CHAPTER 11

Surface areas and volumes

CLASS 9TH

NCERT EXERCISE AND SOLUTIONS - MATHEMATICS

EXERCISE 11.1

1. Diameter of the base of a cone is 10.5 cm, and its slant height is 10 cm. Find its curved surface area. (Assume $\pi = \frac{22}{7}$)

SOLUTION:

Radius of the base of cone = diameter/ 2 = (10.5/2)cm = 5.25cm

The slant height of the cone, say l = 10 cm

CSA of the cone is = πrl

$= \frac{22}{7} \times 5.25 \times 10 = 165\ cm^2$

Therefore, the curved surface area of the cone is 165 cm^2.

2. Find the total surface area of a cone, if its slant height is 21 m and the diameter of its base is 24 m. (Assume $\pi = \frac{22}{7}$)

SOLUTION:

Radius of cone, r = 24/2 m = 12m Slant height, l = 21 m

Formula: Total Surface area of the cone = $\pi r\ (l + r)$

Total Surface area of the cone $= \frac{22}{7} \times 12 \times (21 + 12)\ m^2$

$= 1244.57 m^2$

3. Curved surface area of a cone is 308 cm^2, and its slant height is 14 cm. Find

(i) radius of the base and (ii) total surface area of the cone.

(Assume $\pi = \frac{22}{7}$)

SOLUTION:

The slant height of the cone, l = 14 cm

Let the radius of the cone be r.

(i) We know the CSA of cone = πrl

Given: Curved surface area of a cone is 308 cm^2

$(308\ ) = \frac{22}{7} \times r \times 14$

308 = 44 r

r = 308/44 = 7 cm

The radius of a cone base is 7 cm.

(ii) Total surface area of cone = CSA of cone + Area of base (πr^2)

Total surface area of cone = $308 + \frac{22}{7} \times 72 = 308 + 154 = 462$ cm^2

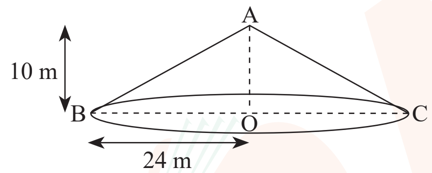
Therefore, the total surface area of the cone is 462 cm^2.

4. A conical tent is 10 m high, and the radius of its base is 24 m. Find

(i) slant height of the tent.

(ii) cost of the canvas required to make the tent, if the cost of 1 m^2 canvas is ₹70.

(Assume $\pi = \frac{22}{7}$)



SOLUTION:

Let ABC be a conical tent.

Height of conical tent, h = 10 m

Radius of conical tent, r = 24m Let the slant height of the tent be l.

(i) In the right triangle ABO, we have

$AB^2 = AO^2 + BO^2$ (using Pythagoras' theorem)

$l^2 = h^2 + r^2$

$l^2 = (10)^2 + (24)^2 = 676$

l = 26 m

Therefore, the slant height of the tent is 26 m.

(ii) CSA of tent = πrl

$= \frac{22}{7} \times 24 \times 26$ m^2

Cost of 1 m^2 canvas = ₹70

Cost of (13728/7)m^2 canvas is equal to ₹(13728/7) × 70 = ₹137280

Therefore, the cost of the canvas required to make such a tent is ₹ 137280.

5. What length of tarpaulin 3 m wide will be required to make a conical tent of height 8 m and base radius 6m? Assume that the extra length of material that will be required for stitching margins and wastage in cutting is approximately 20 cm. [Use $\pi = 3.14$]

SOLUTION:

Height of the conical tent, h = 8m Radius

of the base of the tent, r = 6m Slant height of the tent, $l^2 = (r^2 + h^2)$

$l^2 = (6^2 + 8^2) = (36 + 64) = (100)$

or l = 10 m

Again, CSA of conical tent = πrl

= (3.14 × 6 × 10) m^2

= 188.4m^2

Let the length of the tarpaulin sheet required be L.

As 20 cm will be wasted,

The effective length will be (L − 0.2m).

The breadth of tarpaulin = 3m (given)

Area of sheet = CSA of the tent

[(L – 0.2) × 3] = 188.4

L − 0.2 = 62.8

L = 63 m

Therefore, the length of the required tarpaulin sheet will be 63 m.

6. **The slant height and base diameter of the conical tomb are 25m and 14 m, respectively. Find the cost of whitewashing its curved surface at the rate of ₹. 210 per 100 m^2. (Assume $\pi = \frac{22}{7}$)**

SOLUTION:

Slant height of the conical tomb, l = 25m

Base radius, r = diameter/2 = 14/2 m = 7m

CSA of the conical tomb = πrl

$= \frac{22}{7} \times 7 \times 25 = 550$

CSA of the conical tomb = 550m^2

Cost of whitewashing 550 m^2 area, which is ₹(210×550)/100

= ₹1155

Therefore, the cost will be ₹1155 while whitewashing the tomb.

7. **A joker's cap is in the form of a right circular cone of base radius 7 cm and height 24cm. Find the area of the sheet required to make 10 such caps. (Assume $\pi = \frac{22}{7}$)**

SOLUTION:

Radius of the conical cap, r = 7 cm Height

of the conical cap, h = 24 cm Slant height,

$l^2 = (r^2 + h^2) = (7^2 + 24^2)$

= (49 + 576) = (625)

Or l = 25 cm

CSA of 1 conical cap = πrl

$= \frac{22}{7} \times 7 \times 25 = 550 \text{ cm}^2$

CSA of 10 caps = (10 × 550) cm^2 = 5500 cm^2

Therefore, the area of the sheet required to make 10 such caps is 5500 cm^2.

8. A bus stop is barricaded from the remaining part of the road by using 50 hollow cones made of recycled cardboard. Each cone has a base diameter of 40 cm and height 1 m. If the outer side of each of the cones is to be painted and the cost of painting is ₹12 per m^2, what will be the cost of painting all these cones? (Use $\pi = 3.14$ and take $\sqrt{(1.04)} = 1.02$)

SOLUTION:

Given:

Radius of cone, r = diameter/2 = 40/2 cm = 20cm = 0.2 m

Height of cone, h = 1m

Slant height of cone is l, and $l^2 = (r^2 + h^2)$

Using given values, $l^2 = (0.2^2 + 1^2)$

= (1.04) Or l = 1.02 m

Slant height of the cone is 1.02 m.

Now,

CSA of each cone = πrl

= $(3.14 \times 0.2 \times 1.02) = 0.64056$ m^2

CSA of 50 such cones = $(50 \times 0.64056) = 32.028$

CSA of 50 such cones = 32.028 m^2

Again,

Cost of painting 1 m^2 area = ₹12 (given)

Cost of painting 32.028 m^2 area = ₹(32.028 × 12)

= ₹384.336 = ₹384.34 (approximately)

Therefore, the cost of painting all these cones is ₹384.34.

EXERCISE 11.2

1. Find the surface area of a sphere of radius

(i) 10.5cm **(ii) 5.6cm** **(iii) 14cm**

(Assume $\pi = \frac{22}{7}$)

SOLUTION:

Formula: Surface area of a sphere (SA) = $4\pi r^2$

(i) Radius of a sphere, r = 10.5 cm

$SA = 4 \times \frac{22}{7} \times 10.52 = 1386$

Surface area of a sphere is 1386 cm^2

(ii) Radius of a sphere, r = 5.6cm

Using formula,

$SA = 4 \times \frac{22}{7} \times 5.62 = 394.24$

Surface area of a sphere is 394.24 cm^2

(iii) Radius of a sphere, r = 14cm

SA = $4\pi r^2$

$= 4 \times \frac{22}{7} \times (14)^2$

= 2464

Surface area of a sphere is 2464 cm^2

2. Find the surface area of a sphere of diameter

(i) 14cm **(ii) 21cm** **(iii) 3.5cm**

(Assume π = 22/7)

SOLUTION:

(i) Radius of sphere, r = diameter/2 = 14/2 cm = 7 cm

Formula for the surface area of sphere = $4\pi r^2$

$= 4 \times \frac{22}{7} \times 72 = 616$

Surface area of a sphere is 616 cm^2

(ii) Radius (r) of sphere = 21/2 = 10.5 cm

Surface area of a sphere = $4\pi r^2$

$= 4 \times \frac{22}{7} \times 10.52 = 1386$

Surface area of a sphere is 1386 cm^2

Therefore, the surface area of a sphere having a diameter 21 cm is 1386 cm^2

(iii) Radius(r) of a sphere = 3.5/2 = 1.75 cm

Surface area of a sphere = $4\pi r^2$

$= 4 \times \frac{22}{7} \times 1.752 = 38.5$

Surface area of a sphere is 38.5 cm^2

3. Find the total surface area of a hemisphere of radius 10 cm. [Use π = 3.14]

SOLUTION:

Radius of the hemisphere, r = 10cm

Formula: Total surface area of the hemisphere = $3\pi r^2$

$= 3 \times 3.14 \times 10^2 = 942$

The total surface area of the given hemisphere is 942 cm^2.

4. The radius of a spherical balloon increases from 7cm to 14cm as air is being pumped into it. Find the ratio of surface areas of the balloon in the two cases.

SOLUTION:

Let r_1 and r_2 be the radii of the spherical balloon and spherical balloon when air is pumped into it, respectively.

So, r_1 = 7cm

r_2 = 14 cm

Now, Required ratio = (initial surface area)/(Surface area after pumping air into balloon)

$= 4\pi r_1^2/4\pi r_2^2$

$= (r_1/r_2)^2$

$= (7/14)^2 = (1/2)^2 = 1/4$

Therefore, the ratio between the surface areas is 1:4.

5. **A hemispherical bowl made of brass has an inner diameter 10.5 cm. Find the cost of tin-plating it on the inside at the rate of ₹16 per 100 cm². (Assume $\pi = \frac{22}{7}$)**

SOLUTION:

Inner radius of hemispherical bowl, say r = diameter/2 = (10.5)/2 cm = 5.25 cm

Formula for the surface area of hemispherical bowl = $2\pi r^2$

$= 2 \times \frac{22}{7} \times (5.25)^2 = 173.25$

Surface area of the hemispherical bowl is 173.25 cm^2

Cost of tin-plating 100 cm^2 area = ₹16

Cost of tin-plating 1 cm^2 area = ₹16 /100

Cost of tin-plating 173.25 cm^2 area = ₹(16×173.25)/100 = ₹27.72

Therefore, the cost of tin-plating the inner side of the hemispherical bowl at the rate of ₹16 per 100 cm^2 is ₹27.72.

6. **Find the radius of a sphere whose surface area is 154 cm². (Assume $\pi = \frac{22}{7}$)**

SOLUTION:

Let the radius of the sphere be r.

Surface area of sphere = 154 (given) Now,

$4\pi r^2 = 154$

$r^2 = \frac{(154 \times 7)}{(4 \times 22)} = (49/4)$

$r = (7/2) = 3.5$

The radius of the sphere is 3.5 cm.

7. **The diameter of the moon is approximately one-fourth of the diameter of the earth. Find the ratio of their surface areas.**

SOLUTION:

If the diameter of the earth is said d, then the diameter of the moon will be $\frac{d}{4}$ (as per the given statement).

Radius of earth = $\frac{d}{2}$

Radius of moon = $\frac{1}{2} \times \frac{d}{4} = d/8$

Surface area of moon = $4\pi(d/8)^2$

Surface area of earth = $4\pi(d/2)^2$

Ratio of their Surface areas $= \dfrac{4\pi\left(\dfrac{d}{8}\right)^2}{4\pi\left(\dfrac{d}{2}\right)^2} = 4/64 = 1/16$

The ratio between their surface areas is 1:16.

8. A hemispherical bowl is made of steel, 0.25 cm thick. The inner radius of the bowl is 5cm. Find the outer curved surface of the bowl. (Assume $\pi = \dfrac{22}{7}$)

SOLUTION:

Given:

Inner radius of the hemispherical bowl = 5cm

Thickness of the bowl = 0.25 cm

Outer radius of the hemispherical bowl = (5+0.25) cm = 5.25 cm

Formula for outer CSA of the hemispherical bowl = $2\pi r^2$, where r is the radius of the hemisphere.

$= 2 \times (22/7) \times (5.25)^2 = 173.25\ cm^2$

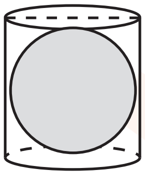
Therefore, the outer curved surface area of the bowl is 173.25 cm^2.

9. A right circular cylinder just encloses a sphere of radius r (see fig. 11.10). Find

(i) surface area of the sphere,

(ii) curved surface area of the cylinder,

(iii) ratio of the areas obtained in(i) and (ii).



SOLUTION:

(i) Surface area of the sphere = $4\pi r^2$, where r is the radius of sphere

(ii) Height of the cylinder, h = r + r = 2r

The radius of the cylinder = r

CSA of the cylinder formula = $2\pi rh = 2\pi r(2r)$ (using value of h)

$= 4\pi r^2$

(iii) Ratio between areas = (Surface area of sphere)/(CSA of Cylinder)

$= 4\pi r^2/4\pi r^2 = 1/1$

The ratio of the areas obtained in (i) and (ii) is 1:1.

EXERCISE 11.3

1. Find the volume of the right circular cone with

(i) radius 6cm, height 7 cm **(ii) radius 3.5 cm, height 12 cm (Assume $\pi = \dfrac{22}{7}$)**

SOLUTION:

Volume of cone = (1/3) $\pi r^2 h$ cube units

Where r be radius and h be the height of the cone

Radius of thecone, r = 6 cm Height of the cone, h = 7cm

Ley V be the volume of the cone, so we have

$V = \frac{1}{3} \times \frac{22}{7} \times 36 \times 7$

= (12 × 22)

= 264

The volume of the cone is 264 cm3.

Radius of the cone, r = 3.5cm Height of the cone, h = 12cm

Volume of the cone = $\frac{1}{3} \times \frac{22}{7} \times 3.5^2 \times 7 = 154$ Hence,

The volume of the cone is 154 cm^3.

2. Find the capacity in litres of a conical vessel with

(i) radius 7cm, slant height 25 cm **(ii) height 12 cm, slant height 13 cm**

(Assume $\pi = \frac{22}{7}$)

SOLUTION:

(i) Radius of the cone, r =7 cm Slant height of the cone, l = 25 cm

Height of cone, $h = \sqrt{l^2 - r^2}$

$h = \sqrt{25^2 - 7^2}$

$h = \sqrt{625 - 49}$

or h = 24

Height of the cone is 24 cm

Now,

Volume of the cone, V = $\frac{1}{3} \pi r^2 h$ (formula)

$V = \frac{1}{3} \times \frac{22}{7} \times 7^2 \times 24$

= (154 × 8)

= 1232

So, the volume of the vessel is 1232 cm^3

Therefore, the capacity of the conical vessel = (1232/1000) liters (because 1L = 1000 cm^3)

= 1.232 Liters.

(ii) Height of the cone, h = 12 cm

Slant height of the cone, l = 13 cm

Radius of cone, $r = \sqrt{l^2 - h^2}$

$r = \sqrt{l^2 - h^2}$

$r = \sqrt{13^2 - 12^2}$

$r = \sqrt{169 - 144}$

r = 5

Hence, the radius of the cone is 5 cm.

Now, Volume of the cone, $V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3} \times \frac{22}{7} \times 52 \times 12\ cm^3$

= 2200/7

Volume of the cone is 2200/7 cm^3

Now, Capacity of the conical vessel= 2200/7000 litres (1L = 1000 cm^3)

= 11/35 litres

3. The height of a cone is 15cm. If its volume is 1570cm^3, find the diameter of its base. (Use π = 3.14)

SOLUTION:

Height of the cone, h = 15 cm

Volume of cone =1570 cm^3

Let r be the radius of the cone

As we know, volume of the cone, $V = \frac{1}{3}\pi r^2 h$

So, $\frac{1}{3}\pi r^2 h = 1570\ \frac{1}{3} \times 3.14 \times r^2 \times 15 = 1570$

$r^2 = 100$

r = 10

Radius of the base of the cone 10 cm.

4. If the volume of a right circular cone of height 9 cm is 48πcm^3, find the diameter of its base.

SOLUTION:

Height of cone, h = 9cm

Volume of cone = 48π cm^3

Let r be the radius of the cone.

As we know, volume of the cone, $V = \frac{1}{3}\pi r^2 h$

So, $\frac{1}{3}\pi r^2(9) = 48\pi$

$r^2 = 16$

r = 4

Radius of the cone is 4 cm.

So, diameter = 2 × Radius = 8

Thus, diameter of the base is 8cm.

5. A conical pit of a top diameter 3.5m is 12 m deep. What is its capacity in kilolitres? (Assume $\pi = \frac{22}{7}$)

SOLUTION:

Diameter of conical pit = 3.5 m

Radius of conical pit, r = diameter/ 2 = (3.5/2)m = 1.75m

Height of pit, h = Depth of pit = 12m

Volume of cone, $V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3} \times \frac{22}{7} \times (1.75)^2 \times 12 = 38.5$

Volume of the cone is 38.5 m^3

Hence, capacity of the pit = (38.5 × 1) kiloliters = 38.5 kiloliters.

6. The volume of a right circular cone is 9856 cm^3. If the diameter of the base is 28 cm, find

(i) height of the cone

(ii) slant height of the cone

(iii) curved surface area of the cone (Assume $\pi = \frac{22}{7}$)

SOLUTION:

Volume of a right circular cone = 9856 cm^3

Diameter of the base = 28 cm

(i) Radius of cone, r = (28/2) cm = 14 cm

Let the height of the cone be h

Volume of cone, $V = \frac{1}{3}\pi r^2 h = 9856$

$\frac{1}{3} \times \frac{22}{7} \times 14 \times 14 \times h = 9856$

h = 48

(ii) Slant height of cone, $l = \sqrt{r^2 + h^2}$

$l = \sqrt{14^2 + 48^2} = \sqrt{196 + 2304} = 50$

The height of the cone is 48 cm.

Slant height of the cone is 50 cm.

(iii) curved surface area of cone = πrl

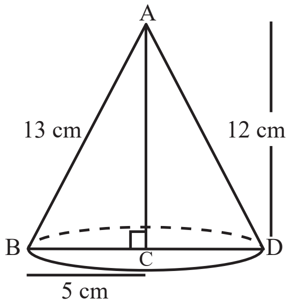
$= \frac{22}{7} \times 14 \times 50$

= 2200

Curved surface area of the cone is 2200 cm^2.

7. A right triangle ABC with sides 5cm, 12cm and 13cm is revolved about the side 12 cm. Find the volume of the solid so obtained.

SOLUTION:



Height (h) = 12 cm

Radius (r) = 5 cm, and

Volume of cone, $V = \frac{1}{3}\pi r^2 h$

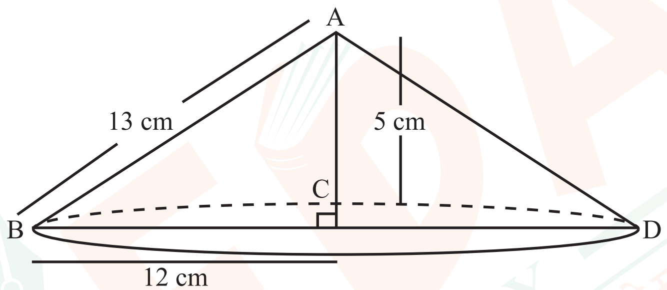
$V = \frac{1}{3} \times \pi \times 5^2 \times 12$

$= 100\pi$

Volume of the cone so formed is 100π cm^3.

8. If the triangle ABC in Question 7 is revolved about the side 5cm, then find the volume of the solids so obtained. Find also the ratio of the volumes of the two solids obtained in Questions 7 and 8.

SOLUTION:



A right-angled ΔABC is revolved about its side 5cm, a cone will be formed of radius as 12 cm, height as 5 cm, and slant height as 13 cm.

Volume of cone $= \frac{1}{3}\pi r^2 h$, where r is the radius and h is the height of the cone.

$= \frac{1}{3} \times \pi \times 12 \times 12 \times 5$

$= 240\pi$

The volume of the cones formed is 240π cm^3.

So, the required ratio = (the result of question 7) / (the result of question 8) = $(100\pi)/(240\pi) = 5/12 = 5:12$.

9. A heap of wheat is in the form of a cone whose diameter is 10.5 m and height is 3 m. Find its volume. The heap is to be covered by canvas to protect it from rain. Find the area of the canvas. (Assume $\pi = \frac{22}{7}$)

SOLUTION:

Radius (r) of heap = (10.5/2) m = 5.25

Height (h) of heap = 3m

Volume of heap $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 5.25 \times 5.25 \times 3 = 86.625$

The volume of the heap of wheat is 86.625 m^3.

Again,

Area of canvas required = CSA of cone = πrl, where $l = \sqrt{r^2 + h^2}$

CSA of cone, $\left[\frac{22}{7} \times 5.25 \times \sqrt{(5.25)^2 + 3^2}\right]$

$= \frac{22}{7} \times 5.25 \times 6.05$

= 99.825

Therefore, the area of the canvas is 99.825 m^2.

EXERCISE 11.4

1. Find the volume of a sphere whose radius is

(i) 7 cm **(ii) 0.63 m (Assume $\pi = \frac{22}{7}$)**

SOLUTION:

(i) Radius of the sphere, r = 7 cm

Using, Volume of the sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 73$

= 4312/3

Hence, volume of the sphere is 4312/3 cm^3

(ii) Radius of the sphere, r = 0.63 m

Using, volume of sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 0.633$

= 1.0478

Hence, volume of the sphere is 1.05 m^3 (approx).

2. Find the amount of water displaced by a solid spherical ball of diameter

(i) 28 cm **(ii) 0.21 m (Assume $\pi = \frac{22}{7}$)**

SOLUTION:

(i) Diameter = 28 cm

Radius, r = 28/2 cm = 14 cm

Volume of the solid spherical ball $= \frac{4}{3}\pi r3$ Volume of the ball $= \frac{4}{3} \times \frac{22}{7} \times 143 = 34496/3$

Hence, volume of the ball is 34496/3 cm^3

(ii) Diameter = 0.21 m

Radius of the ball = 0.21/2 m= 0.105 m

Volume of the ball = $\frac{4}{3}\pi r^3$

Volume of the ball = $\frac{4}{3} \times \frac{22}{7} \times 0.1053\ m^3$

Hence, volume of the ball = 0.004851 m^3

3. The diameter of a metallic ball is 4.2cm. What is the mass of the ball, if the density of the metal is 8.9 g per cm^3? (Assume $\pi = \frac{22}{7}$)

SOLUTION:

Given,

Diameter of a metallic ball = 4.2 cm

Radius(r) of the metallic ball, r = 4.2/2 cm = 2.1 cm

Volume formula = $4/3\ \pi r^3$

Volume of the metallic ball = $\frac{4}{3} \times \frac{22}{7} \times 2.1\ cm^3$

Volume of the metallic ball = 38.808 cm^3

Now, using the relationship between density, mass and volume, Density = Mass/Volume

Mass = Density × volume

= (8.9 × 38.808) g

= 345.3912 g

Mass of the ball is 345.39 g (approx).

4. The diameter of the moon is approximately one-fourth of the diameter of the earth. What fraction of the volume of the earth is the volume of the moon?

SOLUTION:

Let the diameter of the earth be "d". Therefore, the radius of the earth will be d/2.

Diameter of the moon will be d/4, and the radius of the moon will be d/8.

Find the volume of the moon.

Volume of the moon = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi\frac{d^3}{8^3} = \frac{4}{3}\pi\frac{d^3}{512}$

Find the volume of the earth

Volume of the earth = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi\frac{d^3}{2^3} = \frac{4}{3}\pi\frac{d^3}{8}$ Fraction of the volume of the earth is the volume of the moon

Volume of the moon/volume of the earth $= \dfrac{\frac{4}{3}\pi\left(\frac{d^3}{512}\right)}{\frac{4}{3}\pi\left(\frac{d^3}{8}\right)} = 8/512 = 1/64$

Answer: Volume of the moon is of the 1/64 volume of the earth.

5. **How many litres of milk can a hemispherical bowl of diameter 10.5 cm hold? (Assume $\pi = \frac{22}{7}$)**

SOLUTION:

Diameter of the hemispherical bowl = 10.5 cm

Radius of the hemispherical bowl, r = 10.5/2 cm = 5.25 cm

Formula for volume of the hemispherical bowl = $\frac{2}{3}\pi r^3$

Volume of the hemispherical bowl = $\frac{2}{3} \times \frac{22}{7} \times 5.25^3 = 303.1875$

Volume of the hemispherical bowl is 303.1875 cm^3

Capacity of the bowl = (303.1875)/1000 L = 0.303 litres(approx.)

Therefore, the hemispherical bowl can hold 0.303 litres of milk.

6. **A hemispherical tank is made up of an iron sheet 1cm thick. If the inner radius is 1 m, then find the volume of the iron used to make the tank. (Assume $\pi = \frac{22}{7}$)**

SOLUTION:

Inner Radius of the tank, (r) = 1m

Outer Radius (R) = 1.01m

Volume of the iron used in the tank = $\frac{2}{3}\pi(R^3 - r^3)$ Put values,

Volume of the iron used in the hemispherical tank = $\frac{2}{3} \times \frac{22}{7} \times ((1.01^3) - 1^3) = 0.06348$

So, volume of the iron used in the hemispherical tank is 0.06348 m^3.

7. **Find the volume of a sphere whose surface area is 154 cm^2. (Assume $\pi = \frac{22}{7}$)**

SOLUTION:

Let r be the radius of a sphere.

Surface area of the sphere = $4\pi r^2$

$4\pi r^2 = 154\ cm^2$ (given)

$r^2 = \frac{(154 \times 7)}{(4 \times 22)}$

r = 7/2

The radius is 7/2 cm.

Now,

Volume of the sphere = $\frac{4}{3}\pi r^3$

Volume of the sphere = $(4/3)\times(22/7)\times(7/2)^3 = 179\frac{2}{3}$

Volume of the sphere is $179\frac{2}{3} cm^3$

8. A dome of a building is in the form of a hemisphere. From inside, it was whitewashed at the cost of ₹4989.60. If the cost of white-washing is 20 per square metre, find the

(i) inside surface area of the dome

(ii) volume of the air inside the dome (Assume $\pi = \frac{22}{7}$)

SOLUTION:

(i) Cost of whitewashing the dome from inside = ₹4989.60

Cost of whitewashing $1m^2$ area = ₹20

CSA of the inner side of dome = $498.96/2\ m^2 = 249.48\ m^2$ ......(1)

(ii) Let the inner radius of the hemispherical dome be r.

CSA of the inner side of dome = $249.48\ m^2$ (from (i))

Formula to find CSA of a hemisphere = $2\pi r^2$

$2\pi r^2 = 249.48$

$2 \times \frac{22}{7} \times r^2 = 249.48$

$r^2 = \frac{(249.48 \times 7)}{(2 \times 22)}$

$r^2 = 39.69$

$r = 6.3$

So, the radius is 6.3 m.

Volume of air inside the dome = Volume of hemispherical dome

Using the formula, the volume of the hemisphere = $\frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times 6.3 \times 6.3 \times 6.3$

$= 523.908$

$= 523.9$(approx.)

Answer: The volume of air inside the dome is $523.9\ m^3$.

9. Twenty-seven solid iron spheres, each of radius r and surface area S are melted to form a sphere with surface area S'. Find the

(i) radius r' of the new sphere,

(ii) ratio of Sand S'.

SOLUTION:

Volume of the solid sphere = $\frac{4}{3}\pi r^3$

Volume of twenty seven solid sphere = $27 \times \frac{4}{3}\pi r^3 = 36\pi r^3$

(i) New solid iron sphere radius = r'

Volume of this new sphere = $\frac{4}{3}\pi (r')^3$

$\frac{4}{3}\pi (r')^3 = 36\pi r^3$

$(r')3 = 27r^3$

$r' = 3r$

Radius of the new sphere will be 3r (thrice the radius of the original sphere)

(ii) Surface area of the iron sphere of radius r, $S = 4\pi r^2$

Surface area of the iron sphere of radius $r' = 4\pi (r')^2$ Now

$$S/S' = \frac{(4\pi r^2)}{(4\pi (r')^2)}$$

$$S/S' = \frac{r^2}{(3r')^2} = 1/9$$

The ratio of S and S' is 1: 9.

10. A capsule of medicine is in the shape of a sphere of diameter 3.5mm. How much medicine (in mm^3) is needed to fill this capsule? (Assume $\pi = \frac{22}{7}$)

SOLUTION:

Diameter of the capsule = 3.5 mm

Radius of the capsule, say r = diameter/ 2 = (3.5/2) mm = 1.75mm

Volume of the spherical capsule $= \frac{4}{3}\pi r^3$

Volume of the spherical capsule $= \frac{4}{3} \times \frac{22}{7} \times (1.75)^3 = 22.458$

Answer: The volume of the spherical capsule is 22.46 mm^3.