CHAPTER 1

# Real Numbers

CLASS 10TH

NCERT EXERCISE AND SOLUTIONS - MATHEMATICS

## EXERCISE 1.1

**1. Express each number as a product of its prime factors:**

(i) **140**
(ii) **156**
(iii) **3825**
(iv) **5005**
(v) **7429**

**SOLUTION:**

In this question, number is given and we have to express it as a product of prime factors.

**(i)** $140 = 2 \times 2 \times 5 \times 7 = 2^2 \times 5 \times 7$
**(ii)** $156 = 2 \times 2 \times 3 \times 13 = 2^2 \times 3 \times 13$
**(iii)** $3825 = 3 \times 3 \times 5 \times 5 \times 17 = 3^2 \times 5^2 \times 17$
**(iv)** $5005 = 5 \times 7 \times 11 \times 13$
**(v)** $7429 = 17 \times 19 \times 23$

**2. Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of the two numbers.**

(i) **26 and 91**
(ii) **510 and 92**
(iii) **336 and 54**

**SOLUTION:**

In this question, pair of number is given so, first we have to find the LCM and HCF then we have to verify the relationship LCM × HCF = product of the two numbers.

**(i)** $26 = 2 \times 13$
$91 = 7 \times 13$
HCF = 13
LCM = $2 \times 7 \times 13 = 182$
Product of two numbers 26 and 91 = $26 \times 91 = 2366$
HCF × LCM = $13 \times 182 = 2366$
Hence, product of two numbers = HCF × LCM

**(ii)** 510 and 92

$510 = 2 \times 3 \times 5 \times 17$

$92 = 2 \times 2 \times 23$

HCF = 2

LCM = $2 \times 2 \times 3 \times 5 \times 17 \times 23 = 23460$

Product of two numbers 510 and 92 = $510 \times 92 = 46920$

HCF × LCM = $2 \times 23460 = 46920$

Hence, product of two numbers = HCF × LCM

**(iii)** 336 and 54

$336 = 2 \times 2 \times 2 \times 2 \times 3 \times 7$

$54 = 2 \times 3 \times 3 \times 3$

HCF = $2 \times 3 = 6$

LCM = $2^4 \times 3^3 \times 7 = 3024$

Product of two numbers 336 and 54 = $336 \times 54 = 18144$

HCF × LCM = $6 \times 3024 = 18144$

Hence, product of two numbers = HCF × LCM

**3. Find the LCM and HCF of the following integers by applying the prime factorization method.**

**(i) 12, 15 and 21**

**(ii) 17, 23 and 29**

**(iii) 8, 9 and 25**

**SOLUTION:**

In this question ,numbers are given and we have to find the LCM and HCF by prime factorization method.

**(i)** 12, 15 and 21

$12 = 2 \times 2 \times 3 = 2^2 \times 3$

$15 = 3 \times 5$

$21 = 3 \times 7$

HCF = 3

LCM = $2^2 \times 3 \times 5 \times 7 = 420$

**(ii)** 17, 23 and 29

Since the three numbers are prime, we have

$17 = 1 \times 17$

$23 = 1 \times 23$

$29 = 1 \times 29$

HCF = 1

LCM = $17 \times 23 \times 29 = 11339$

**(iii)** 8, 9 and 25

$8 = 2 \times 2 \times 2 = 2^3$

$9 = 3 \times 3 = 3^2$

$25 = 5 \times 5 = 5^2$

HCF = 1

LCM = $2^3 \times 3^2 \times 5^2 = 1800$

**4. Given that HCF (306, 657) = 9, find LCM (306, 657).**

**SOLUTION:**

HCF × LCM = Product of two numbers

$$LCM\ (306,\ 657)\ =\ \frac{306\ \times\ 657}{HCF(306,657)}$$

$$=\ \frac{306\ \times\ 657}{9}$$

= 22338

**5. Check whether $6^n$ can end with the digit 0 for any natural number n.**

**SOLUTION:**

If any number ends with the digit 0, it should be divisible by 10 or in other words, it will also be divisible by 2 and 5 as 10 = 2 × 5

Prime factorization of $6^n = (2 \times 3)^n$

It can be observed that 5 is not in the prime factorization of .

Hence, for any value of n, $6^n$ will not be divisible by 5.

Therefore, $6^n$ cannot end with the digit 0 for any natural number n.

**6. Explain why 7 × 11 × 13 + 13 and 7 × 6 × 5 × 4 × 3 × 2 × 1 + 5 are composite numbers.**

**SOLUTION:**

Numbers are of two types - prime and composite.

Prime numbers can be divided by 1 and only itself, whereas composite numbers have factors other than 1 and itself.

It can be observed that

$7 \times 11 \times 13 + 13$

$= 13 \times (7 \times 11 + 1)$

$= 13 \times (77 + 1)$

$= 13 \times 78 = 13 \times 13 \times 6$

The given expression has 6 and 13 as its factors.

Therefore, it is a composite number.

$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$

$= 5 \times (7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1)$

$= 5 \times (1008 + 1)$

$= 5 \times 1009$

1009 cannot be factorized further

Therefore, the given expression has 5 and 1009 as its factors.

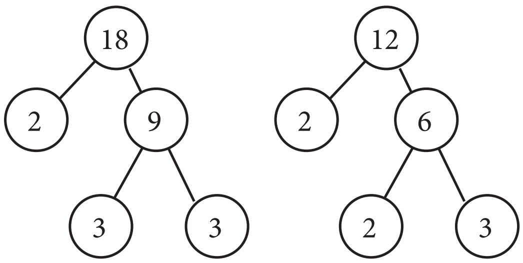
Hence, it is a composite number.

**7. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?**

**SOLUTION:**

Sonia drive around the circular field and takes 18 minutes to complete it. Similarly, Ravi drive around the circular field and takes 12 minutes to complete it.

By taking LCM of time taken (in minutes) by Sonia and Ravi, We can get the actual number of minutes after which they meet again at the starting point after both start at the same point and at the same time, and go in the same direction.




$18 = 2 \times 3 \times 3$

$12 = 2 \times 2 \times 3$

LCM $(18, 12) = 2 \times 2 \times 3 \times 3$

Therefore, both Sonia and Ravi will meet again at the starting point after 36 minutes.

## EXERCISE 1.2

**1. Prove that $\sqrt{5}$ is irrational.**

**SOLUTION:**

Let us prove irrational by contradiction.

Let us suppose that $\sqrt{5}$ is rational. It means that we have co-prime integers a and b ($b \neq 0$)

Such that $\sqrt{5} = \frac{a}{b}$

$\Rightarrow b\sqrt{5} = a$

Squaring both sides, we get

$5b^2 = a^2$ …... (1)

It means that 5 is factor of $a^2$

Hence, 5 is also factor of a ……. (2)

If 5 is factor of a, it means that we can write,

$a = 5c$ for some integer c.

Substituting value of a in (1) ,

$5b^2 = 25c^2$

$\Rightarrow b^2 = 5c^2$

It means that 5 is factor of $b^2$.

Hence, 5 is also factor of b ……. (3)

From (2) and (3), we can say that 5 is factor of both a and b.

But, a and b are co-prime.

Therefore, our assumption was wrong.

$\sqrt{5}$ cannot be rational. Hence, it is irrational.

**2. Prove that $3 + 2\sqrt{5}$ is irrational.**

**SOLUTION:**

Let us assume that $3 + 2\sqrt{5}$ be rational.

Then, it will be of the form $\frac{a}{b}$,where $a$, $b$ are integers and $b \neq 0$.

Again, let a and b have no common factor other than 1.

$\Rightarrow 3+2\sqrt{5}=\frac{a}{b}$, where and are coprime integers.

$\Rightarrow 2\sqrt{5}=\frac{a}{b}-3$

$\Rightarrow \sqrt{5}=\frac{1}{2}\left(\frac{a}{b}-3\right)$

The right-hand side of the equation $\frac{1}{2}\left(\frac{a}{b}-3\right)$ is a fraction where both the numerator and denominator are integers.

$\Rightarrow$ $\sqrt{5}$ is a rational number, because it can be expressed as a ratio of two integers.

But it contradicts the fact that $\sqrt{5}$ is irrational and this contradiction has arisen due to our wrong initial assumption.

$\therefore$ $3 + 2\sqrt{5}$ is irrational.

**3. Prove that the following are irrationals.**

(i) $\frac{1}{\sqrt{2}}$ (ii) $7\sqrt{5}$ (iii) $6 + \sqrt{2}$

**SOLUTION:**

(i) Let us suppose that $\frac{1}{\sqrt{2}}$ is rational.

It means we have some co-prime integers a and b (b $\neq$ 0)

Such that

$\frac{1}{\sqrt{2}}=\frac{a}{b}$

$\Rightarrow \sqrt{2}=\frac{b}{a}$ ..........(1)

R.H.S of (1) is rational but we know that $\sqrt{2}$ is irrational.

But it is not possible which means our supposition was wrong.

Therefore, $\frac{1}{\sqrt{2}}$ cannot be rational.

Hence, $\frac{1}{\sqrt{2}}$ is irrational.

(ii) Let us suppose that $7\sqrt{5}$ is rational.

It means we have some co-prime integers a and b (b $\neq$ 0)

Such that

$$7\sqrt{5}=\frac{a}{b}$$

$$\Rightarrow \sqrt{5}=\frac{b}{7a} \qquad ..........(1)$$

R.H.S of (1) is rational but we know that $\sqrt{5}$ is irrational.
But It is not possible which means our supposition was wrong.

Therefore, $7\sqrt{5}$ cannot be rational.

Hence, $7\sqrt{5}$ is irrational.

**(iii)** Let us suppose that $6+\sqrt{2}$ is rational.

It means we have some co-prime integers a and b ($b \neq 0$)

$$6+\sqrt{2}=\frac{a}{b}$$

$$\Rightarrow \sqrt{2}=\frac{a}{b}-6$$

$$\Rightarrow \sqrt{2}=\frac{a-6b}{b} \ ......(1)$$

R.H.S of (1) is rational but we know that $\sqrt{2}$ is irrational.
But it is not possible which means our supposition was wrong.

Therefore, $6+\sqrt{2}$ cannot be rational.

Hence, $6+\sqrt{2}$ is irrational.