GRAVITATION

7.1 Answer the following:

(a) You can shield a charge from electrical forces by putting it inside a hollow conductor. Can you shield a body from the gravitational influence of nearby matter by putting it inside a hollow sphere or by some other means?

(b) An astronaut inside a small spaceship orbiting around the earth cannot detect gravity. If the space station orbiting around the earth has a large size, can he hope to detect gravity?

(c) If you compare the gravitational force on the earth due to the sun to that due to the moon, you would find that the Sun's pull is greater than the moon's pull. (you can check this yourself using the data available in the succeeding exercises). However, the tidal effect of the moon's pull is greater than the tidal effect of sun. Why?

SOLUTION:

(a) No. Gravitational forces are independent of medium. A body cannot be shielded from the gravitational influence of nearby matter.

(b) Yes. If the size of the spaceship is extremely large, then the gravitational effect of the spaceship may become measurable. The variation in g can also be detected.

(c) Tidal effect depends inversely on the cube of the distance, unlike force which depends inversely on the square of the distance. Since the distance of moon from the ocean water is very small as compared to the distance of sun from the ocean water on earth. Therefore, the tidal effect of Moon's pull is greater than the tidal effect of the sun.

7.2 Choose the correct alternative :

(a) Acceleration due to gravity increases/decreases with increasing altitude.

(b) Acceleration due to gravity increases/decreases with increasing depth (assume the earth to be a sphere of uniform density).

(c) Acceleration due to gravity is independent of mass of the earth/mass of the body.

(d) The formula - GMm $(1/r_2 - 1/r_1)$ is more/less accurate than the formula mg $(r_2 - r_1)$ for the difference of potential energy between two points r_2 and r_1 distance away from the centre of the earth.

SOLUTION:

(a) decreases
(b) decreases
(c) mass of the body
(d) more

7.3 Suppose there existed a planet that went around the Sun twice as fast as the earth. What would be its orbital size as compared to that of the earth?

SOLUTION:

T_e :time taken by earth to revolve around earth, T_p :time taken by earth to revolve around earth

$T_e = 1$ year

$$T_p = \frac{T_e}{2} = \frac{1}{2} \text{ year}; r_c = 1\text{A.U.}$$

Using Kepler's third law, we have: $r_p = r_e\left(\frac{T_p}{T_e}\right)^{2/3}$

$$\Rightarrow r_p = 1\left(\frac{1/2}{1}\right)^{2/3} = 0.63AU$$

7.4 One of the satellites of Jupiter, has an orbital period of 1.769 days and the radius of the orbit is 4.22×10^8 m. Show that the mass of Jupiter is about one-thousandth that of the sun.

SOLUTION:

Given –T_1 = 1.769 days =1.769 × 24 × 60 × 60 s, Radius of the orbit of satellite, $r_1 = 4.22 \times 10^8$ m

Need to find – mass of Jupiter is 1/1000 times that of the sun

For a satellite of Jupiter, orbital period, Mass of Jupiter, M_1 is given by

$$M_1 = \frac{4\pi^2 r_1^3}{GT_1^2} = \frac{4\pi^2 \times \left(4.22\times10^8\right)^3}{G\times(1.769\times24\times60\times60)^2}$$

orbital radius,. r =1 ^ U. = 1.496×10^{11} m.

Mass of sun is given by $M = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2 \times \left(1.496\times10^{11}\right)^3}{G\times(365.25\times24\times60\times60)^2}$

Dividing M by M_1

$$\frac{M}{M_1} = \frac{4\pi^2 \times \left(1.496\times10^{11}\right)^3}{G\times(365.25\times24\times60\times60)^2} \times \frac{G\times(1.769\times24\times60\times60)^2}{4\pi^2 \times \left(4.22\times10^8\right)^3} = 1046$$

$$\frac{M_1}{M} = \frac{1}{1046} \simeq \frac{1}{1000} \Rightarrow M_1 = \frac{1}{1000}\text{M}$$

Hence proved.

7.5 Let us assume that our galaxy consists of 2.5×10^{11} stars each of one solar mass. How long will a star at a distance of 50,000ly from the galactic centre take to complete one revolution? Take the diameter of the Milky Way to be 10^5ly.

SOLUTION:

Given – distance of the star (r) = 50000ly = $50000 \times 9.46 \times 10^{15}$ m = 4.73×10^{20} m

Need to find – time(T) taken for one revolution

Answer:

$M = 2.5 \times 10^{11}$ x solar mass.

$M = 2.5 \times 10^{11} \times (2 \times 10^{30})$ kg $= 5.0 \times 10^{41}$ kg

Also-

$$M = \frac{4\pi^2 r^3}{GT^2}$$

$$T = \left(\frac{4\pi^2 r^3}{GM}\right)^{1/2} = \left[\frac{4 \times (22/7)^2 \times \left(4.73 \times 10^{20}\right)^3}{\left(6.67 \times 10^{-11}\right) \times \left(5.0 \times 10^{41}\right)}\right]^{1/2}$$

$$T = 1.12 \times 10^{16}\,\text{s}$$

7.6 Choose the correct alternative:

(a) If the zero of potential energy is at infinity, the total energy of an orbiting satellite is negative of its kinetic/potential energy.

(b) The energy required to launch an orbiting satellite out of earth's gravitational influence is more/less than the energy required to project a stationary object at the same height (as the satellite) out of earth's influence.

SOLUTION:

(a) If the zero of potential energy is at infinity, the total energy of an orbiting satellite is negative of its kinetic energy.

(b) The energy required to launch an orbiting satellite out of Earth's gravitational influence is less than the energy required to project a stationary object at the same height (as the satellite) out of Earth's influence.

7.7 Does the escape speed of a body from the earth depend on

(a) the mass of the body,

(b) the location from where it is projected,

(c) the direction of projection,

(d) the height of the location from where the body is launched?

SOLUTION:

The escape speed $v_{es} = \sqrt{\dfrac{2GM}{R}} = \sqrt{2gR}$

(a) The escape speed of a body from the Earth does not depend on the mass of the body.

(b) The escape speed does not depend on the location from where a body is projected.

(c) The escape speed does not depend on the direction of projection of a body.

(d) The escape speed of a body depends upon the height of the location from where the body is projected, because the escape velocity depends upon the gravitational potential at the point from which it is projected and this potential depends upon height also.

7.8 A comet orbits the sun in a highly elliptical orbit. Does the comet have a constant

(a) linear speed,

(b) angular speed,

(c) angular momentum,

(d) kinetic energy,

(e) potential energy,

(f) total energy throughout its orbit? Neglect any mass loss of the comet when it comes very close to the Sun.

SOLUTION:

(a) The linear speed of the comet is variable in accordance with Kepler's second law. When comet is near the sun, its speed is higher. When the comet is far away from the sun, its speed is very less.

(b) Angular speed also varies slightly.

(c) Comet has constant angular momentum.

(d) Kinetic energy does not remain constant.

(e) Potential energy varies along the path.

(f) Total energy throughout the orbit remains constant.

7.9 Which of the following symptoms is likely to afflict an astronaut in space

(a) swollen feet,

(b) swollen face,

(c) headache,

(d) orientational problem.

SOLUTION:

(a) The blood flow in feet would be lesser in zero gravity. So, the astronaut will not get swollen feet.

(b) In the conditions of weightlessness, the face of the astronaut is expected to get more supply. Due to it, the astronaut may develop swollen face.

(c) Due to more blood supply to face, the astronaut may get headache.

(d) Space has diverse orientations. Therefore, orientational difficulty can affect an astronaut in space.

7.10 In the following two exercises, choose the correct answer from among the given ones: The gravitational intensity at the centre of a hemispherical shell of uniform mass density has the direction indicated by the arrow (see Fig) (i) a, (ii) b, (iii) c, (iv) 0 .

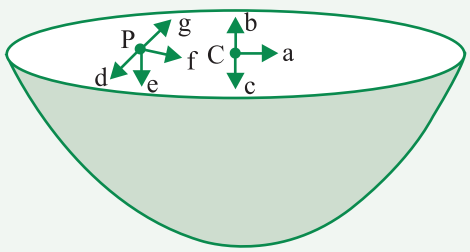

Fig.

SOLUTION:

At all points inside a hollow spherical shell, potential is same. So, gravitational intensity, which is negative of gravitational potential gradient, is zero. Due to zero gravitational intensity, the gravitational forces acting on any particle at any point inside a spherical shell will be symmetrically placed. It follows from here that if we remove the upper hemispherical shell, the net gravitational force acting on a particle at will be downwards. Since gravitational intensity is gravitational force per unit mass therefore, the direction of gravitational intensity will be along c. So, option (iii) is correct.

7.11 For the above problem, the direction of the gravitational intensity at an arbitrary point *P* is indicated by the arrow (i) d, (ii) e, (iii) f, (iv) g.

SOLUTION:

Using the explanation given in the solution of the previous problem, the direction of the gravitational field intensity at P will be along e. So, option (ii) is correct.

7.12 A rocket is fired from the earth towards the sun. At what distance from the earth's centre is the gravitational force on the rocket zero? Mass of the sun = 2×10^{30} kg, mass of the earth = 6×10^{24} kg Neglect the effect of other planets etc. (orbital radius = 1.5×10^{11} m).

SOLUTION:

Given – Mass of Sun, M = 2×10^{30} kg; Mass of Earth, m = 6×10^{24} kg Distance between Sim and Earth, r = 1.5×10^{11} m

Need to find – distance from the earth's centre (d)

Let at the point P, the gravitational force on the rocket due to Earth = gravitational force on the rocket due to Sun

Let x = distance of the point P from the Earth

Then $\dfrac{Gm}{x^2} = \dfrac{GM}{(r-x)^2}$

$$\Rightarrow \frac{(r-x)^2}{x^2} = \frac{M}{m} = \frac{2\times10^{30}}{6\times10^{24}} = \frac{10^6}{3}$$

$$\frac{r-x}{x} = \frac{10^3}{\sqrt{3}} \Rightarrow \frac{r}{x} = \frac{10^3}{\sqrt{3}} + 1 \simeq \frac{10^3}{\sqrt{3}}$$

$$x = \frac{\sqrt{3}r}{10^3} = \frac{1.732\times1.5\times10^{11}}{10^3} = 2.6\times10^8\,\text{m}$$

7.13 How will you 'weigh the sun', that is estimate its mass? The mean orbital radius of the earth around the sun is 1.5 × 10^8 km.

SOLUTION:

Given – The mean orbital radius of the Earth around the Sun (R) = 1.5 × 10^8 km (R) = 1.5 × 10^{11} m

Need to find – mass of the sun (M)

Time period, T = 365.25 × 24 × 60 × 60 s

Let the mass of the Sun be M and that of Earth be .

According to law of gravitation

$$F = G\frac{Mm}{R^2}$$

Centripetal force,

$$F = \frac{mv^2}{R} = m\cdot R\cdot\omega^2$$

Combining both equations-

$$\frac{GMm}{R^2} = m\cdot R\cdot\omega^2$$

$$= \frac{mR\cdot4\pi^2}{T^2}$$

$$\therefore M = \frac{4\pi^2R^3}{G\cdot T^2} \qquad \left[\because \omega = \frac{2\pi}{T}\right]$$

$$M = \frac{4\times(3.14)^2\times\left(1.5\times10^{11}\right)^3}{6.67\times10^{-11}\times(365.25\times24\times60\times60)^2}$$

$$M = 2.009\times10^{30}\,kg = 2.0\times10^{30}\,kg.$$

7.14 A Saturn year is 29.5 times the earth year. How far is the Saturn from the sun if the earth is 1.50 × 10^8 km away from the sun?

SOLUTION:

We know that $T^2 \propto R^3$

$$\therefore \frac{T_s^2}{T_e^2} = \frac{R_s^3}{R_e^3}$$

where subscripts s and e refer to the Saturn and Earth respectively.

Now

$$\frac{T_s}{T_e} = 29.5 \text{ [given]}; \; R_e = 1.50 \times 10^8 \, km$$

$$\left(\frac{R_s}{R_e}\right)^3 = \left(\frac{T_s}{T_e}\right)^2$$

$$R_s = R_e \times \left[(29.5)^2\right]^{1/3} = 1.50 \times 10^8 \times (870.25)^{1/3} \, km$$

$$R_s = 1.43 \times 10^9 \, km = 1.43 \times 10^{12} \, \text{m}$$

$\therefore$ Distance of Saturn from Sun = 1.43×10^{12} m.

7.15 A body weighs 63 N on the surface of the earth. What is the gravitational force on it due to the earth at a height equal to half the radius of the earth?

SOLUTION:

Given – weight of the body = 63N

Need to find – gravitational force (F)

Answer: Let g_h be the acceleration due to gravity at a height equal to half the radius of the Earth (h = R/2) and g its value on Earth's surface. Let the body have mass m .

We know that

$$\frac{g_h}{g} = \left(\frac{R}{R+h}\right)^2 \text{ or } \frac{g_h}{g} = \left(\frac{R}{R+\frac{R}{2}}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

Let W be the weight of body on the surface of Earth and W_h the weight of the body at height h.

Then,

$$\frac{W_h}{W} = \frac{mg_h}{mg} = \frac{g_h}{g} = \frac{4}{9}$$

$$W_h = \frac{4}{9} \text{W} = \frac{4}{9} \times 63\text{N} = 28\text{N}$$

7.16 Assuming the earth to be a sphere of uniform mass density, how much would a body weigh halfway down to the centre of the earth if it weighed 250 N on the surface?

SOLUTION:

Using

$$g_d = g\left(1 - \frac{d}{R}\right)$$

$$\Rightarrow mg_d = mg\left(1 - \frac{d}{R}\right)$$

Here

$$d = \frac{R}{2}$$

$$\therefore mg_d = (250) \times \left(1 - \frac{R/2}{R}\right) = 250 \times \frac{1}{2} = 125\text{N}$$

7.17 A rocket is fired vertically with a speed of 5 km s^{-1} from the earth's surface. How far from the earth does the rocket go before returning to the earth? Mass of the earth = 6.0 × 10^{24} kg; mean radius of the earth = 6.4 ×10^6 m; G = 6.67×10^{-11}Nm^2 kg^{-2}).

SOLUTION:

Given – speed of the rocket (v) = 5 km/s Mass of the earth = 6.0 × 10^{24} kg; mean radius of the earth = 6.4 ×10^6 m; G = 6.67×10^{-11} Nm^2 kg^{-2}

Need to find – distance rocket go before returning to the earth (s)

Initial kinetic energy of rocket = 1/2 mv^2 = 1/2 × m × $(5000)^2$ = 1.25 × 10^7 mJ

At distance r from centre of earth, kinetic energy becomes zero

∴ Change in kinetic energy 1.25 × 10^7 – 0 = 1.25 × 10^7 mJ

This energy changes into potential energy.

Initial potential energy at the surface of earth = GM_e m/r

$$= \frac{-\left(6.67\times10^{-11}\right)\times\left(6\times10^{24}\right)\text{m}}{6.4\times10^{6}} = -6.25\text{m}\times10^{7}\,\text{J}$$

Final potential energy at distance, $r = -\frac{GM_e m}{r}$

$$r = \frac{-\left(6.67\times10^{-11}\right)\times\left(6\times10^{24}\right)m}{r} = -4\times10^{14}\frac{\text{m}}{r}\text{J}$$

∴ Change in potential energy $= 6.25\times10^{7}\,\text{m} - 4\times10^{14}\frac{\text{m}}{r}$

Using law of conservation of energy,

Change in potential energy = change in kinetic energy

$$6.25\times10^{7}\,\text{m} - \frac{4\times10^{14}\,\text{m}}{r} = 1.25\times10^{7}\,\text{m}$$

$$r = \frac{4\times10^{14}}{5\times10^{7}}m = 8\times10^{16}\,\text{m}$$

7.18 The escape speed of a projectile on the earth's surface is. 11.2 km s^{-1} A body is projected out with thrice this speed. What is the speed of the body far away from the earth? Ignore the presence of the sun and other planets.

SOLUTION:

Given – escape speed from surface of Earth

(v_{es}) = 11.2 kg s^{-1} = 11.2 × 10^3 m s^{-1}

Need to find – speed of the body far away from the earth(s)

Using expression-

$$\frac{1}{2}mv_e^2 = \frac{GMm}{R^2}$$

When a body is projected with a speed $v_i = 3v_{es} = 3 \times 11.2 \times 10^3$ *m/s*, then it will have a final speed v_f such that

$$\frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 - \frac{GMm}{R^2} = \frac{1}{2}mv_i^2 - \frac{1}{2}mv_e^2$$

$$\Rightarrow v_f = \sqrt{v_i^2 - v_e^2}$$

$$v_f = \sqrt{\left(3\times 11.2\times 10^3\right) - \left(11.2\times 10^3\right)^2}$$

$$v_f = 11.2\times 10^3 \times \sqrt{8}$$

$$v_f = 31.7\times 10^3 ms^{-1} \text{ or } 31.7kms^{-1}$$

7.19 A satellite orbits the earth at a height of 400 km above the surface. How much energy must be expended to rocket the satellite out of the earth's gravitational influence? Mass of the satellite; mass of the earth = 6.4 × 10^{24} kg; radius of the earth = 6.4 × 10^6 m; *G* = 6.67 × 10^{-11} N m^2 kg^{-2}

SOLUTION:

Given – height (h) = 400 km, Mass of the satellite = 200 kg; mass of the earth = 6.4 × 10^{24} kg radius of the earth = 6.4 × 10^6 m; *G* = 6.67 × 10^{-11} N m^2 kg^{-2}.

Need to find – Energy(E)

Total energy of orbiting satellite at a height h, E(o)

$$E(o) = -\frac{GMm}{(R+h)} + \frac{1}{2}mv^2 = -\frac{GMm}{(R+h)} + \frac{1}{2}m\frac{GM}{(R+h)}$$

$$E(o) = -\frac{GMm}{2(R+h)}$$

Energy expended to rocket the satellite out of the earth's gravitational field (E)

E = - (total energy of the orbiting satellite)

$$E = \frac{GMm}{2(R+h)} = \frac{\left(6.67\times 10^{-11}\right)\times\left(6\times 10^{24}\right)\times 200}{2\times\left(6.4\times 10^6 + 4\times 10^5\right)}$$

E = 5.9 × 10^{9}J

7.20 **Two stars each of one solar mass (= 2 × 10^{30} kg) are approaching each other for a head on collision. When they are a distance 10^9 km, their speeds are negligible. What is the speed with which they collide? The radius of each star is 10^4 km. Assume the stars to remain undistorted until they collide. (Use the known value of *G*).**

SOLUTION:

Given – mass of each star, M = 2 × 10^{30} kg, Initial potential between two stars, r = 10^9 km = 10^{12} m

Need to find – speed with which they collide (v)

Initial potential energy of the system = -GMm/r

Total K.E. of the stars = 1/2 Mv^2 + 1/2 Mv^2

When the stars are about to collide, the distance between their centres, $r' = 2R$.

∴ Final potential energy of two stars = -GMm/2R

Since gain in K.E. is at the cost of loss in P.E

$$Mv^2 = -\frac{GMM}{r} - \left(-\frac{GMM}{2R}\right)$$

$$Mv^2 = -\frac{GMM}{r} + \frac{GMM}{2R}$$

$$2\times10^{30}v^2 = -\frac{6.67\times10^{-11}\times\left(2\times10^{30}\right)^2}{10^{12}} + \frac{6.67\times10^{-11}\times\left(2\times10^{30}\right)^2}{2\times10^7}$$

$2 \times 10^{30}\ v^2 = -2.668 \times 10^{38} + 1.334 \times 10^{43}$

$$v = \sqrt{\frac{1.334\times10^{43}}{2\times10^{30}}} = 2.583\times10^6\, ms^{-1}.$$

7.21 **Two heavy spheres each of mass 100 kg and radius 0.10 m are placed 1.0 m apart on a horizontal table. What is the gravitational force and potential at the mid-point of the line joining the centres of the spheres? Is an object placed at that point in equilibrium? If so, is the equilibrium stable or unstable?**

SOLUTION:

Given – -G = 6.67 × 10^{-11} $Nm^2\ kg^{-2}$; M (mass of spheres) = 100 kg; R (radius) = 0.1 m,
distance between the two spheres, d =1.0 m

Need to find – gravitational force (F)

Answer: Suppose that the distance of either sphere from the mid-point of the line joining their centre is r. Then r = d/2 = 0.5 m. The gravitational field at the mid-point due to two spheres will be equal and opposite.

Hence, the resultant gravitational field at the mid-point = 0

The gravitational potential at the mid point $= \left(-\frac{GM}{r}\right)\times 2$

$$V = -\frac{6.67\times10^{-11}\times100\times2}{0.5} = -2.668\times10^{-8}\, Jkg^{-1}$$

The object placed at that point would be in stable equilibrium.