, a circular hole of radius R/2 is cut out. The centre of the hole is at R/2 from the centre of the original disc. Locate the centre of gravity of the resulting flat body.

SOLUTION:

Let from a bigger uniform disc of radius R with centre 0 a smaller circular hole of radius R/2 with its centre at O_1 (where $R0O_1 = R/2$) is cut out. Let centre of gravity or the centre of mass of remaining flat body be at O_2, where $OO_2 = x$. If σ be mass per unit area, then mass of whole disc $M_1 = \pi R^2\sigma$ and mass of cut out part

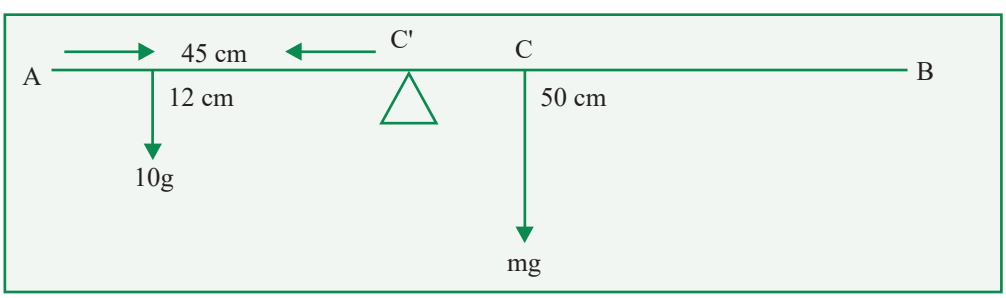
$$M_2 = \pi\left(\frac{R}{2}\right)^2\sigma = \frac{1}{4}\pi R^2\sigma = \frac{M_1}{4}$$

$$\therefore \quad x = \frac{M_1 \times (0) - M_2(OO_1)}{M_1 - M_2} = \frac{0 - \frac{M_1}{4} \times \frac{R}{2}}{M_1 - \frac{M_1}{4}} = -\frac{R}{6}$$

i.e., O_2 is at a distance R/6 from centre of disc on diametrically opposite side to centre of hole.

6.16 A metre stick is balanced on a knife edge at its centre. When two coins, each of mass 5 g are put one on top of the other at the 12.0 cm mark, the stick is found to be balanced at 45.0 cm. What is the mass of the metre stick?

SOLUTION:



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Given – mass of each coin (m)= 5g,

Need to find – mass of the metre stick (M)

Let m be the mass of the stick concentrated at C, the 50 cm mark, see fig.

For equilibrium about C, the 45 cm mark,

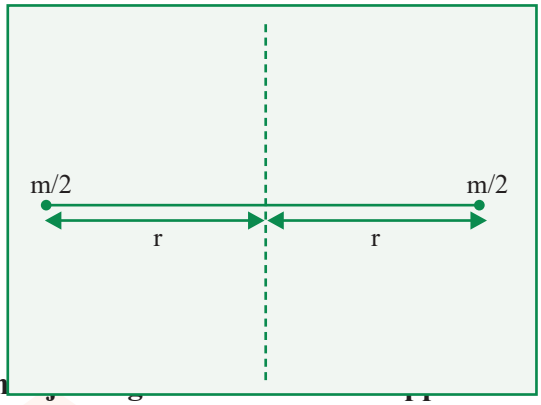
Anticlockwise toque = clockwise torque

10g(45 – 12) = mg(50 – 45)

10g × 33 = mg × 5

m =10 × 33/5

m = 66 grams



6.17 The oxygen molecule has a mass of 5.30 × 10^{-26} kg and a about an axis through its centre perpendicular to the lin mean speed of such a molecule in a gas is 500 m/s and that its kinetic energy of rotation is two thirds of its kinetic energy of translation. Find the average angular velocity of the molecule.

SOLUTION:

Given – mass of oxygen molecule (m) = 5.30 × 10^{-26} kg moment of inertia (I) =, 1.94 × 10^{-46} kg m^2 speed (v) = 500 m/s

Need to find – average angular velocity (ω)

If $\frac{m}{2}$ is mass of each atom of oxygen and 2r is distance between the two atoms as shown in Fig, then

$$I = \frac{m}{2}r^2 + \frac{m}{2}r^2 = mr^2$$

$$r = \sqrt{\frac{I}{m}} = \sqrt{\frac{1.94\times10^{-46}}{5.30\times10^{-26}}}$$

$$= 0.61\times10^{-10}\,\text{m}$$

As K.E. of rotation $= \frac{2}{3}$ K.E. of translation

$$\therefore \frac{1}{2}I\omega^2 = \frac{2}{3}\times\frac{1}{2}m\omega^2$$

$$\omega = \sqrt{\frac{2}{3}}\frac{v}{r}$$

$$\omega = \sqrt{\frac{2}{3}}\times\frac{500}{0.61\times10^{-10}} = 6.7\times10^{12}\,rad/\text{s}$$

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