CHAPTER 8

MECHANICAL PROPERTIES OF SOLIDS

CLASS 11TH

NCERT EXERCISE AND SOLUTIONS - PHYSICS

8.1 A steel wire of length 4.7 m and cross-sectional area 3.0×10^{-5} m² stretches by the same amount as a copper wire of length 3.5 m and cross-sectional area of 4.0×10^{-5} m² under a given load. What is the ratio of the Young's modulus of steel to that of copper?

SOLUTION:

Given – FOR STEEL: l_1 (length) = 4.7 m, A_1 (cross sectional area) = 3.0×10^{-5} m²

FOR COPPPER: l_2 (length) = 3.5 m, A_2 (cross sectional area) = 4.0×10^{-5} m²

Need to find – ratio of the Young's modulus of steel to that of copper

If F newton is the stretching force and Δl metre the extension in each case, then

For steel-

$$Y_1 = -\frac{Fl_1}{A_1 \Delta l}$$

$$\Rightarrow Y_1 = \frac{F \times 4.7}{3.0 \times 10^{-5} \times \Delta l}$$

For copper-

$$Y_2 = \frac{F \times 3.5}{4.0 \times 10^{-5} \times \Delta l}$$

Dividing Y_1 by Y_2:

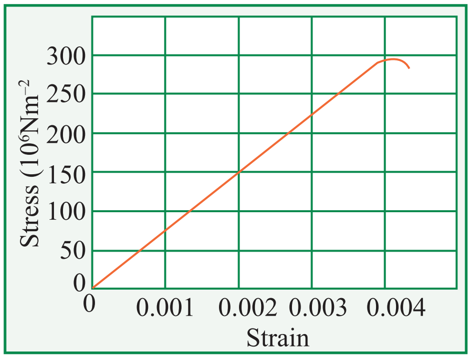
$$\frac{Y_1}{Y_2} = \frac{4.7}{3.0 \times 10^{-5}} \times \frac{4.0 \times 10^{-5}}{3.5} = \frac{4.7 \times 4.0}{3.0 \times 3.5} = 1.79$$

Therefore, the ratio is 1.79.

8.2 Figure shows the strain-stress curve for a given material. What are

(a) Young's modulus and

(b) approximate yield strength for this material?



SOLUTION:

(a) Young's modulus of the material (Y) is given by

Y = Stress/Strain

$Y = 150 \times 10^6 / 0.002$

$Y = 75 \times 10^9\ Nm^{-2}$

$Y = 7.5 \times 10^{10}\ Nm^{-2}$

(b) Yield strength of a material is defined as the maximum stress it can sustain. From graph, the approximate yield strength of the given material $300 \times 10^6\ Nm^{-2}$.

8.3 The stress-strain graphs for materials A and B are shown in Fig.

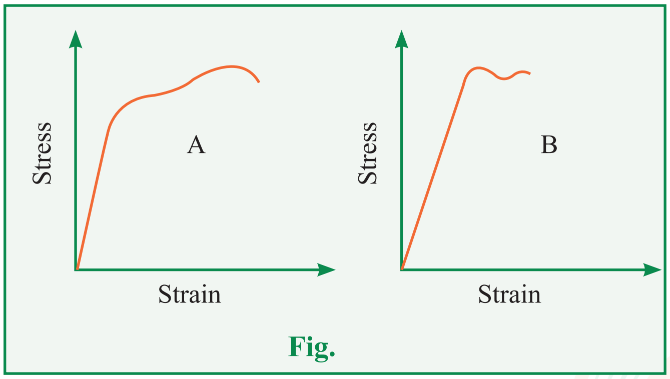


Fig.

The graphs are drawn to the same scale.

(a) Which of the materials has the greater Young's modulus?

(b) Which of the two is the stronger material?

SOLUTION

(a) From the two graphs we note that for a given strain, stress for A is more than that of B. Hence Young's modulus = (Stress / Strain is greater for A than that of B).

(b) Strength of a material is determined by the amount of stress required to cause fracture. This stress corresponds to the point of fracture. The stress corresponding to the point of fracture in A is more than for B. So, material A is stronger than material B.

8.4 Read the following two statements below carefully and state, with reasons, if it is true or false.

(a) The Young's modulus of rubber is greater than that of steel;

(b) The stretching of a coil is determined by its shear modulus.

SOLUTION:

(a) False. The-Young's modulus is defined as the ratio of stress to the strain within elastic limit. For a given stretching force elongation is more in rubber and quite less in steel. Hence, rubber is less elastic than steel.

(b) True. Stretching of a coil is determined by its shear modulus. When equal and opposite forces are applied at opposite ends of a coil, the distance, as well as shape of helicals of the coil change and it, involves shear modulus.

8.5 Two wires of diameter 0.25 cm, one made of steel and the other made of brass are loaded as shown in Fig. The unloaded length of steel wire is 1.5 m and that of brass wire is 1.0 m. Compute the elongations of the steel and the brass wires.

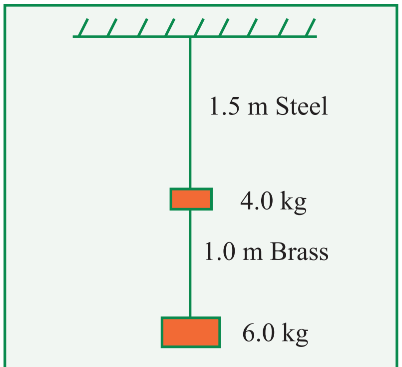


Fig.

SOLUTION:

Given – STEEL: diameter (d_1) = 0.25cm, radius (r_1) = d/2 = 0.125cm, unloaded length (L) = 1.5m

BRASS: diameter (d_2) = 0.25cm, radius (r_2) = d/2 = 0.125cm, unloaded length (L) = 1m

Need to find – elongations for steel(Δl_1) and brass(Δl_2)

For steel wire; total force on steel wire:

$F_1 = 4 + 6 = 10\ \text{kg}f = 10 \times 9.8$ N

$Y_1 = 2.0 \times 10^{11}$ Pa

For brass wire,

$F_2 = 6.0\ \text{kg}f = 6 \times 9.8$ N

$Y_2 = 0.91 \times 10^{11}$ Pa

Since,

$$Y_1 = \frac{F_1 \times l_1}{A_1 \times \Delta l_1} = \frac{F_1 \times l_1}{\pi r_1^2 \times \Delta l_1}$$

$$\Rightarrow \Delta l_1 = \frac{F_1 \times l_1}{\pi r_1^2 \times \Delta l_1}$$

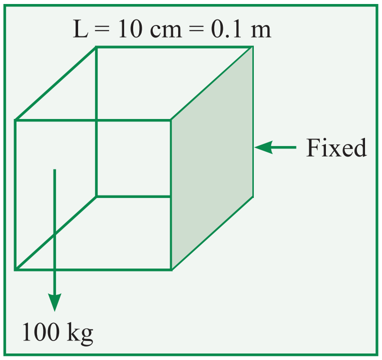
$$\Delta l_1 = \frac{(10 \times 9.8) \times 1.5 \times 7}{22 \times \left(0.125 \times 10^{-2}\right)^2 \times 2 \times 10^{11}} = 1.49 \times 10^{-4}\,\text{m}$$

Similarly,

$$\Delta l_2 = \frac{F_2 \times l_2}{\pi r_2^2 \times Y_2} = \frac{(6 \times 9.8) \times 1 \times 7}{22 \times \left(0.125 \times 10^{-2}\right)^2 \times \left(0.91 \times 10^{11}\right)} = 1.3 \times 10^{-4}\,\text{m}$$

8.6 The edge of an aluminium cube is 10 cm long. One face of the cube is firmly fixed to a vertical wall. A mass of 100 kg is then attached to the opposite face of the cube. The shear modulus of aluminium is 25 GPa . What is the vertical deflection of this face?

SOLUTION:



Given – side of cube, $L = 10\ cm = 10/100 = 0.1m$, mass(m)=100kg

shear modulus of aluminium = 25 GPa

Need to find – vertical deflection

$\therefore$ Area of each face, $A = (0.1)^2 = 0.01\ m^2 A$

Tangential force acting on the face,

F = 100 kg = 100 × 9.8 = 980 N

Shear modulus, $\eta = 25GPa = 25 \times 10^9\ Nm^{-2}$

Since shear modulus is given as:

$$\eta = \frac{\text{Tangential stress}}{\text{Shearing strain}}$$

$$\therefore \text{ Shearing strain} = \frac{\text{Tangential stress}}{\text{Shear modulus}}$$

$$\text{Shearing strain} = \frac{F}{A\eta} = \frac{980}{0.01 \times 25 \times 10^9} = 3.92 \times 10^{-6}$$

$$\frac{\text{Lateral Strain}}{\text{Side of cube}} = \text{Shearing strain}$$

Lateral Strain = Shearing strain × Side of the cube

$= 3.92 \times 10^{-6} \times 0.1$

$= 3.92 \times 10^{-7}$ m $\approx 4 \times 10^{-7}$ m.

8.7 **Four identical hollow cylindrical columns of mild steel support a big structure of mass 50,000 kg. The inner and outer radii of each column are 30 and 60 cm respectively. Assuming the load distribution to be uniform, calculate the compressional strain of each column.**

SOLUTION:

Given – total mass to be supported, M = 50,000 kg, Inner radius of a column,

$r_1 = 30$ cm = 0.3 m, Outer radius of a column, $r_2 = 60$ cm = 0.6 m.

Need to find – compressional strain

$\therefore$ Total weight of the structure to be supported-

(W) = Mg

W = 50,000 × 9.8 N

Since this weight is to be supported by 4 columns,

$\therefore$ Compressional force on each column (F) is given by

$$F = \frac{Mg}{4} = \frac{50,000 \times 9.8}{4}$$

$\therefore$ Area of cross-section of each column is given by

$$A = \pi\left(r_2^2 - r_1^2\right)$$

$$A = \pi\left[(0.6)^2 - (0.3)^2\right] = 0.27\pi \text{m}^2$$

Young's modulus, $Y = 2 \times 10^{11}$ Pa

$$Y = \frac{\text{Compressional force /area}}{\text{Compressional Strain}}$$

$$Y = \frac{F/A}{\text{Compressional Strain}}$$

or Compressional strain of each column

$$= \frac{F}{AY} = \frac{50,000 \times 9.8 \times 7}{4 \times 0.27 \times 22 \times 2 \times 10^{11}}$$

$$= 0.722 \times 10^{-6}$$

∴ Compressional strain of all columns is given by

$= 0.722 \times 10^{-6} \times 4 = 2.88 \times 10^{-6}$

$= 2.88 \times 10^{-6}$

8.8 A piece of copper having a rectangular cross-section of 15.2 mm × 19.1 mm is pulled in tension with 44, 500 N force, producing only elastic deformation. Calculate the resulting strain?

SOLUTION:

Given– A(cross sectional area) = $15.2 \times 19.2 \times 10^{-6}$ m^2; F = 44500 N; $\eta = 42 \times 10^9$ Nm^{-2}

Need to find – strain

$$\text{Strain} = \frac{\text{Stress}}{\text{modulus of elasticity}} = \frac{F/A}{\eta}$$

$$\text{Strain} = \frac{F}{A\eta} = \frac{44500}{\left(15.2 \times 19.2 \times 10^{-6}\right) \times 42 \times 10^{9}} = 3.65 \times 10^{-3}$$

8.9 A steel cable with a radius of 1.5 cm supports a chairlift at a ski area. If the maximum stress is not to exceed 10^8 Nm^{-2}, what is the maximum load the cable can support?

SOLUTION:

Given – radius (r) = 1.5cm, maximum stress = 10^8 Nm^{-2}

Need to find – maximum load

Maximum load = Maximum stress × Cross-sectional area

$$\text{Maximum load} = 10^8\, Nm^{-2} \times \frac{22}{7} \times \left(1.5 \times 10^{-2}\,\text{m}\right)^2$$

Maximum load = 7.07×10^4 N.

8.10 A rigid bar of mass 15 kg is supported symmetrically by three wires each 2.0 m long. Those at each end are of copper and the middle one is of iron. Determine the ratios of their diameters if each is to have the same tension.

SOLUTION:

Given – mass of the rigid bar(m) = 15kg, length of each wire (l) = 2m.

Need to find – ratio of diameters

Since each wire is to have same tension therefore, each wire has same extension. Moreover, each wire has the same initial length. So, strain is same for each wire.

Now, or

$$Y \propto \frac{1}{D^2} \Rightarrow D \propto \frac{1}{\sqrt{Y}}$$

$$\frac{D_{copper}}{D_{iron}} = \sqrt{\frac{Y_{iron}}{Y_{copper}}} = \sqrt{\frac{190\times10^9}{110\times10^9}} = \sqrt{\frac{19}{11}} = 1.314$$

8.11 A 14.5 kg mass, fastened to the end of a steel wire of unstretched length 1.0 m , is whirled in a vertical circle with an angular velocity of 2rev/s at the bottom of the circle. The cross-sectional area of the wire is 0.065 cm². Calculate the elongation of the wire when the mass is at the lowest point of its path.

SOLUTION:

Given – m (mass) = 14.5 kg; l ((length) = r = 1m; v = 2 rps; A (cross sectional area) = 0.065×10^{-4} m²

Need to find – elongation of the wire

Total pulling force on mass, when it is at the lowest position of the vertical circle is:

$F = mg + mrw^2 = mg + mr4\pi^2 v^2$

$F = 14.5 \times 9.8 + 14.5 \times 1 \times 4 \times 2^2$

$F = 142.1 + 2291.6 = 2433.9$ N

$$Y = \frac{F}{A}\times\frac{l}{\Delta l}$$

$$\Delta l = \frac{Fl}{AY} = \frac{2433.7\times1}{\left(0.065\times10^{-4}\right)\times\left(2\times10^{11}\right)} = 1.87\times10^{-3}\,\text{m} = 1.87\ \text{mm}$$

8.12 Compute the bulk modulus of water from the following data: Initial volume = 100.0 litre, Pressure increase = 100.0 atm (1 atm = 1.013 × 10⁵ Pa), Final volume = 100.5 litre. Compare the bulk modulus of water with that of air (at constant temperature). Explain in simple terms why the ratio is so large.

SOLUTION:

Given: P = 100 atmosphere = $100 \times 1.013 \times 10^5$ Pa ($\because$ 1 atm = 1.013×10^5 Pa), Initial volume, V_1 = 100 litre = 100×10^{-3} m³, Final volume, V_2 = 100.5 litre = 100.5×10^{-3} m³

Need to find – compare bulk modulus (B)

$\therefore$ Change in volume:

$\Delta V = V_2 - V_1$

$\Delta V = (100.5 - 100) \times 10^{-3}\ m^3$

$\Delta V = 0.5 \times 10^{-3}\ m^3$

Using formula of bulk modulus,

$$B = \frac{P}{\frac{\Delta V}{V}} = \frac{PV}{\Delta V}$$

$$B = \frac{100 \times 1.013 \times 10^5 \times 100 \times 10^{-3}}{0.5 \times 10^{-3}}$$

$B = 2.023 \times 10^9$ Pa

Also, we know that the bulk modulus of air = 1.0×10^5 Pa

$$\frac{\text{Bulk modulus of water}}{\text{Bulk modulus of air}} == \frac{2.026 \times 10^9}{1.0 \times 10^5}$$

$= 2.026 \times 10^4$

The ratio is too large. This is due to the fact that the strain for air is much larger than for water at the same temperature. In other words, the intermolecular distances in case of liquids are very small as compared to the corresponding distances in the case of gases. Hence there are larger interatomic forces in liquids than in gases.

8.13 What is the density of water at a depth where pressure is 80.0 atm, given that its density at the surface is 1.03×10^3 kg m^{-3} ?

SOLUTION:

Given – density at the surface (ρ) = 1.03×10^3 kg m^{-3}

Need to find – density of water at a depth where pressure is 80.0 atm (ρ')

Compressibility of water,

$$k = \frac{1}{B} = 45.8 \times 10^{-11}\ Pa^{-1}$$

Change in pressure,

$\Delta p = 80$ atm – 1atm

$\Delta p = 79$ atm $= 79 \times 1.013 \times 10^5$ Pa

$$B = \frac{\Delta p \cdot V}{\Delta V} \text{ or } \frac{\Delta V}{V} = \frac{\Delta p}{B} = \Delta p \times \frac{1}{B} = \Delta p \times k$$

$$\frac{\Delta V}{V} = 79 \times 1.013 \times 10^5 \times 45.8 \times 10^{-11} = 3.665 \times 10^{-5}$$

Now,

$$\frac{\Delta V}{V} = \frac{(M/\rho) - (M/\rho')}{(M/\rho)} = 1 - \frac{\rho}{\rho'}$$

$$\frac{\rho}{\rho'} = 1 - \frac{\Delta V}{V}$$

$$\rho' = \frac{\rho}{1 - (\Delta V / V)}$$

$$\rho' = \frac{1.03 \times 10^3}{1 - 3.665 \times 10^{-3}} = \frac{1.03 \times 10^3}{0.996}$$

$$\rho' = 1.034 \times 10^3 \, kg / \text{m}^3$$

8.14 Compute the fractional change in volume of a glass slab, when subjected to a hydraulic pressure of 10 atm .

SOLUTION:

Given - $P = 100$ atm $= 10 \times 1.013 \times 10^5$ Pa; $k = 37 \times 10^9$ Nm^{-2}

Need to find – fractional change in volume $\frac{\Delta V}{V}$

Volumetric strain $= \frac{\Delta V}{V} = \frac{P}{K}$

$$\frac{\Delta V}{V} = \frac{10 \times 1.013 \times 10^5}{37 \times 10^9} = 2.74 \times 10^{-5}$$

Fractional change in volume $= \frac{\Delta V}{V} = 2.74 \times 10^{-5}$

8.15 Determine the volume contraction of a solid copper cube, 10 cm on an edge, when subjected to a hydraulic pressure of 7.0 × 10⁶ Pa.

SOLUTION:

Given - (side of copper cube) a = 10 cm, volume $V = a^3 = 10^{-3}$ m^3, (hydraulic pressure applied) $p = 7.0 \times 10^6$ Pa from table, bulk modulus of copper B = 140GPa = 140×10^9 Pa.

Need to find – volume contraction ΔV

Using the relation $B = -\frac{P}{\frac{\Delta V}{V}}$, we have decrease in

Volume $\Delta V = \frac{PV}{B}$,

$$\therefore \Delta V = \frac{7.0 \times 10^6 \times 10^{-3}}{140 \times 10^9}$$

$\Delta V = 5 \times 10^{-8}$ m^3 = 5×10^{-2} cm^3

8.16 How much should the pressure on a litre of water be changed to compress it by? carry one quarter of the load.

SOLUTION:

$V = 1$ litre $= 10^{-3}$ m^3; $\Delta V/V = 0.10/100 = 10^{-3}$

$$K = \frac{pV}{\Delta V}$$

$$P = K\frac{\Delta V}{V} = \left(2.2\times10^{9}\right)\times10^{-3} = 2.2\times10^{6}\,Pa.$$