

CHAPTER 2

Polynomials

VEDA
ACADEMY

CLASS 9TH

NCERT EXERCISE AND SOLUTIONS - MATHEMATICS

EXERCISE 2.1

1. Which of the following expressions are polynomials in one variable, and which are not? State reasons for your answer.

(i) $4x^2 - 3x + 7$ (ii) $y^2 + \sqrt{2}$ (iii) $3\sqrt{t} + t\sqrt{2}$ (iv) $y + 2/y$ (v) $x^{10} + y^3 + t^{50}$

SOLUTION:

(i) $4x^2 - 3x + 7$

The equation $4x^2 - 3x + 7$ can be written as $4x^2 - 3x^1 + 7x^0$

Since x is the only variable in the given equation and the powers of x (i.e. 2, 1 and 0) are whole numbers, we can say that the expression $4x^2 - 3x + 7$ is a polynomial in one variable.

(ii) $y^2 + \sqrt{2}$

The equation $y^2 + \sqrt{2}$ can be written as $y^2 + \sqrt{2}y^0$

Since y is the only variable in the given equation and the powers of y (i.e., 2 and 0) are whole numbers, we can say that the expression $y^2 + \sqrt{2}$ is a polynomial in one variable.

(iii) $3\sqrt{t} + t\sqrt{2}$

The equation $3\sqrt{t} + t\sqrt{2}$ can be written as $3t^{1/2} + \sqrt{2}t$

Though t is the only variable in the given equation, the power of t (i.e., 1/2) is not a whole number. Hence, we can say that the expression $3\sqrt{t} + t\sqrt{2}$ is not a polynomial in one variable.

(iv) $y + 2/y$

The equation $y + \frac{2}{y}$ can be written as $y + 2y^{-1}$

Though y is the only variable in the given equation, the power of y (i.e., -1) is not a whole number. Hence, we can say that the expression $y + \frac{2}{y}$ is not a polynomial in one variable.

(v) $x^{10} + y^3 + t^{50}$

Here, in the equation $x^{10} + y^3 + t^{50}$

Though the powers, 10, 3, 50, are whole numbers, there are 3 variables used in the expression $x^{10} + y^3 + t^{50}$. Hence, it is not a polynomial in one variable.



2. Write the coefficients of x^2 in each of the following:

- (i) $2 + x^2 + x$ (ii) $2 - x^2 + x^3$ (iii) $(\pi/2)x^2 + x$ (iv) $\sqrt{2}x - 1$

SOLUTION:

- (i) $2 + x^2 + x$

The equation $2 + x^2 + x$ can be written as $2 + (1)x^2 + x$

We know that the coefficient is the number which multiplies the variable. Here, the number that multiplies the variable x^2 is 1

Hence, the coefficient of x^2 in $2 + x^2 + x$ is 1.

- (ii) $2 - x^2 + x^3$

The equation $2 - x^2 + x^3$ can be written as $2 + (-1)x^2 + x^3$

We know that the coefficient is the number (along with its sign, i.e. - or +) which multiplies the variable. Here, the number that multiplies the variable x^2 is -1

Hence, the coefficient of x^2 in $2 - x^2 + x^3$ is -1.

- (iii) $(\pi/2)x^2 + x$

The equation $(\pi/2)x^2 + x$ can be written as $(\pi/2)x^2 + x$

We know that the coefficient is the number (along with its sign, i.e. - or +) which multiplies the variable. Here, the number that multiplies the variable x^2 is $\pi/2$.

Hence, the coefficient of x^2 in $(\pi/2)x^2 + x$ is $\pi/2$.

- (iv) $\sqrt{2}x - 1$

The equation $\sqrt{2}x - 1$ can be written as $0x^2 + \sqrt{2}x - 1$ [Since $0x^2$ is 0]

We know that the coefficient is the number (along with its sign, i.e. - or +) which multiplies the variable. Here, the number that multiplies the variable x^2 is 0

Hence, the coefficient of x^2 in $\sqrt{2}x - 1$ is 0.

3. Give one example each of a binomial of degree 35, and of a monomial of degree 100.

SOLUTION:

Binomial of degree 35: A polynomial having two terms and the highest degree 35 is called a binomial of degree 35.

For example, $3x^{35} + 5$

Monomial of degree 100: A polynomial having one term and the highest degree 100 is called a monomial of degree 100.

For example, $4x^{100}$

4. Write the degree of each of the following polynomials:

- (i) $5x^3 + 4x^2 + 7x$ (ii) $4 - y^2$ (iii) $5t - \sqrt{7}$ (iv) 3

SOLUTION:

- (i) $5x^3 + 4x^2 + 7x$

The highest power of the variable in a polynomial is the degree of the polynomial.

Here, $5x^3 + 4x^2 + 7x = 5x^3 + 4x^2 + 7x^1$



The powers of the variable x are: 3, 2, 1

The degree of $5x^3 + 4x^2 + 7x$ is 3, as 3 is the highest power of x in the equation.

(ii) $4 - y^2$

The highest power of the variable in a polynomial is the degree of the polynomial. Here, in $4 - y^2$,

The power of the variable y is 2

The degree of $4 - y^2$ is 2, as 2 is the highest power of y in the equation.

(iii) $5t - \sqrt{7}$

The highest power of the variable in a polynomial is the degree of the polynomial. Here, in $5t - \sqrt{7}$

The power of the variable t is 1

The degree of $5t - \sqrt{7}$ is 1, as 1 is the highest power of t in the equation.

(iv) 3

The highest power of the variable in a polynomial is the degree of the polynomial. Here, $3 = 3 \times 1 = 3 \times x^0$

The power of the variable here is: 0 Hence, the degree of 3 is 0.

5. Classify the following as linear, quadratic and cubic polynomials:

(i) $x^2 + x$

(ii) $x - x^3$

(iii) $y + y^2 + 4$

(iv) $1 + x$

(v) $3t$

(vi) r^2

(vii) $7x^3$

SOLUTION:

We know that,

Linear polynomial: A polynomial of degree one is called a linear polynomial. Quadratic polynomial: A polynomial of degree two is called a quadratic polynomial. Cubic polynomial: A polynomial of degree three is called a cubic polynomial.

(i) $x^2 + x$

The highest power of $x^2 + x$ is 2

The degree is 2

Hence, $x^2 + x$ is a quadratic polynomial

(ii) $x - x^3$

The highest power of $x - x^3$ is 3

The degree is 3

Hence, $x - x^3$ is a cubic polynomial

(iii) $y + y^2 + 4$

The highest power of $y + y^2 + 4$ is 2

The degree is 2

Hence, $y + y^2 + 4$ is a quadratic polynomial

(iv) $1 + x$

The highest power of $1 + x$ is 1

The degree is 1

Hence, $1 + x$ is a linear polynomial.



(v) $3t$

The highest power of $3t$ is 1 The degree is 1
Hence, $3t$ is a linear polynomial.

(vi) r^2

The highest power of r^2 is 2 The degree is 2
Hence, r^2 is a quadratic polynomial.

(vii) $7x^3$

The highest power of $7x^3$ is 3 The degree is 3
Hence, $7x^3$ is a cubic polynomial.

EXERCISE 2.2

1. Find the value of the polynomial $(x) = 5x - 4x^2 + 3$.

(i) $x = 0$

(ii) $x = -1$

(iii) $x = 2$

SOLUTION:

Let $f(x) = 5x - 4x^2 + 3$

(i) $x = 0$

When $x = 0$

$$f(0) = 5(0) - 4(0)^2 + 3$$

$$= 3$$

(ii) When $x = -1$

$f(x) = 5x - 4x^2 + 3$

$$f(-1) = 5(-1) - 4(-1)^2 + 3$$

$$= -5 - 4 + 3$$

$$= -6$$

(iii) When $x = 2$

$f(x) = 5x - 4x^2 + 3$

$$f(2) = 5(2) - 4(2)^2 + 3$$

$$= 10 - 16 + 3$$

$$= -3$$

2. Find $p(0)$, $p(1)$ and $p(2)$ for each of the following polynomials:

(i) $p(y) = y^2 - y + 1$

(ii) $p(t) = 2 + t + 2t^2 - t^3$

(iii) $p(x) = x^3$

(iv) $p(x) = (x - 1)(x + 1)$

SOLUTION:

(i) $p(y) = y^2 - y + 1$

$\therefore p(0) = (0)^2 - (0) + 1 = 1$

$p(1) = (1)^2 - (1) + 1 = 1$

$p(2) = (2)^2 - (2) + 1 = 3$



$$\begin{aligned} \therefore p(-1) &= (-1 + 1)(-1 - 2) \\ &= (0)(-3) = 0 \\ p(2) &= (2 + 1)(2 - 2) = (3)(0) = 0 \\ \therefore -1, 2 &\text{ are zeros of } p(x). \end{aligned}$$

(v) $p(x) = x^2, x = 0$

$$\begin{aligned} \text{For, } x = 0 \quad p(x) &= x^2 \\ p(0) &= 0^2 = 0 \\ \therefore 0 &\text{ is a zero of } p(x). \end{aligned}$$

(vi) $p(x) = lx + m, x = -m/l$

$$\begin{aligned} \text{For, } x = -m/l; \quad p(x) &= lx + m \\ \therefore p(-m/l) &= l(-m/l) + m = -m + m = 0 \\ \therefore -m/l &\text{ is a zero of } p(x). \end{aligned}$$

(vii) $p(x) = 3x^2 - 1, x = -1/\sqrt{3}, 2/\sqrt{3}$

$$\begin{aligned} \text{For, } x = -1/\sqrt{3}, 2/\sqrt{3}; \quad p(x) &= 3x^2 - 1 \\ \therefore p(-1/\sqrt{3}) &= 3(-1/\sqrt{3})^2 - 1 = 3(1/3) - 1 = 1 - 1 = 0 \\ \therefore p(2/\sqrt{3}) &= 3(2/\sqrt{3})^2 - 1 = 3(4/3) - 1 = 4 - 1 = 3 \neq 0 \\ \therefore 1/\sqrt{3} &\text{ is zero of } p(x), \text{ but } 2/\sqrt{3} \text{ is not a zero of } p(x) \end{aligned}$$

(viii) $p(x) = 2x + 1, x = 1/2$

$$\begin{aligned} \text{For, } x = 1/2 \quad p(x) &= 2x + 1 \\ \therefore p(1/2) &= 2(1/2) + 1 = 1 + 1 = 2 \neq 0 \\ \therefore 1/2 &\text{ is not a zero of } p(x). \end{aligned}$$

4. Find the zero of the polynomials in each of the following cases:

(i) $p(x) = x + 5$

(ii) $p(x) = x - 5$

(iii) $p(x) = 2x + 5$

(iv) $p(x) = 3x - 2$

(v) $p(x) = 3x$

(vi) $p(x) = ax, a \neq 0$

(vii) $p(x) = cx + d, c \neq 0, c, d$ are real numbers.

SOLUTION:

(i) $p(x) = x + 5$

$$\Rightarrow x + 5 = 0$$

$$\Rightarrow x = -5$$

$\therefore -5$ is a zero of the polynomial $p(x)$.

(ii) $p(x) = x - 5$

$$\Rightarrow x - 5 = 0$$

$$\Rightarrow x = 5$$

$\therefore 5$ is a zero of the polynomial $p(x)$.



- (iii) $p(x) = 2x + 5$
 $\Rightarrow 2x + 5 = 0$
 $\Rightarrow 2x = -5$
 $\Rightarrow x = -5/2$
 $\therefore x = -5/2$ is a zero of the polynomial $p(x)$.
- (iv) $p(x) = 3x - 2$
 $\Rightarrow 3x - 2 = 0$
 $\Rightarrow 3x = 2$
 $\Rightarrow x = 2/3$
 $\therefore x = 2/3$ is a zero of the polynomial $p(x)$.
- (v) $p(x) = 3x$
 $\Rightarrow 3x = 0$
 $\Rightarrow x = 0$
 $\therefore 0$ is a zero of the polynomial $p(x)$.
- (vi) $p(x) = ax, a \neq 0$
 $p(x) = ax$
 $\Rightarrow ax = 0$
 $\Rightarrow x = 0$
 $\therefore x = 0$ is a zero of the polynomial $p(x)$.
- (vii) $p(x) = cx + d, c \neq 0, c, d$ are real numbers.
 $p(x) = cx + d$
 $\Rightarrow cx + d = 0$
 $\Rightarrow x = -d/c$
 $\therefore x = -d/c$ is a zero of the polynomial $p(x)$.

EXERCISE 2.4

1. Determine which of the following polynomials has $(x + 1)$ a factor:

- (i) $x^3 + x^2 + x + 1$ (ii) $x^4 + x^3 + x^2 + x + 1$ (iii) $x^4 + 3x^3 + 3x^2 + x + 1$
 (iv) $x^3 - x^2 - (2 - \sqrt{2})x + \sqrt{2}$

SOLUTION:

(i) $x^3 + x^2 + x + 1$

Let $p(x) = x^3 + x^2 + x + 1$

The zero of $x + 1$ is -1 . [$x + 1 = 0$ means $x = -1$] $p(-1) = (-1)^3 + (-1)^2 + (-1) + 1$

$= -1 + 1 - 1 + 1$

$= 0$

\therefore By factor theorem, $x + 1$ is a factor of $x^3 + x^2 + x + 1$



(ii) $x^4 + x^3 + x^2 + x + 1$

Let $p(x) = x^4 + x^3 + x^2 + x + 1$

The zero of $x+1$ is -1 . [$x + 1 = 0$ means $x = -1$]
 $p(-1) = (-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1$
 $= 1 - 1 + 1 - 1 + 1$
 $= 1 \neq 0$

∴ By factor theorem, $x + 1$ is not a factor of $x^4 + x^3 + x^2 + x + 1$

(iii) $x^4 + 3x^3 + 3x^2 + x + 1$

Let $p(x) = x^4 + 3x^3 + 3x^2 + x + 1$ The zero of $x + 1$ is -1 .

$p(-1) = (-1)^4 + 3(-1)^3 + 3(-1)^2 + (-1) + 1$
 $= 1 - 3 + 3 - 1 + 1$
 $= 1 \neq 0$

∴ By factor theorem, $x + 1$ is not a factor of $x^4 + 3x^3 + 3x^2 + x + 1$

(iv) $x^3 - x^2 - (2 - \sqrt{2})x + \sqrt{2}$

Let $p(x) = x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$ The zero of $x + 1$ is -1 .

$p(-1) = (-1)^3 - (-1)^2 - (2 + \sqrt{2})(-1) + \sqrt{2} = -1 - 1 + 2 + \sqrt{2} + \sqrt{2}$
 $= 2\sqrt{2} \neq 0$

∴ By factor theorem, $x+1$ is not a factor of $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

2. Use the Factor Theorem to determine whether $g(x)$ is a factor of $p(x)$ in each of the following cases:

(i) $p(x) = 2x^3 + x^2 - 2x - 1$, $g(x) = x + 1$

(ii) $p(x) = x^3 + 3x^2 + 3x + 1$, $g(x) = x + 2$

(iii) $p(x) = x^3 - 4x^2 + x + 6$, $g(x) = x - 3$

SOLUTION:

(i) $p(x) = 2x^3 + x^2 - 2x - 1$, $g(x) = x + 1$

$g(x) = 0$

$\Rightarrow x + 1 = 0$

$\Rightarrow x = -1$

∴ Zero of $g(x)$ is -1 .

Now,

$p(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1$
 $= -2 + 1 + 2 - 1$
 $= 0$

∴ By factor theorem, $g(x)$ is a factor of $p(x)$.

(ii) $p(x) = x^3 + 3x^2 + 3x + 1$, $g(x) = x + 2$

$g(x) = 0$

$\Rightarrow x + 2 = 0$

$\Rightarrow x = -2$



∴ Zero of $g(x)$ is -2 .

Now,

$$\begin{aligned} p(-2) &= (-2)^3 + 3(-2)^2 + 3(-2) + 1 \\ &= -8 + 12 - 6 + 1 \\ &= -1 \neq 0 \end{aligned}$$

∴ By factor theorem, $g(x)$ is not a factor of $p(x)$.

(iii) $p(x) = x^3 - 4x^2 + x + 6$, $g(x) = x - 3$

$$g(x) = 0$$

$$\Rightarrow x - 3 = 0$$

$$\Rightarrow x = 3$$

∴ Zero of $g(x)$ is 3 .

Now,

$$\begin{aligned} p(3) &= (3)^3 - 4(3)^2 + (3) + 6 \\ &= 27 - 36 + 3 + 6 \\ &= 0 \end{aligned}$$

∴ By factor theorem, $g(x)$ is a factor of $p(x)$.

3. Find the value of k , if $x - 1$ is a factor of $p(x)$ in each of the following cases:

(i) $p(x) = x^2 + x + k$

(ii) $p(x) = 2x^2 + kx + \sqrt{2}$

(iii) $p(x) = kx^2 - \sqrt{2}x + 1$

(iv) $p(x) = kx^2 - 3x + k$

SOLUTION:

(i) $p(x) = x^2 + x + k$

If $x - 1$ is a factor of $p(x)$, then $p(1) = 0$ By Factor Theorem

$$\Rightarrow (1)^2 + (1) + k = 0$$

$$\Rightarrow 1 + 1 + k = 0$$

$$\Rightarrow 2 + k = 0$$

$$\Rightarrow k = -2$$

(ii) $p(x) = 2x^2 + kx + \sqrt{2}$

If $x - 1$ is a factor of $p(x)$, then $p(1) = 0$

$$\Rightarrow 2(1)^2 + k(1) + \sqrt{2} = 0$$

$$\Rightarrow 2 + k + \sqrt{2} = 0$$

$$\Rightarrow k = -(2 + \sqrt{2})$$

(iii) $p(x) = kx^2 - \sqrt{2}x + 1$

If $x - 1$ is a factor of $p(x)$, then $p(1) = 0$ By Factor Theorem



$$\Rightarrow k(1)^2 - \sqrt{2}(1) + 1 = 0$$

$$\Rightarrow k = \sqrt{2} - 1$$

(iv) $p(x) = kx^2 - 3x + k$

If $x - 1$ is a factor of $p(x)$, then $p(1) = 0$

By Factor Theorem

$$\Rightarrow k(1)^2 - 3(1) + k = 0$$

$$\Rightarrow k - 3 + k = 0$$

$$\Rightarrow 2k - 3 = 0$$

$$\Rightarrow k = 3/2$$

4. Factorise:

(i) $12x^2 - 7x + 1$

(ii) $2x^2 + 7x + 3$

(iii) $6x^2 + 5x - 6$

(iv) $3x^2 - x - 4$

SOLUTION:

(i) $12x^2 - 7x + 1$

Using the splitting the middle term method,

We have to find a number whose sum = -7 and product = $1 \times 12 = 12$

We get -3 and -4 as the numbers [$-3 + -4 = -7$ and $-3 \times -4 = 12$]

$$12x^2 - 7x + 1 = 12x^2 - 4x - 3x + 1$$

$$= 4x(3x - 1) - 1(3x - 1)$$

$$= (4x - 1)(3x - 1)$$

(ii) $2x^2 + 7x + 3$

Using the splitting the middle term method,

We have to find a number whose sum = 7 and product = $2 \times 3 = 6$ We get 6 and 1 as the numbers [$6 + 1 = 7$ and $6 \times 1 = 6$]

$$2x^2 + 7x + 3 = 2x^2 + 6x + 1x + 3$$

$$= 2x(x + 3) + 1(x + 3)$$

$$= (2x + 1)(x + 3)$$

(iii) $6x^2 + 5x - 6$

Using the splitting the middle term method,

We have to find a number whose sum = 5 and product = $6 \times -6 = -36$ We get -4 and 9 as the numbers [$-4 + 9 = 5$ and $-4 \times 9 = -36$]

$$6x^2 + 5x - 6 = 6x^2 + 9x - 4x - 6$$

$$= 3x(2x + 3) - 2(2x + 3)$$

$$= (2x + 3)(3x - 2)$$

(iv) $3x^2 - x - 4$



Using the splitting the middle term method,

We have to find a number whose sum = -1 and product = $3 \times -4 = -12$ We get -4 and 3 as the numbers [$-4 + 3 = -1$ and $-4 \times 3 = -12$]

$$\begin{aligned} 3x^2 - x - 4 &= 3x^2 - 4x + 3x - 4 \\ &= x(3x - 4) + 1(3x - 4) \\ &= (3x - 4)(x + 1) \end{aligned}$$

5. Factorise:

(i) $x^3 - 2x^2 - x + 2$ (ii) $x^3 - 3x^2 - 9x - 5$ (iii) $x^3 + 13x^2 + 32x + 20$

(iv) $2y^3 + y^2 - 2y - 1$

SOLUTION:

Let $p(x) = x^3 - 2x^2 - x + 2$ Factors of 2 are ± 1 and ± 2 Now,

$$p(x) = x^3 - 2x^2 - x + 2$$

$$p(-1) = (-1)^3 - 2(-1)^2 - (-1) + 2$$

$$= -1 - 2 + 1 + 2$$

$$= 0$$

Therefore, $(x + 1)$ is the factor of $p(x)$

$$\begin{array}{r} x^2 - 3x + 2 \\ x+1 \overline{) x^3 - 2x^2 - x + 2} \\ \underline{x^3 + x^2} \\ -3x^2 - x + 2 \\ \underline{-3x^2 - 3x} \\ 2x + 2 \\ \underline{2x + 2} \\ 0 \end{array}$$

Now, Dividend = Divisor \times Quotient + Remainder

$$(x + 1)(x^2 - 3x + 2) = (x + 1)(x^2 - x - 2x + 2)$$

$$= (x + 1)(x(x - 1) - 2(x - 1))$$

$$= (x + 1)(x - 1)(x - 2)$$

(ii) $x^3 - 3x^2 - 9x - 5$

Let $p(x) = x^3 - 3x^2 - 9x - 5$ Factors of 5 are ± 1 and ± 5

By the trial method, we find that $p(5) = 0$

So, $(x - 5)$ is factor of $p(x)$ Now,

$$p(x) = x^3 - 3x^2 - 9x - 5$$

$$p(5) = (5)^3 - 3(5)^2 - 9(5) - 5$$



$$= 125 - 75 - 45 - 5$$

$$= 0$$

Therefore, $(x - 5)$ is the factor of $p(x)$

$$\begin{array}{r}
 x^2 + 2x + 1 \\
 x - 5 \overline{) x^3 - 3x^2 - 9x - 5} \\
 \underline{- + 5x^2} \\
 2x^2 - 9x - 5 \\
 \underline{- 2x^2 + 10x} \\
 x - 5 \\
 \underline{- x + 5} \\
 0
 \end{array}$$

Now, Dividend = Divisor \times Quotient + Remainder

$$(x - 5)(x^2 + 2x + 1) = (x - 5)(x^2 + x + x + 1)$$

$$= (x - 5)(x(x + 1) + 1(x + 1))$$

$$= (x - 5)(x + 1)(x + 1)$$

(iii) $x^3 + 13x^2 + 32x + 20$

Let $p(x) = x^3 + 13x^2 + 32x + 20$

Factors of 20 are $\pm 1, \pm 2, \pm 4, \pm 5, \pm 10$ and ± 20

By the trial method, we find that

$$p(-1) = 0$$

So, $(x + 1)$ is factor of $p(x)$

Now,

$$p(x) = x^3 + 13x^2 + 32x + 20$$

$$p(-1) = (-1)^3 + 13(-1)^2 + 32(-1) + 20$$

$$= -1 + 13 - 32 + 20$$

$$= 0$$

Therefore, $(x + 1)$ is the factor of $p(x)$

$$\begin{array}{r}
 x^2 + 12x + 20 \\
 x + 1 \overline{) x^3 + 13x^2 + 32x + 20} \\
 \underline{- x^3 + x^2} \\
 12x^2 + 32x + 20 \\
 \underline{- 12x^2 + 12x} \\
 20x + 20 \\
 \underline{- 20x + 20} \\
 0
 \end{array}$$



Now, Dividend = Divisor \times Quotient + Remainder

$$\begin{aligned}(x + 1)(x^2 + 12x + 20) &= (x + 1)(x^2 + 2x + 10x + 20) \\ &= (x + 1)(x(x + 2) + 10(x + 2)) \\ &= (x + 1)(x + 2)(x + 10)\end{aligned}$$

(iv) $2y^3 + y^2 - 2y - 1$

Let $p(y) = 2y^3 + y^2 - 2y - 1$

Factors = $2 \times (-1) = -2$ are ± 1 and ± 2 By the trial method, we find that $p(1) = 0$

So, $(y - 1)$ is factor of $p(y)$ Now,

$p(y) = 2y^3 + y^2 - 2y - 1$

$p(1) = 2(1)^3 + (1)^2 - 2(1) - 1$

$= 2 + 1 - 2$

$= 0$

Therefore, $(y - 1)$ is the factor of $p(y)$

$$\begin{array}{r} 2y^2 + 3y + 1 \\ y-1 \overline{) 2y^3 + y^2 - 2y - 1} \\ \underline{- 2y^3 + 2y^2} \\ 3y^2 - 2y - 1 \\ \underline{- 3y^2 + 3y} \\ y - 1 \\ \underline{- y + 1} \\ 0 \end{array}$$

Now, Dividend = Divisor \times Quotient + Remainder

$$\begin{aligned}(y - 1)(2y^2 + 3y + 1) &= (y - 1)(2y^2 + 2y + y + 1) \\ &= (y - 1)(2y(y + 1) + 1(y + 1)) \\ &= (y - 1)(2y + 1)(y + 1)\end{aligned}$$

EXERCISE 2.5

1. Use suitable identities to find the following products:

- (i) $(x + 4)(x + 10)$ (ii) $(x + 8)(x - 10)$ (iii) $(3x + 4)(3x - 5)$
- (iv) $(y^2 + 3/2)(y^2 - 3/2)$

SOLUTION:

(i) $(x + 4)(x + 10)$

Using the identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$



[Here, $a = 4$ and $b = 10$]

We get,

$$\begin{aligned}(x + 4)(x + 10) &= x^2 + (4 + 10)x + (4 \times 10) \\ &= x^2 + 14x + 40\end{aligned}$$

(ii) $(x + 8)(x - 10)$

Using the identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$

[Here, $a = 8$ and $b = -10$]

We get,

$$\begin{aligned}(x + 8)(x - 10) &= x^2 + (8 + (-10))x + (8 \times (-10)) \\ &= x^2 + (8 - 10)x - 80 \\ &= x^2 - 2x - 80\end{aligned}$$

(iii) $(3x + 4)(3x - 5)$

Using the identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$

[Here, $x = 3x$, $a = 4$ and $b = -5$]

We get,

$$\begin{aligned}(3x + 4)(3x - 5) &= (3x)^2 + [4 + (-5)]3x + 4 \times (-5) \\ &= 9x^2 + 3x(4 - 5) - 20 \\ &= 9x^2 - 3x - 20\end{aligned}$$

(iv) $(y^2 + 3/2)(y^2 - 3/2)$

Using the identity, $(x + y)(x - y) = x^2 - y^2$

[Here, $x = y^2$ and $y = 3/2$] We get,

$$\begin{aligned}(y^2 + 3/2)(y^2 - 3/2) &= (y^2)^2 - (3/2)^2 \\ &= y^4 - 9/4\end{aligned}$$

2. Evaluate the following products without multiplying directly:

(i) 103×107

(ii) 95×96

(iii) 104×96

SOLUTION:

(i) 103×107

$$103 \times 107 = (100 + 3) \times (100 + 7)$$

Using identity, $[(x + a)(x + b) = x^2 + (a + b)x + ab]$ Here, $x = 100$

$$a = 3$$

$$b = 7$$

$$\text{We get, } 103 \times 107 = (100 + 3) \times (100 + 7)$$

$$= (100)^2 + (3 + 7)100 + (3 \times 7)$$

$$= 10000 + 1000 + 21$$

$$= 11021$$



(ii) 95×96

$$95 \times 96 = (100 - 5) \times (100 - 4)$$

Using identity, $[(x - a)(x - b) = x^2 - (a + b)x + ab]$ Here, $x = 100$

$$a = -5$$

$$b = -4$$

$$\text{We get, } 95 \times 96 = (100 - 5) \times (100 - 4)$$

$$= (100)^2 + 100(-5 + (-4)) + (-5 \times -4)$$

$$= 10000 - 900 + 20$$

$$= 9120$$

(iii) 104×96

$$104 \times 96 = (100 + 4) \times (100 - 4)$$

Using identity, $[(a + b)(a - b) = a^2 - b^2]$ Here, $a = 100$ $b = 4$

$$\text{We get, } 104 \times 96 = (100 + 4) \times (100 - 4)$$

$$= (100)^2 - (4)^2$$

$$= 10000 - 16$$

$$= 9984$$

3. Factorise the following using appropriate identities:

(i) $9x^2 + 6xy + y^2$

(ii) $4y^2 - 4y + 1$

(iii) $x^2 - y^2/100$

SOLUTION:

(i) $9x^2 + 6xy + y^2$

$$9x^2 + 6xy + y^2 = (3x)^2 + (2 \times 3x \times y) + y^2$$

Using identity, $x^2 + 2xy + y^2 = (x + y)^2$ Here, $x = 3x$

$$y = y$$

$$9x^2 + 6xy + y^2 = (3x)^2 + (2 \times 3x \times y) + y^2$$

$$= (3x + y)^2$$

$$= (3x + y)(3x + y)$$

(ii) $4y^2 - 4y + 1$

$$4y^2 - 4y + 1 = (2y)^2 - (2 \times 2y \times 1) + 1$$

Using identity, $x^2 - 2xy + y^2 = (x - y)^2$ Here, $x = 2y$

$$y = 1$$



$$4y^2 - 4y + 1 = (2y)^2 - (2 \times 2y \times 1) + 1^2$$

$$= (2y - 1)^2$$

$$= (2y - 1)(2y - 1)$$

(iii) $x^2 - y^2/100$

$$x^2 - y^2/100 = x^2 - (y/10)^2$$

Using identity, $x^2 - y^2 = (x - y)(x + y)$

Here, $x = x$

$y = y/10$

$$x^2 - y^2/100 = x^2 - (y/10)^2$$

$$= (x - y/10)(x + y/10)$$

4. Expand each of the following using suitable identities:

(i) $(x + 2y + 4z)^2$

(ii) $(2x - y + z)^2$

(iii) $(-2x + 3y + 2z)^2$

(iv) $(3a - 7b - c)^2$

(v) $(-2x + 5y - 3z)^2$

(vi) $((1/4)a - (1/2)b + 1)^2$

SOLUTION:

(i) $(x + 2y + 4z)^2$

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x = x$

$y = 2y$

$z = 4z$

$$(x + 2y + 4z)^2 = x^2 + (2y)^2 + (4z)^2 + (2 \times x \times 2y) + (2 \times 2y \times 4z) + (2 \times 4z \times x)$$

$$= x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8xz$$

(ii) $(2x - y + z)^2$

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x = 2x$

$y = -y$

$z = z$

$$(2x - y + z)^2 = (2x)^2 + (-y)^2 + z^2 + (2 \times 2x \times -y) + (2 \times -y \times z) + (2 \times z \times 2x)$$

$$= 4x^2 + y^2 + z^2 - 4xy - 2yz + 4xz$$

(iii) $(-2x + 3y + 2z)^2$

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x = -2x$

$y = 3y$

$z = 2z$

$$(-2x + 3y + 2z)^2 = (-2x)^2 + (3y)^2 + (2z)^2 + (2 \times -2x \times 3y) + (2 \times 3y \times 2z) + (2 \times 2z \times -2x)$$

$$= 4x^2 + 9y^2 + 4z^2 - 12xy + 12yz - 8xz$$

(iv) $(3a - 7b - c)^2$



Using identity $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$ Here, $x = 3a$

$$y = -7b$$

$$z = -c$$

$$(3a - 7b - c)^2 = (3a)^2 + (-7b)^2 + (-c)^2 + (2 \times 3a \times -7b) + (2 \times -7b \times -c) + (2 \times -c \times 3a)$$

$$= 9a^2 + 49b^2 + c^2 - 42ab + 14bc - 6ca$$

(v) $(-2x + 5y - 3z)^2$

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x = -2x$

$$y = 5y$$

$$z = -3z$$

$$(-2x + 5y - 3z)^2 = (-2x)^2 + (5y)^2 + (-3z)^2 + (2 \times -2x \times 5y) + (2 \times 5y \times -3z) + (2 \times -3z \times -2x)$$

$$= 4x^2 + 25y^2 + 9z^2 - 20xy - 30yz + 12zx$$

(vi) $((\frac{1}{4})a - (\frac{1}{2})b + 1)^2$

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x = (\frac{1}{4})a$

$$y = (-\frac{1}{2})b$$

$$z = 1$$

$$\left(\left(\frac{1}{4} \right) a - \left(\frac{1}{2} \right) b + 1 \right)^2 = \left(\frac{1}{4} a \right)^2 + \left(-\frac{1}{2} b \right)^2 + (1)^2 + \left(2 \times \frac{1}{4} a \times \frac{1}{2} b \right) + \left(2 \times -\frac{1}{2} b \times 1 \right) + \left(2 \times 1 \times \frac{1}{4} a \right)$$

$$= \frac{1}{16} a^2 + \frac{1}{4} b^2 + 1^2 - \frac{2}{8} ab - \frac{2}{2} b + \frac{2}{4} a$$

$$= \frac{1}{16} a^2 + \frac{1}{4} b^2 + 1 - \frac{1}{4} ab - b + \frac{1}{2} a$$

5. Factorise:

(i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

SOLUTION:

(i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

We can say that, $x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = (x + y + z)^2$

$$4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz = (2x)^2 + (3y)^2 + (-4z)^2 + (2 \times 2x \times 3y) + (2 \times 3y \times -4z) + (2 \times -4z \times 2x)$$

$$= (2x + 3y - 4z)^2$$

$$= (2x + 3y - 4z)(2x + 3y - 4z)$$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

We can say that, $x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = (x + y + z)^2$



$$\begin{aligned}
 & 2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz \\
 &= (-\sqrt{2}x)^2 + (y)^2 + (2\sqrt{2}z)^2 + (2 \times -\sqrt{2}x \times y) + (2 \times y \times 2\sqrt{2}z) + (2 \times 2\sqrt{2} \times -\sqrt{2}x) \\
 &= (-\sqrt{2}x + y + 2\sqrt{2}z)^2 \\
 &= (-\sqrt{2}x + y + 2\sqrt{2}z)(-\sqrt{2}x + y + 2\sqrt{2}z)
 \end{aligned}$$

6. Write the following cubes in expanded form:

(i) $(2x + 1)^3$

(ii) $(2a - 3b)^3$

(iii) $((3/2)x + 1)^3$

(iv) $(x - (2/3)y)^3$

SOLUTION:

(i) $(2x + 1)^3$

Using identity, $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$

$$(2x + 1)^3 = (2x)^3 + 1^3 + (3 \times 2x \times 1)(2x + 1)$$

$$= 8x^3 + 1 + 6x(2x + 1)$$

$$= 8x^3 + 12x^2 + 6x + 1$$

(ii) $(2a - 3b)^3$

Using identity, $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

$$(2a - 3b)^3 = (2a)^3 - (3b)^3 - (3 \times 2a \times 3b)(2a - 3b)$$

$$= 8a^3 - 27b^3 - 18ab(2a - 3b)$$

$$= 8a^3 - 27b^3 - 36a^2b + 54ab^2$$

(iii) $((3/2)x + 1)^3$

Using identity, $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$

$$((3/2)x + 1)^3 = ((3/2)x)^3 + 1^3 + (3 \times (3/2)x \times 1)((3/2)x + 1)$$

$$= \frac{27}{8}x^3 + 1 + \frac{9}{2}x \left(\frac{3}{2}x + 1 \right)$$

$$= \frac{27}{8}x^3 + 1 + \frac{27}{4}x^2 + \frac{9}{2}x$$

$$= \frac{27}{8}x^3 + \frac{27}{4}x^2 + \frac{9}{2}x + 1$$

(iv) $(x - (2/3)y)^3$

Using identity, $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

$$\left(x - \frac{2}{3}y \right)^3 = (x)^3 - \left(\frac{2}{3}y \right)^3 - \left(3 \times x \times \frac{2}{3}y \right) \left(x - \frac{2}{3}y \right)$$

$$= (x)^3 - \frac{8}{27}y^3 - 2xy \left(x - \frac{2}{3}y \right)$$

$$= (x)^3 - \frac{8}{27}y^3 - 2x^2y + \frac{4}{3}xy^2$$



7. Evaluate the following using suitable identities:

- (i) $(99)^3$ (ii) $(102)^3$ (iii) $(998)^3$

SOLUTIONS:

- (i) $(99)^3$

We can write 99 as $100 - 1$

Using identity, $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

$$\begin{aligned} (99)^3 &= (100 - 1)^3 \\ &= (100)^3 - 1^3 - (3 \times 100 \times 1)(100 - 1) \\ &= 1000000 - 1 - 300(100 - 1) \\ &= 1000000 - 1 - 30000 + 300 \\ &= 970299 \end{aligned}$$

- (ii) $(102)^3$

We can write 102 as $100 + 2$

Using identity, $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$

$$\begin{aligned} (100 + 2)^3 &= (100)^3 + 2^3 + (3 \times 100 \times 2)(100 + 2) \\ &= 1000000 + 8 + 600(100 + 2) \\ &= 1000000 + 8 + 60000 + 1200 \\ &= 1061208 \end{aligned}$$

- (iii) $(998)^3$

We can write 99 as $1000 - 2$

Using identity, $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

$$\begin{aligned} (998)^3 &= (1000 - 2)^3 \\ &= (1000)^3 - 2^3 - (3 \times 1000 \times 2)(1000 - 2) \\ &= 1000000000 - 8 - 6000(1000 - 2) \\ &= 1000000000 - 8 - 6000000 + 12000 \\ &= 994011992 \end{aligned}$$

8. Factorise each of the following:

- (i) $8a^3 + b^3 + 12a^2b + 6ab^2$ (ii) $8a^3 - b^3 - 12a^2b + 6ab^2$
 (iii) $27 - 125a^3 - 135a + 225a^2$ (iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$
 (v) $27p^3 - (1/216) - (9/2)p^2 + (1/4)p$

SOLUTIONS:

- (i) $8a^3 + b^3 + 12a^2b + 6ab^2$

The expression, $8a^3 + b^3 + 12a^2b + 6ab^2$ can be written as $(2a)^3 + b^3 + 3(2a)^2b + 3(2a)(b)^2$

$$\begin{aligned} 8a^3 + b^3 + 12a^2b + 6ab^2 &= (2a)^3 + b^3 + 3(2a)^2b + 3(2a)(b)^2 \\ &= (2a + b)^3 \end{aligned}$$



$$= (2a + b)(2a + b)(2a + b)$$

Here, the identity, $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$ is used.

(ii) $8a^3 - b^3 - 12a^2b + 6ab^2$

The expression, $8a^3 - b^3 - 12a^2b + 6ab^2$ can be written as $(2a)^3 - b^3 - 3(2a)^2b + 3(2a)(b)^2$

$$8a^3 - b^3 - 12a^2b + 6ab^2 = (2a)^3 - b^3 - 3(2a)^2b + 3(2a)(b)^2$$

$$= (2a - b)^3$$

$$= (2a - b)(2a - b)(2a - b)$$

Here, the identity, $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$ is used.

(iii) $27 - 125a^3 - 135a + 225a^2$

The expression, $27 - 125a^3 - 135a + 225a^2$ can be written as $3^3 - (5a)^3 - 3(3)^2(5a) + 3(3)(5a)^2$

$$27 - 125a^3 - 135a + 225a^2 = 3^3 - (5a)^3 - 3(3)^2(5a) + 3(3)(5a)^2$$

$$= (3 - 5a)^3$$

$$= (3 - 5a)(3 - 5a)(3 - 5a)$$

Here, the identity, $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$ is used.

(iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

The expression, $64a^3 - 27b^3 - 144a^2b + 108ab^2$ can be written as $(4a)^3 - (3b)^3 - 3(4a)^2(3b) + 3(4a)(3b)^2$

$$64a^3 - 27b^3 - 144a^2b + 108ab^2 = (4a)^3 - (3b)^3 - 3(4a)^2(3b) + 3(4a)(3b)^2$$

$$= (4a - 3b)^3$$

$$= (4a - 3b)(4a - 3b)(4a - 3b)$$

Here, the identity, $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$ is used.

(v) $27p^3 - (1/216) - (9/2)p^2 + (1/4)p$

The expression, $27p^3 - (1/216) - (9/2)p^2 + (1/4)p$ can be written as

$$(3p)^3 - (1/6)^3 - (9/2)p^2 + (1/4)p = (3p)^3 - (1/6)^3 - 3(3p)(1/6)(3p - 1/6)$$

Using $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

$$27p^3 - (1/216) - (9/2)p^2 + (1/4)p = (3p)^3 - (1/6)^3 - 3(3p)(1/6)(3p - 1/6)$$

Taking $x = 3p$ and $y = 1/6$

$$= (3p - 1/6)^3$$

$$= (3p - 1/6)(3p - 1/6)(3p - 1/6)$$

9. Verify:

(i) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

(ii) $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

SOLUTIONS:

(i) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

We know that, $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$

$$\Rightarrow x^3 + y^3 = (x + y)^3 - 3xy(x + y)$$

$$\Rightarrow x^3 + y^3 = (x + y)[(x + y)^2 - 3xy]$$

Taking $(x + y)$ common $\Rightarrow x^3 + y^3 = (x + y)[(x^2 + y^2 + 2xy) - 3xy]$



$$\Rightarrow x^3 + y^3 = (x + y)(x^2 + y^2 - xy)$$

(ii) $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

We know that, $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

$$\Rightarrow x^3 - y^3 = (x - y)^3 + 3xy(x - y)$$

$$\Rightarrow x^3 - y^3 = (x - y)[(x - y)^2 + 3xy]$$

Taking $(x + y)$ common $\Rightarrow x^3 - y^3 = (x - y)[(x^2 + y^2 - 2xy) + 3xy]$

$$\Rightarrow x^3 - y^3 = (x - y)(x^2 + y^2 + xy)$$

10. Factorise each of the following:

(i) $27y^3 + 125z^3$

(ii) $64m^3 - 343n^3$

SOLUTIONS:

(i) $27y^3 + 125z^3$

The expression, $27y^3 + 125z^3$ can be written as $(3y)^3 + (5z)^3$

$$27y^3 + 125z^3 = (3y)^3 + (5z)^3$$

We know that, $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

$$27y^3 + 125z^3 = (3y)^3 + (5z)^3$$

$$= (3y + 5z)[(3y)^2 - (3y)(5z) + (5z)^2]$$

$$= (3y + 5z)(9y^2 - 15yz + 25z^2)$$

(ii) $64m^3 - 343n^3$

The expression, $64m^3 - 343n^3$ can be written as $(4m)^3 - (7n)^3$

$$64m^3 - 343n^3 = (4m)^3 - (7n)^3$$

We know that, $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

$$64m^3 - 343n^3 = (4m)^3 - (7n)^3$$

$$= (4m - 7n)[(4m)^2 + (4m)(7n) + (7n)^2]$$

$$= (4m - 7n)(16m^2 + 28mn + 49n^2)$$

11. Factorise: $27x^3 + y^3 + z^3 - 9xyz$.

SOLUTION:

The expression $27x^3 + y^3 + z^3 - 9xyz$ can be written as $(3x)^3 + y^3 + z^3 - 3(3x)(y)(z)$

$$27x^3 + y^3 + z^3 - 9xyz = (3x)^3 + y^3 + z^3 - 3(3x)(y)(z)$$

We know that, $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$$27x^3 + y^3 + z^3 - 9xyz = (3x)^3 + y^3 + z^3 - 3(3x)(y)(z)$$

$$= (3x + y + z)[(3x)^2 + y^2 + z^2 - 3xy - yz - 3xz]$$

$$= (3x + y + z)(9x^2 + y^2 + z^2 - 3xy - yz - 3xz)$$

12. Verify that:

$$x^3 + y^3 + z^3 - 3xyz = (1/2)(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$$



SOLUTION:

We know that,

$$\begin{aligned} x^3 + y^3 + z^3 - 3xyz &= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - xz) \\ \Rightarrow x^3 + y^3 + z^3 - 3xyz &= (1/2)(x + y + z)[2(x^2 + y^2 + z^2 - xy - yz - xz)] \\ &= (1/2)(x + y + z)(2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2xz) \\ &= (1/2)(x + y + z)[(x^2 + y^2 - 2xy) + (y^2 + z^2 - 2yz) + (x^2 + z^2 - 2xz)] \\ &= (1/2)(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2] \end{aligned}$$

13. If $x + y + z = 0$, show that $x^3 + y^3 + z^3 = 3xyz$.

SOLUTION:

We know that,

$$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - xz)$$

Now, according to the question, let $(x + y + z) = 0$,

$$\text{Then, } x^3 + y^3 + z^3 - 3xyz = (0)(x^2 + y^2 + z^2 - xy - yz - xz)$$

$$\Rightarrow x^3 + y^3 + z^3 - 3xyz = 0$$

$$\Rightarrow x^3 + y^3 + z^3 = 3xyz$$

Hence Proved

14. Without actually calculating the cubes, find the value of each of the following:

(i) $(-12)^3 + (7)^3 + (5)^3$

(ii) $(28)^3 + (-15)^3 + (-13)^3$

SOLUTION:

(i) $(-12)^3 + (7)^3 + (5)^3$

Let $a = -12$

$b = 7$

$c = 5$

We know that if $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$.

Here, $-12 + 7 + 5 = 0$

$$(-12)^3 + (7)^3 + (5)^3 = 3xyz$$

$$= 3 \times -12 \times 7 \times 5$$

$$= -1260$$

(ii) $(28)^3 + (-15)^3 + (-13)^3$

Let $a = 28$

$b = -15$

$c = -13$

We know that if $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$.

Here, $x + y + z = 28 - 15 - 13 = 0$



$$\begin{aligned}(28)^3 + (-15)^3 + (-13)^3 &= 3xyz \\ &= 3(28)(-15)(-13) \\ &= 16380\end{aligned}$$

15. Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:

(i) Area: $25a^2 - 35a + 12$

(ii) Area: $35y^2 + 13y - 12$

SOLUTION:

(i) Area: $25a^2 - 35a + 12$

Using the splitting the middle term method,

We have to find a number whose sum = -35 and product = $25 \times 12 = 300$

We get -15 and -20 as the numbers [$-15 + -20 = -35$ and $-15 \times -20 = 300$]

$$25a^2 - 35a + 12 = 25a^2 - 15a - 20a + 12$$

$$= 5a(5a - 3) - 4(5a - 3)$$

$$= (5a - 4)(5a - 3)$$

Possible expression for length = $5a - 4$

Possible expression for breadth = $5a - 3$

(ii) Area: $35y^2 + 13y - 12$

Using the splitting the middle term method,

We have to find a number whose sum = 13 and product = $35 \times -12 = 420$

We get -15 and 28 as the numbers [$-15 + 28 = 13$ and $-15 \times 28 = 420$]

$$35y^2 + 13y - 12 = 35y^2 - 15y + 28y - 12$$

$$= 5y(7y - 3) + 4(7y - 3)$$

$$= (5y + 4)(7y - 3)$$

Possible expression for length = $(5y + 4)$

Possible expression for breadth = $(7y - 3)$

16. What are the possible expressions for the dimensions of the cuboids whose volumes are given below?

(i) Volume: $3x^2 - 12x$

(ii) Volume: $12ky^2 + 8ky - 20k$

SOLUTION:

(i) Volume: $3x^2 - 12x$

$3x^2 - 12x$ can be written as $3x(x - 4)$ by taking $3x$ out of both the terms.

Possible expression for length = 3

Possible expression for breadth = x

Possible expression for height = $(x - 4)$

(ii) Volume:



$$12ky^2 + 8ky - 20k$$

$12ky^2 + 8ky - 20k$ can be written as $4k(3y^2 + 2y - 5)$ by taking $4k$ out of both the terms.

$$12ky^2 + 8ky - 20k = 4k(3y^2 + 2y - 5)$$

[Here, $3y^2 + 2y - 5$ can be written as $3y^2 + 5y - 3y - 5$ using splitting the middle term method.]

$$= 4k(3y^2 + 5y - 3y - 5)$$

$$= 4k[y(3y + 5) - 1(3y + 5)]$$

$$= 4k(3y + 5)(y - 1)$$

Possible expression for length = $4k$

Possible expression for breadth = $(3y + 5)$

Possible expression for height = $(y - 1)$



