CHAPTER 9

MECHANICAL PROPERTIES OF FLUIDS

CLASS 11TH

NCERT EXERCISE AND SOLUTIONS - PHYSICS

9.1 Explain why

(a) The blood pressure in humans is greater at the feet than at the brain

(b) Atmospheric pressure at a height of about 6 km decreases to nearly half of its value at the sea level, though the height of the atmosphere is more than 100 km

(c) Hydrostatic pressure is a scalar quantity even though pressure is force divided by area.

SOLUTION:

(a) The height of the blood column is more for the feet as compared to that for the brain. Consequently, the blood pressure in humans is greater at the feet than at the brain.

(b) The variation of air-density with height is not linear. So, pressure also does not reduce linearly with height. The air pressure at a height h is given by $P = P_0 e^{-ah}$ where P_0 represents the pressure of air at sealevel and is a constant.

(c) Due to applied force on liquid, the pressure is transmitted equally in all directions inside the liquid. That is why there is no fixed direction for the pressure due to liquid. Hence hydrostatic pressure is a scalar quantity.

9.2 Explain why

(a) The angle of contact of mercury with glass is obtuse, while that of water with glass is acute.

(b) Water on a clean glass surface tends to spread out while mercury on the same surface tends to form drops. (Put differently, water wets glass while mercury does not.)

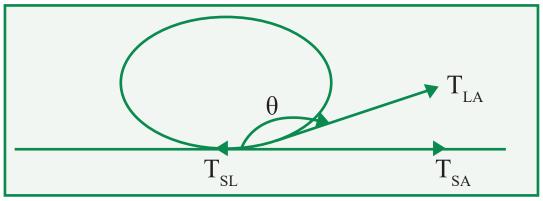
(c) Surface tension of a liquid is independent of the area of the surface

(d) Water with detergent disolved in it should have small angles of contact.

(e) A drop of liquid under no external forces is always spherical in shape

SOLUTION:

(a) Let a drop of a liquid L be poured on a solid surface S placed in air A. If T_{SL} and T_{SA} be the surface tensions corresponding to solid-liquid layer, liquid-air layer and solid-air layer respectively and θ be the angle of contact between the liquid and solid, then



$$T_{LA} \cos\theta + T_{SL} = T_{SA}$$

$$\Rightarrow \text{Cos } \theta = T_{SA} - T_{SL}/T_{LA}$$

For the mercury-glass interface, $T_{SA} < T_{SL}$. Therefore, is negative. Thus θ is an obtuse angle. For the water-glass interface, $T_{SA} > T_{SL}$. Therefore cos 0 is positive. Thus, θ is an acute angle.

(b) Water on a clean glass surface tends to spread out i.e., water wets glass because force of cohesion of water is much less than the force of adhesion due to glass. In case of mercury force of cohesion due to mercury molecules is quite strong as compared to adhesion force due to glass. Consequently, mercury does not wet glass and tends to form drops.

(c) Surface tension of liquid is the force acting per unit length on a line drawn tangentially to the liquid surface at rest. Since h as force is independent of the area of liquid surface therefore, surface tension is also independent of the area of the liquid surface.

(d) We know that the clothes have narrow pores or spaces which act as capillaries. Also, we know that the rise of liquid in a capillary tube is directly proportional to cos θ (Here θ is the angle of contact). As θ is small for detergent, therefore cos θ will be large. Due to this, the detergent will penetrate more in the narrow pores of the clothes.

(e) We know that any system tends to remain in a state of minimum energy. In the absence of any external force for a given volume of liquid its surface area and consequently. Surface energy is least for a spherical shape. It is due to this reason that a liquid drop, in the absence of an external force is spherical in shape.

9.3 Fill in the blanks using the word(s) from the list appended with each statement:

(a) Surface tension of liquids generally ... with temperatures (increases / decreases)

(b) Viscosity of gases ... with temperature, whereas viscosity of liquids ... with temperature (increases / decreases)

(c) For solids with elastic modulus of rigidity, the shearing force is proportional to ... , while for fluids it is proportional to ... (shear strain / rate of shear strain)

(d) For a fluid in a steady flow, the increase in flow speed at a constriction follows (conservation of mass / Bernoulli's principle)

(e) For the model of a plane in a wind tunnel, turbulence occurs at a ... speed for turbulence for an actual plane (greater / smaller)

SOLUTION:

(a) Surface tension of liquids generally **decreases** with temperatures.

(b) Viscosity of gases **increases** with temperature, whereas viscosity of liquids decreases with temperature.

(c) For solids with elastic modulus of rigidity, the shearing force is proportional to **shear strain**, while for fluids it is proportional to rate of shear strain.

(d) For a fluid in a steady flow, the increase in flow speed at a constriction follows **Bernoulli's principle**.

(e) For the model of a plane in a wind tunnel, turbulence occurs at a **greater** speed for turbulence for an actual plane.

9.4 Explain why

(a) To keep a piece of paper horizontal, you should blow over, not under, it

(b) When we try to close a water tap with our fingers, fast jets of water gush through the openings between our fingers

(c) The size of the needle of a syringe controls flow rate better than the thumb pressure exerted by a doctor while administering an injection

(d) A fluid flowing out of a small hole in a vessel results in a backward thrust on the vessel

(e) A spinning cricket ball in air does not follow a parabolic trajectory

SOLUTION:

(a) When we blow over the piece of paper, the velocity of air increases. As a result, the pressure on it decreases in accordance with the Bernoulli's theorem whereas the pressure below remains the same (atmospheric pressure). Thus, the paper remains horizontal.

(b) By doing so the area of outlet of water jet is reduced, so velocity of water increases according to equation of continuity av = constant.

(c) For a constant height, Bernoulli's theorem is expressed as $P + 1/2\rho v^2$ = Constant

In this equation, the pressure P occurs with a single power whereas the velocity occurs with a square power. Therefore, the velocity has more effect compared to the pressure. It is for this reason that needle of the syringe controls flow rate better than the thumb pressure exerted by the doctor.

(d) This is because of principle of conservation of momentum. While the flowing fluid carries forward momentum, the vessel gets a backward momentum.

(e) A spinning cricket ball would have followed a parabolic trajectory has there been no air. But because of air the Magnus effect takes place. Due to the Magnus effect the spinning cricket ball deviates from its parabolic trajectory.

9.5 A 50 kg girl wearing high heel shoes balances on a single heel. The heel is circular with a diameter 1.0 cm. What is the pressure exerted by the heel on the horizontal floor?

SOLUTION:

Given – Mass of girl, m = 50kg. Diameter, D = 1.0 cm = 1×10^{-2} m

Need to find – pressure (P)

Force on the heel, $F = mg = 50 \times 9.8 = 490$ N

$$\therefore \quad \text{Area, } A = \frac{\pi D^2}{4} = \frac{3.14 \times \left(1 \times 10^{-2}\right)^2}{4} = 7.85 \times 10^{-5}\,\text{m}^2$$

$$\text{Pressure, } P = \frac{F}{A}$$

$$\text{P} = \frac{490}{7.85 \times 10^{-5}} = 6.24 \times 10^6\,Pa$$

9.6 Torricelli's barometer used mercury. Pascal duplicated it using French wine of density 984 kg m^{-3}. Determine the height of the wine column for normal atmospheric pressure.

SOLUTION:

Given – atmospheric pressure, $P = 1.01 \times 10^5$ Pa. French wine of density, $\rho = 984$ kg m^{-3}

Need to find – height of wine column should be h_m,

Using - $P = h\rho g$

$$\Rightarrow h_m = \frac{P}{\rho g}$$

$$h_m = \frac{1.01\times10^5}{984\times9.8} = 10.47\text{m} \simeq 10.5\text{m}$$

Hence height of wine column is 10.5.

9.7 A vertical offshore structure is built to withstand a maximum stress of 10^9 Pa. Is the structure suitable for putting up on top of an oil well in the ocean? Take the depth of the ocean to be roughly 3 km, and ignore ocean currents.

SOLUTION:

Given - Maximum stress = 10^9 Pa, (depth of the ocean) h = 3 km = 3×10^3 m, p (water) = 10^3 kg/m^3 and g = 9.8 m/s^2.

The structure will be suitable for putting upon top of an oil well provided the pressure exerted by sea water is less than the maximum stress it can bear.

Pressure due to sea water, P = $h\rho g = 3 \times 10^3 \times 10^3 \times 9.8$ Pa = 2.94×10^7 Pa

Since the pressure of sea water is less than the maximum

9.8 A hydraulic automobile lift is designed to lift cars with a maximum mass of 3000 kg. The area of cross-section of the piston carrying the load is 425 cm^2. What maximum pressure would the smaller piston have to bear?

SOLUTION:

Given – maximum mass (m)= 3000 kg, cross section area = 425 cm^2

Need to find – maximum pressure piston will bear (P)

$$\text{Pressure on the piston due to car} = \frac{\text{Weight of car}}{\text{Area of piston}}$$

$$P = \frac{3000\times9.8}{425\times10^{-4}} Nm^{-2} = 6.92\times10^5\, Pa$$

This is also the maximum pressure that the smaller piston would have to bear.

9.9 A U-tube contains water and methylated spirit separated by mercury. The mercury columns in the two arms are in level with 10.0 cm of water in one arm and 12.5 cm of spirit in the other. What is the specific gravity of spirit?

SOLUTION:

Given – For water column in one arm of *U* tube, h_1 =10.0 cm; ρ_1 (density) =1g cm^{-3}, For spirit column in other arm of *U* tube, h_2 = 12.5 cm

Need to find – specific gravity $\rho_{2;}$

As the mercury columns in the two arms of *U* tube is in level, therefore pressure exerted by each is equal.

Hence

$h_1\rho_1 g = h_2\rho_2 g$

$\rho_1 = h_1\rho_1/h_2 = 10 \times 1/12.5 = 0.8$ g cm^{-3}

Therefore, relative density of spirit = $\rho_1/\ \rho_2 = 0.8/1 = 0.8$

9.10 In the previous problem, if 15.0 cm of water and spirit each are further poured into the respective arms of the tube, what is the difference in the levels of mercury in the two arms? (Specific gravity of mercury = 13.6)

SOLUTION:

Given – Height of the water column, $h_1 = 10 + 15 = 25$ cm, Height of the spirit column,

$h_2 = 12.5 + 15 = 27.5$ cm, Density of water, $\rho_1 = 1$ g cm^{-3}, Density of spirit, $\rho_2 = 0.8$ g cm^{-3}

Density of mercury = 13.6 g cm^{-3}

Need to find – *h* (difference between the levels of mercury in the two arms)

Pressure exerted by height *h* of the mercury column:

$P = h\rho g$

$P = h \times 13.6$ g …………(i)

Difference between the pressures exerted by water and spirit:

$Pw - Ps = \rho_1 h_1 g - \rho_2 h_2 g$

$Pw - Ps = g(25 \times 1 - 27.5 \times 0.8)$

$Pw - Ps = 3g$ ………….(ii)

Equating equations (i) and (ii), we get:

$13.6\ hg = 3g$

$h = 0.220588 \approx 0.221$ cm

Hence, the difference between the levels of mercury in the two arms is 0.221 cm .

9.11 Can Bernoulli's equation be used to describe the flow of water through a rapid in a river? Explain.

SOLUTION:

Bernoulli's theorem is applicable only for there it ideal fluids in streamlined motion. Since the flow of water in a river is rapid, way cannot be treated as streamlined motion, the theorem cannot be used.

9.12 Does it matter if one uses gauge instead of absolute pressures in applying Bernoulli's equation? Explain.

SOLUTION:

Using Bernoulli's theorem: $P+\rho gh+\frac{1}{2}\rho v^2$ = constant

where P is the absolute pressure at a point, ρ is the density of the fluid, h is the height of that point above a reference point and v is the velocity of fluid at that point.

Thus $P_1+\rho gh_1+\frac{1}{2}\rho v_1^2=P_2+\rho gh_2+\frac{1}{2}\rho v_2^2$

Subtracting atmospheric pressure from both sides.

We get $P_1-P_o+\rho gh_1+\frac{1}{2}\rho v_1^2=P_2-P_o+\rho gh_2+\frac{1}{2}\rho v_2^2$

Gauge pressure $P_i^{'}=P_i-P_o\ (i=1,2)$

$\Rightarrow P_1^{'}+\rho gh_1+\frac{1}{2}\rho v_1^2=P_2^{'}+\rho gh_2+\frac{1}{2}\rho v_2^2$

Thus the Bernoulli's equation remains in the same form.

Hence, it does not matter if one uses gauge instead of absolute pressure in applying Bernoulli's equation.

9.13 Glycerine flows steadily through a horizontal tube of length 1.5 m and radius 1.0 cm. If the amount of glycerine collected per second at one end is 4.0 × 10^{-3} kg s^{-1}, what is the pressure difference between the two ends of the tube? (Density of glycerine = 1.3 × 10^{-3} kg m^{-3} and viscosity of glycerine = 0.83 Pa s). [You may also like to check if the assumption of laminar flow in the tube is correct].

SOLUTION:

Given – length (l) = 1.5 m, radius (r) = 1 × 10^{-2} m, η = 0.83 Pa s.

Need to find –

$$V=\frac{\text{Mass / s}}{\text{Density}}=\frac{4\times10^{-3}}{1.3\times10^{3}}\text{m}^3\text{s}^{-1}$$

$$V=\frac{4}{1.3}\times10^{-6}\text{m}^3\text{s}^{-1}$$

where p is the pressure difference across the capillary

$$p=\frac{8V\eta l}{\pi r^4}$$

Substituting values, $p = 8 \times \frac{4}{1.3} \times 10^{-6} \times 0.83 \times 1.5 \times \frac{7}{22} \times \frac{1}{10^{-8}} Pa$

$p = 9.75 \times 10^2\ Pa$

The Reynolds number is 0.3. So, the flow is laminar.

9.14 In a test experiment on a model aeroplane in a wind tunnel, the flow speeds on the upper and lower surfaces of the wing are 70 m s^{-1} and 63 m s^{-1} respectively. What is the lift on the wing if its area is 2.5 m^2? Take the density of air to be 1.3 kg m^{-3}.

SOLUTION:

Let v_1, v_2 be the speeds on the upper and lower surfaces of the wing of aeroplane, and P_1 and P_2 be the pressures on upper and lower surfaces of the wing respectively.

Then $v_1 = 70$ ms^{-1}; $v_2 = 63$ ms^{-1}; $\rho = 1.3$ kg m^{-3}.

From Bernoulli's theorem

$$\frac{P_1}{\rho} + gh + \frac{1}{2}v_1^2 = \frac{P_2}{\rho} + gh + \frac{1}{2}v_2^2$$

$$\therefore \frac{P_1}{\rho} - \frac{P_2}{\rho} = \frac{1}{2}\left(v_2^2 - v_1^2\right)$$

Or

$$P_1 - P_2 = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) = \frac{1}{2} \times 1.3\left[(70)^2 - (63)^2\right] Pa = 605.15 Pa$$

This difference of pressure provides the lift to the aeroplane. So, lift on the aeroplane = pressure difference × area of wings

= 605.15 × 2.5 N = 1512.875 N

= 1.51×10^3 N

9.15 Figures (a) and (b) refer to the steady flow of a (non-viscous) liquid. Which of the two figures is incorrect? Why?

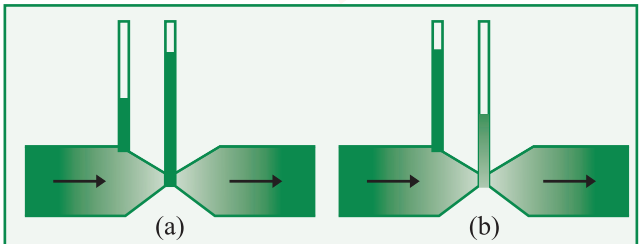


Fig.

SOLUTION:

Figure (a) is incorrect. It is because of the fact that at the kink, the velocity of flow of liquid is large and hence using the Bernoulli's theorem the pressure is less. As a result, the water should not rise higher in the tube where there is a kink (i.e., where the area of cross-section is small).

9.16 The cylindrical tube of a spray pump has a cross-section of 8.0 cm² one end of which has 40 fine holes each of diameter 1.0 mm. If the liquid flow inside the tube is 1.5 m min^{-1}, what is the speed of ejection of the liquid through the holes?

SOLUTION:

Given – cross-section = 8.0 cm², number of holes = 40, diameter (d) = 1mm, Speed inside the tube,

$$v_1 = 1.5 \text{m min}^{-1} = \frac{1.5}{60} ms^{-1}$$

Need to find – Speed of ejection, v_2

Answer:

Total cross-sectional area of 40 holes, a_2

$$= 40 \times \frac{22}{7} \times \frac{(1\times10^{-3})^2}{4} \text{m}^2$$

$$= \frac{22}{7} \times 10^{-5} \text{m}^2$$

Cross-sectional area of tube, $a_1 = 8 \times 10^{-4}$ m²

Using $a_2 v_2 = a_1 v_1$,

we get

$$v_2 = \frac{a_1 v_1}{a_2}$$

$$v_2 = \frac{8\times10^{-4} \times \frac{1.5}{60} \times 7}{22\times10^{-5}} ms^{-1}$$

$v_2 = 0.64\ ms^{-1}$

9.17 A U-shaped wire is dipped in a soap solution and removed. The thin soap film formed between the wire and the light slider supports a weight of 1.5 × 10^{-2} N (which includes the small weight of the slider). The length of the slider is 30 cm . What is the surface tension of the film?

SOLUTION:

Given – force of surface tension is balancing the weight of 1.5 × 10^{-2} N

Need to find – surface tension of the film (T)

Total length of liquid film,

$I = 2 \times 30$ cm = 60 cm = 0.6 m because the liquid film has two surfaces.

Surface tension,

$T = F/I$

$$T = 1.5 \times \frac{10^{-2}\text{N}}{0.6\text{m}}$$

T = 2.5 × 10^{-2} Nm^{-1}

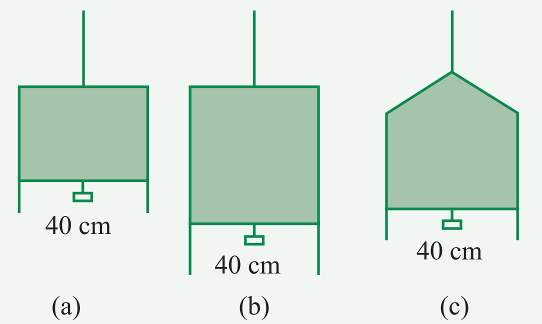


Fig.

9.18 Figure (a) shows a thin liquid film supporting a small weight = 4.5 ×10^{-2} N. What is the weight supported by a film of the same liquid at the same temperature in Fig. (b) and (c)?
Explain your answer physically.

SOLUTION:

(a) Here, length of the film supporting the weight = 40 cm = 0.4 m. Total weight supported (or force) = 4.5 × 10^{-2} N.

Film has two free surfaces, Surface tension, S = 4.5 × 10^{-2}/2 × 0.4 = 5.625 × 10^{-2} Nm^{-1}

Since the liquid is same for all the cases (a), (b) and (c), and temperature is also same, therefore surface tension for cases (b) and (c) will also be the same = 5.625 × 10^{-2}. In Fig. 7(b), 38(b) and (c), the length of the film supporting the weight is also the saihe as that of (a), hence the total weight supported in each case is = 4.5 × 10^{-2} N

9.19 What is the pressure inside the drop of mercury of radius 3.00 mm at room temperature? Surface tension of mercury at that temperature (20°C) is 4.65 × 10^{-1} N m^{-1}. The atmospheric pressure is 1.01 × 10^5 Pa. Also give the excess pressure inside the drop.

SOLUTION:

Given – radius (r) = 3 mm, surface tension (T) = 4.65 ×10^{-1} N m^{-1}

Need to find – Pressure inside the drop (P)

Excess pressure $= \frac{2\sigma}{R} = \frac{2\times4.65\times10^{-1}}{3\times10^{-3}} = 310Pa$

Total pressure (P) $= 1.01\times10^5 + \frac{2\sigma}{R}$

$P = 1.01 \times 10^5 + 310 = 1.0131 \times 10^5\ Pa$

Since data is correct up to three significant figures, we should write total pressure inside the drop as 1.01 x 10^5 Pa.

9.20 What is the excess pressure inside a bubble of soap solution of radius 5.00 mm , given that the surface tension of soap solution at the temperature (20°C) is 2.50 × 10^{-2} N m^{-1}? If an air bubble of the same dimension were formed at depth of 40.0 cm inside a container containing the soap solution (of relative density 1.20), what would be the pressure inside the bubble? (1 atmospheric pressure is 1.01 × 10^5 Pa).

SOLUTION:

Given – surface tension of soap solution at room temperature (T) = 2.50 × 10^{-2} Nm^{-1}, radius of soap bubble, r = 5.00 mm = 5.00 × 10^{-3}m.

Need to find – Pressure inside the bubble (P_i)

Excess pressure inside soap bubble,

$$P = P_i - P_0 = \frac{4T}{r}$$

$$P = \frac{4\times 2.50\times 10^{-2}}{5.00\times 10^{-3}} = 20.0\,Pa$$

When an air bubble of radius r = 5.00 × 10^{-3} m is formed at a depth h = 40.0 cm = 0.4 m inside a container containing a soap solution of relative density 1.20 or density, ρ = 1.20 × 10^{3} kg m^{-3} m then excess pressure h = 40.0 cm = 0.4 m

$$P = P_i - P_0 = \frac{2T}{r}$$

$$P_i = P_0 + \frac{2T}{r} = (P_a + h\rho g) + \frac{2T}{r}$$

$$P_i = \left[1.01\times 10^{5} + 0.4\times 1.2\times 10^{3}\times 9.8 + \frac{2\times 2.50\times 10^{-2}}{5.00\times 10^{-3}}\right] Pa$$

$P_i = (1.01 \times 10^5 + 4.7 \times 10^3 + 10.0)$ Pa

$P_i \simeq (1.057 \times 10^5$ Pa)