WAVES

14.1 A string of mass 2.50 kg is under a tension of 200 N. The length of the stretched string is 20.0 m. If the transverse jerk is struck at one end of the string, how long does the disturbance take to reach the other end?

SOLUTION:

Given – Tension, T = 200 N; Length, l = 20.0 m; Mass, M =2.50 kg

Need to find – time taken for disturbance to take (T)

Mass per unit length, $\mu = \frac{2.50}{20.0} kgm^{-1} = 0.125 kgm^{-1}$

Wave velocity (v),

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{200N}{0.125kgm^{-1}}}$$

$$v = \sqrt{1600}ms^{-1} = 40ms^{-1}$$

Calculating Time,

$$t = \frac{l}{v} = \frac{20.0}{40}s = \frac{1}{2}s = 0.5\text{s}$$

14.2 A stone dropped from the top of a tower of height 300 m splashes into the water of a pond near the base of the tower. When is the splash heard at the top given that the speed of sound in air is 340 m s^{-1}? ($g = 9.8$ m s^{-2}).

SOLUTION:

Given – h = 300 m, g = 9.8 ms^{-2} and velocity of sound, v = 340 ms^{-1}

Need to find – time when the splash heard at the top (t)

Let t_1 be the time taken by the stone to reach at the surface of pond.

Using, equation of motion

$$s = ut + \frac{1}{2}at^2 \frac{1}{2}at^2$$

$$h = 0 \times t + \frac{1}{2}gt_1^2$$

$$t_1 = \sqrt{\frac{2\times 300}{9.8}} = 7.82s$$

Also, if t_2 is the time taken by the sound to reach at a height h, then

$$t_2 = \frac{h}{v} = \frac{300}{340} = 0.88s$$

$\therefore$ Total time after which sound of splash is heard (T) = $t_1 + t_2$

T = 7.82 + 0.88 = 8.7 s

time taken when the splash heard at the top (t) is 8.7s

14.3 A steel wire has a length of 12.0 m and a mass of 2.10 kg. What should be the tension in the wire so that speed of a transverse wave on the wire equals the speed of sound in dry air at 20°C = 343 m s^{-1}.

SOLUTION:

Given – length of the wire (l) =12.0 m, Mass of the wire (M) = 2.10 kg speed of sound in dry air (v) = 343 ms^{-1}

Need to find – tension in the wire (T)

Mass per unit length (m) $= \frac{M}{l} = \frac{2.10}{12.0} = 0.175 kgm^{-1}$

$$v = \sqrt{\frac{T}{m}}$$

T = v^2. m

T = $(343)^2 \times 0.175$

T = 2.06×10^4 N

Hence tension in the wire is = 2.06×10^4 N

14.4 Use the formula $v = \sqrt{\frac{\gamma P}{\rho}}$ to explain why the speed of sound in air

(a) is independent of pressure,

(b) increases with temperature,

(c) increases with humidity.

SOLUTION:

We are given that $v = \sqrt{\frac{\gamma P}{\rho}}$

PV = nRT (for *n* moles of ideal gas)

$$\Rightarrow PV = \frac{m}{M}RT$$

where *m* is the total mass and M is the molecular mass of the gas.

$$\therefore P = \frac{m}{M}\cdot\frac{RT}{M} = \frac{\rho RT}{M} \Rightarrow \frac{P}{\rho} = \frac{\mathrm{R}T}{M}$$

(a) For a gas at constant temperature, $\frac{P}{\rho}$ = constant

$\therefore$ As P increase, ρ also increases and vice versa. This implies that $v = \sqrt{\frac{\gamma P}{\rho}}$ = constant, i.e., velocity is independent of pressure of the gas.

(b) Since $\frac{P}{\rho} = \frac{RT}{M}$, therfore $v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}}$

Clearly $v \propto \sqrt{T}$ i.e., speed of sound in air increases with increase in temperature.

(c) Increase in humidity decreases the effective density of air.

Therefore, the velocity $\left(v \propto \frac{1}{\sqrt{\rho}}\right)$ increases.

14.5 You have learnt that a travelling wave in one dimension is represented by a function y = f(x, t) where x and t must appear in the combination x - vt or x + vt, i.e. y = f(x ± vt). Is the converse true? Examine if the following functions for y can possibly represent a travelling wave:

(a) $(x – vt)^2$

(b) $\log[(x + vt)/x_0]$

(c) 1/(x + vt)

SOLUTION:

Answer: No, the converse is not true. The basic requirement for a wave function to represent a travelling wave is that for all values of x and t , wave function must have a finite value. Out of the given functions for y none satisfies this condition. Therefore, none can represent a travelling wave.

14.6 A bat emits ultrasonic sound of frequency 1000 kHz in air. If the sound meets a water surface, what is the wavelength of (a) the reflected sound, (b) the transmitted sound? Speed of sound in air is 340 m s^{-1} and in water 1486 m s^{-1}.

SOLUTION:

Given – ultrasonic sound frequency (v) = =1000 × 10^3 Hz = 10^6 Hz, speed of the sound in air (v_a) = 340 ms^{-1}, speed of the sound in air (v_a) = 1486 ms^{-1}

Need to find – reflected sound wavelength (λ_a), wavelength of the transmitted sound (λ_ω)

Wavelength of reflected sound, $\lambda_a = \frac{v_a}{v} = \frac{340}{10^6}\mathrm{m} = 3.4\times10^{-4}\mathrm{m}$

Wavelength of transmitted sound, $\lambda_\omega = \frac{v_w}{v} = \frac{1486}{10^6}\mathrm{m} = 1.486\times10^{-3}\mathrm{m}$

14.7 A hospital uses an ultrasonic scanner to locate tumours in a tissue. What is the wavelength of sound in the tissue in which the speed of sound is 1.7 km s^{-1} ? The operating frequency of the scanner is 4.2 MHz.

SOLUTION:

Given – speed of sound ⇒v =1.7 km s^{-1} =1700 ms^{-1}, frequency u = 4.2 MHz = 4.2 × 10^6 Hz

Need to find – Wavelength (A)

Wavelength,

$$A = \frac{u}{V}$$

$$A = \frac{1700}{4.2 \times 106}$$

A = 4.1 × 10^{-4} m.

Hence wavelength of sound in the tissue in which the speed of sound is 1.7 k m^{-1} is 4.1 × 10^{-4} m.

14.8 A transverse harmonic wave on a string is described by

y(x, t) = 3.0sin(36t + 0.018x + π/4)

where x and y are in cm and t in s. The positive direction of x is from left to right.

(a) Is this a travelling wave or a stationary wave?

If it is travelling, what are the speed and direction of its propagation ?

(b) What are its amplitude and frequency?

(c) What is the initial phase at the origin?

(d) What is the least distance between two successive crests in the wave?

SOLUTION:

Given $-y(x,t) = 3.0\sin\left(36t + 0.018x + \frac{\pi}{4}\right)$

The given equation is $y(x,t) = 3.0\sin\left(36t + 0.018x + \frac{\pi}{4}\right)$, where x and y are in cm and t in s.

(a) The equation is the equation of a travelling wave, travelling from right to left (i.e., along -ve direction of x because it is an equation of the type

y(x, t) = Asin(ωt + kx + ϕ)

Here,

$$A = 3.0cm, \omega = 36rads^{-1}, k = 0.018cm \text{ and } \phi = \frac{\pi}{4}$$

∴ Speed of wave propagation,

$$v = \frac{\omega}{k} = \frac{36rads^{-1}}{0.018cm^{-1}}$$

$$v = \frac{36rads^{-1}}{0.018 \times 10^2 ms^{-1}} = 20ms^{-1}$$

(b) Amplitude of wave, A = 3.0 cm = 0.03 m

Frequency of wave(v)

$$v = \frac{\omega}{2\pi}$$

$$v = \frac{36}{2\pi} = 5.7\,Hz$$

(c) Initial phase at the origin, $\phi = \frac{\pi}{4}$

(d) Least distance between two successive crests in the wave

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.018}$$

λ =349 cm =3.5 m

14.9 For the wave described in Exercise 14.8, plot the displacement (y) versus (t) graphs for x = 0,2 and 4 cm . What are the shapes of these graphs? In which aspects does the oscillatory motion in travelling wave differ from one point to another: amplitude, frequency or phase?

SOLUTION:

The transverse harmonic wave is

$$y(x,t) = 3.0\sin\left(36t + 0.018x + \frac{\pi}{4}\right)$$

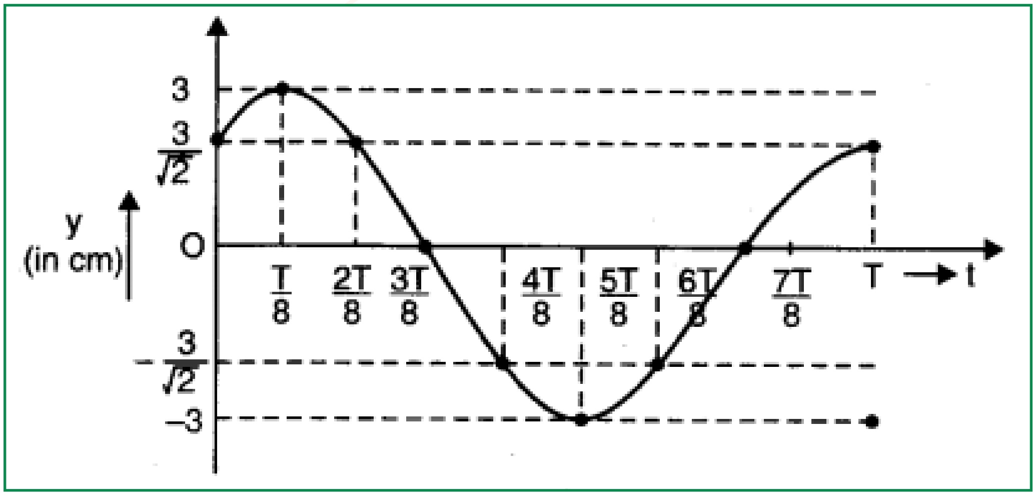
For: x = 0

$$y(0,t) = 3\sin\left(36t + 0 + \frac{\pi}{4}\right) = 3\sin\left(36t + \frac{\pi}{4}\right)$$

Here

$$\omega = \frac{2\pi}{T} = 36 \Rightarrow T = \frac{2\pi}{36}$$

To plot a (y) versus (t) graph, different values of y corresponding to the values of t may be tabulated as under (by making use of eqn. (1)).



t	0	$\frac{T}{8}$	$\frac{2T}{8}$	$\frac{3T}{8}$	$\frac{4T}{8}$	$\frac{5T}{8}$	$\frac{6T}{8}$	$\frac{7T}{8}$	T
y	$\frac{3}{\sqrt{2}}$	3	$\frac{3}{\sqrt{2}}$	0	$\frac{-3}{\sqrt{2}}$	-3	$\frac{-3}{\sqrt{2}}$	0	$\frac{3}{\sqrt{2}}$

14.10 For the travelling harmonic wave

$$y(x, t) = 2.0\cos 2\pi(10t - 0.0080x + 0.35)$$

where x and y are in cm and t in s. Calculate the phase difference between oscillatory motion of two points separated by a distance of

(a) 4 m ,

(b) 0.5 m ,

(c) λ/2,

(d) 3λ/4

SOLUTION:

Given – (x, t) = 2.0cos 2π(10t – 0.0080x + 0.35)

Need to find – phase difference (Δϕ)

The given equation can be rewritten as under:

y(x, t) = 2.0 cos[2π(10t- 0.0080x) + 2π × 0.35]

$$y(x,t) = 2.0\cos\left[2\pi \times 0.0080\left(\frac{10t}{0.0080} - x\right) + 0.7\pi\right]$$

Comparing this equation with the standard equation of a travelling harmonic wave,

$$\frac{2\pi}{\lambda} = 2\pi \times 0.0080$$

$$\lambda = \frac{1}{0.0080} cm = 125 cm$$

The phase difference between oscillatory motion of two points separated by a distance Δx is given by

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x$$

(a) When, Δx = 4 m =400 cm, then

$$\Delta\phi = \frac{2\pi}{125} \times 400 = 6.4\pi rad$$

(b) When, Δx = 0.5 m = 50 cm, then

$$\Delta\phi = \frac{2\pi}{125} \times 50 = 0.8\pi rad$$

(c) When, $\Delta x = \frac{\lambda}{2} = \frac{125}{2} cm$, then

$$\Delta\phi = \frac{2\pi}{125} \times \frac{125}{2} = \pi rad$$

(d) When, $\Delta x = \frac{3\lambda}{4} = \frac{3 \times 125}{4} cm$ then

$$\Delta\phi = \frac{2\pi}{125} \times \frac{3 \times 125}{4} = \frac{3\pi}{2} rad$$

14.11 The transverse displacement of a string (clamped at its both ends) is given by

$$y(x, t) = 0.06 \sin\left(\frac{2\pi}{3} x\right) \cos(120\pi t)$$

where x and y are in m and t in s. The length of the string is 1.5 m and its mass is 3.0 ×10^{-2} kg. Answer the following :

(a) Does the function represent a travelling wave or a stationary wave?

(b) Interpret the wave as a superposition of two waves travelling in opposite directions. What is the wavelength, frequency, and speed of each wave?

(c) Determine the tension in the string.

SOLUTION:

Given- $y(x,t) = 0.06 \sin\left(\frac{2\pi}{3} x\right) \cos(120\pi t)$ length (l)=1.5m, mass(m)= 3.0×10^{-2} kg.

Need to find – tension (T)

y(x, t) = 0.06sin 2π/3xcos120 πt…(1)

(i) As the equation involves harmonic functions of x and t separately, it represents a stationary wave.

(ii) We know that when a wave pulse

$y_1 = r \sin \frac{2\pi}{\lambda}(vt - x)$, travelling along + direction of x-axis is superimposed by the reflected wave

$y_2 = -r \sin \frac{2\pi}{\lambda}(vt + x)$, travelling in opposite direction, a stationary wave

$y = y_1 + y_2 = -2r \sin \frac{2\pi}{\lambda} x \cos \frac{2\pi}{\lambda} vt$ is formed ……….(2)

Comparing eqns. (1) and (2), we find that

$$\frac{2\pi}{\lambda} = \frac{2\pi}{3}$$

$\Rightarrow \lambda = 3$ m

Also,

$$\frac{2\pi}{\lambda} v = 120\pi$$

v = 60λ = 60 × 3 =180 ms^{-1}

Frequency, $\nu = \frac{v}{\lambda} = \frac{180}{3} = 60 Hz$

(iii) Velocity of transverse waves is

$$V = \sqrt{\frac{T}{m}v^2}$$

$$V = \frac{T}{m}$$

T = mv², where $m = \frac{3\times10^{-2}}{1.5} = 2\times10^{-2}\,kg/\text{m}$

$T = (180)^2 \times 2 \times 10^{-2}$ T

T = 648 N

14.12 (i) For the wave on a string described in Exercise 15.11, do all the points on the string oscillate with the same (a) frequency, (b) phase, (c) amplitude? Explain your answers.

(ii) What is the amplitude of a point 0.375 m away from one end?

SOLUTION:

(i) For the wave on the string described in questions we have seen that I =1.5 m and λ = 3 m. So, it is clear that λ = λ/2 and for a string clamped at both ends, it is possible only when both ends behave as nodes and there is only one antinode in between i.e., whole string is vibrating in one segment only.

(a) Yes, all the sring particles, except nodes, vibrate with the same frequency v = 60 Hz.

(b) As all string particles lie in one segment, all of them are in same phase.

(c) Amplitude varies from particle to particle. At antinode, amplitude = 2A = 0.06 m. It gradually falls on going towards nodes and at nodes, and at nodes, it is zero.

(ii) Amplitude at a point x = 0.375 m will be obtained by putting cos(120πt) as +1 in the wave equation.

$$\therefore A(x) = 0.06\sin\left(\frac{2\pi}{3}\times0.375\right)\times1 = 0.06\sin\frac{\pi}{4} = 0.042\text{m}$$

14.13 Given below are some functions of x and t to represent the displacement (transverse or longitudinal) of an elastic wave. State which of these represent (i) a travelling wave, (ii) a stationary wave or (iii) none at all:

(a) y = 2cos (3x)sin (10t)

(b) $y = 2\sqrt{x - vt}$

(c) y = 3sin(5x – 0.5t) + 4cos(5x – 0.5t)

(d) y = cosxsint + cos2xsin2t

SOLUTION:

(a) It represents a stationary wave.

(b) It does not represent either a travelling wave or a stationary wave.

(c) It is a representation for the travelling wave.

(d) It is a superposition of two stationary wave.

14.14 A wire stretched between two rigid supports vibrates in its fundamental mode with a frequency of 45 Hz. The mass of the wire is 3.5×10^{-2} kg and its linear mass density is 4.0×10^{-2} kg m^{-1}. What is (a) the speed of a transverse wave on the string, and (b) the tension in the string?

SOLUTION:

Given – frequency (n) = 45 Hz, mass of the wire (M) = 3.5×10^{-2} kg, Mass per unit length = 4.0×10^{-2} kg m^{-1}.

Need to find – speed of the transverse wave (v), tension in the string (T)

Length of the wire is given by-

$$l = \frac{M}{m} = \frac{3.5\times10^{-2}}{4.0\times10^{-2}}$$

$$l = \frac{7}{8}$$

Now relation between length and the wavelength-

$$\frac{l}{2} = \lambda$$

$$\lambda = \frac{7}{4}m$$

λ =1.75 m

(a) The speed of the transverse wave, v = vλ = 45 × 1.75 = 78.75 m/s

(b) Using the formula-

$$v = \sqrt{\frac{T}{m}}$$

$T = v^2 \times m$

$T = (78.75)^2 \times 4.0 \times 10^{-2}$

T = 248.06 N

14.15 A metre-long tube open at one end, with a movable piston at the other end, shows resonance with a fixed frequency source (a tuning fork of frequency 340 Hz) when the tube length is 25.5 cm or 79.3 cm . Estimate the speed of sound in air at the temperature of the experiment. The edge effects may be neglected.

SOLUTION:

Frequency of nth mode of vibration of the closed organ pipe of length

$$l_1 = (2n-1)\frac{v}{4l_1}$$

Frequency of (n + 1)th mode of vibration of closed pipe of length

$$'l_2' = [2(n+1)-1]\frac{v}{4l_2} = (2n+1)\frac{v}{4l_2}$$

Both the modes are given to resonate with a frequency of 340 Hz.

$$\therefore \quad (2n-1)\frac{v}{4l_1} = (2n+1)\frac{v}{4l_2}$$

$$\text{or} \quad \frac{2n-1}{2n+2} = \frac{l_1}{l_2} = \frac{25.5}{79.3} = \frac{1}{3}$$

[Approximation has been used because edge effect is being ignored. Moreover, we know that in the case of a closed organ pipe, the second resonance length is three times the first resonance length.]

On simplification, n = 1

Now, $(2n-1)v/4l_1 = 340$.

$$(2\times1-1)v\times\frac{100}{4}\times25.5 = 340$$

v = 346.8 ms $^{-1}$

14.16 A steel rod 100 cm long is clamped at its middle. The fundamental frequency of longitudinal vibrations of the rod are given to be 2.53 kHz . What is the speed of sound in steel?

SOLUTION:

Given – length (L) = 100 cm =1m, frequency (ν) = 2.53kHz = 2.53×10^3 Hz

Need to find – speed of the sound in steel(s)

Answer: Here,

When the rod is clamped at the middle, then in the fundamental mode of vibration of the rod, a node is formed at the middle and ant mode is formed at each end.

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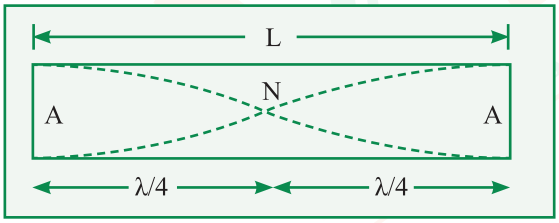

$$L = \frac{\lambda}{4} + \frac{\lambda}{4} = \frac{\lambda}{2}$$

λ = 2L= 2 m

s = νλ

s = $2.53 \times 10^3 \times 2$

s = 5.06×10^3 ms^{-1}

Hence, speed of the sound in steel(s) is = 5.06×10^3 ms^{-1}

14.17 A pipe 20 cm long is closed at one end. Which harmonic mode of the pipe is resonantly excited by a 430 Hz source? Will the same source be in resonance with the pipe if both ends are open? (speed of sound in air is 340 m s^{-1}).

SOLUTION:

Given – length of pipe, 1= 20 cm = 0.20 m, frequency v = 430 Hz and speed of sound in air u = 340 m s^{-1}.

For closed end pipe,

$$v = \frac{(2n-1)v}{4l} \text{ ,where n = 1,2,3……...}$$

$$2n-1 = \frac{4vl}{v} = \frac{4\times 430\times 0.20}{340} = 1.02$$

2n = 1.02 + 1= 2.02

$$n = \frac{0.20}{2} = 1.01$$

Hence, resonance can occur only for first (or fundamental) mode of vibration.

As for an open pipe $v = \frac{nv}{2l}$, where n = 1, 2, 3.

$$\therefore n = \frac{2lv}{v} = \frac{2\times 430\times 0.20}{340} = 0.51$$

As n < 1, hence, in this case resonance position cannot be obtained.

14.18 Two sitar strings A and B playing the note 'Ga' are slightly out of tune and produce beats of frequency 6 Hz. The tension in the string A is slightly reduced and the beat frequency is found to reduce to 3 Hz. If the original frequency of A is 324 Hz , what is the frequency of B ?

SOLUTION:

Given – Let U_1 and U_2 be the frequencies of strings A and B respectively.

Then, u_1 = 324 Hz, Number of beats, b = 6

Need to find – U_2

$u_2 = u_1 \pm b = 324 \pm 6$!. e., u_2 = 330 Hz or 318 Hz

Since the frequency is directly proportional to square root of tension, on decreasing the tension in the string A, its frequency u_1 will be reduced i.e., number of beats will increase if u_2=330 Hz. This is not so because number of beats become 3.

Therefore, it is concluded that the frequency U_2 = 318 Hz. because on reducing the tension in the string A, its frequency may be reduced to 321 Hz, thereby giving 3 beats with U_2 = 318 Hz.

14.19 Explain why (or how):

(a) in a sound wave, a displacement node is a pressure antinode and vice versa,

(b) bats can ascertain distances, directions, nature, and sizes of the obstacles without any "eyes",

(c) a violin note and sitar note may have the same frequency, yet we can distinguish between the two notes,

(d) solids can support both longitudinal and transverse waves, but only longitudinal waves can propagate in gases, and

(e) the shape of a pulse gets distorted during propagation in a dispersive medium.

SOLUTION:

(a) In a sound wave, a decrease in displacement i.e., displacement node causes an increase in the pressure there i.e., a pressure antinode is formed. Also, an increase in displacement is due to the decrease in pressure.

(b) Bats emit ultrasonic waves of high frequency from their mouths. These waves after being reflected back from the obstacles on their path are observed by the bats. These waves give them an idea of distance, direction, nature and size of the obstacles.

(c) The quality of a violin note is different from the quality of sitar. Therefore, they emit different harmonics which can be observed by human ear and used to differentiate between the two notes.

(d) This is due to the fact that gases have only the bulk modulus of elasticity whereas solids have both, the shear modulus as well as the bulk modulus of elasticity.

(e) A pulse of sound consists of a combination of waves of different wavelength. In a dispersive medium, these waves travel with different velocities giving rise to the distortion in the wave.