CHAPTER 2

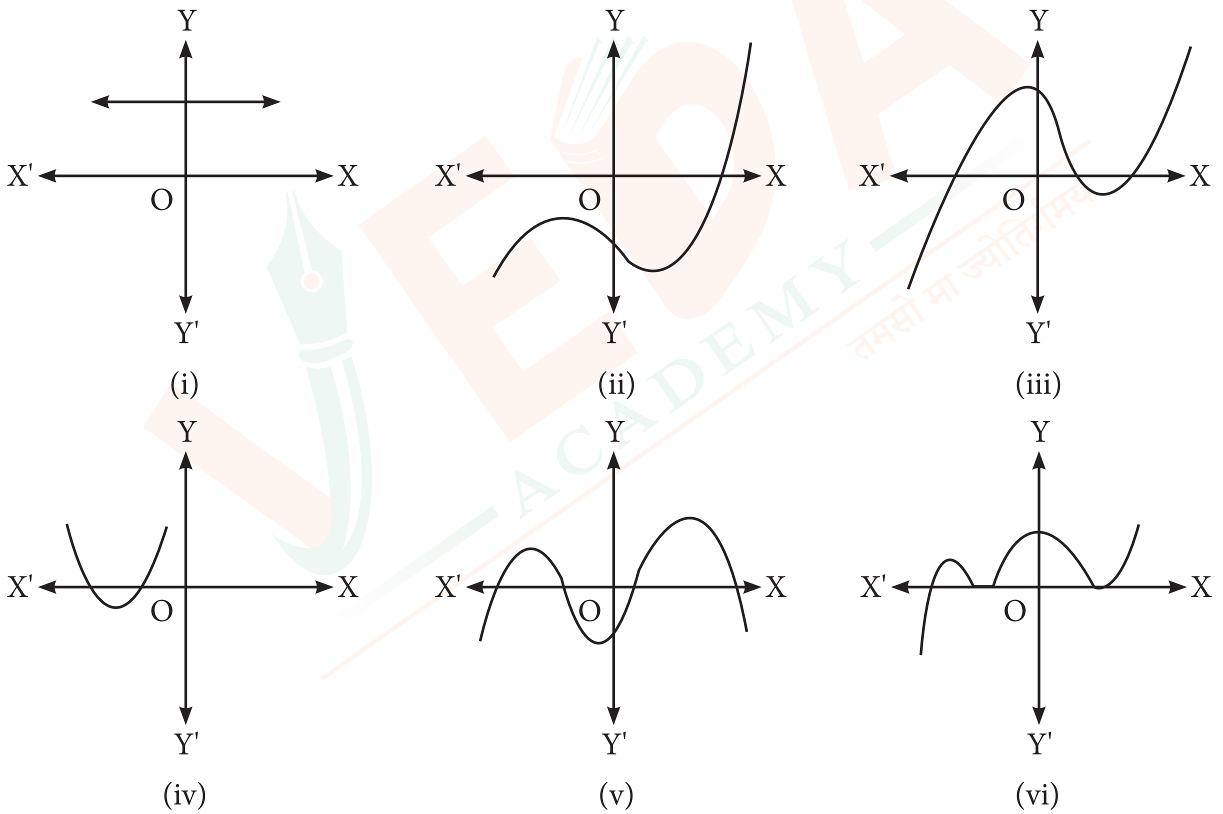
Polynomials

CLASS 10TH

NCERT EXERCISE AND SOLUTIONS - MATHEMATICS

EXERCISE 2.1

1. The graphs of y = p(x) are given below, for some polynomials p(x). Find the number of zeroes of p(x), in each case.



(i) In the given graph, the number of zeroes of p(x) is 0 because the graph is parallel to x-axis does not cut it at any point.

(ii) The number of zeroes of p(x) is 1 because the graph intersects the x-axis at only one point.

(iii) The number of zeroes of p(x) is 3 because the graph intersects the x-axis at any three points.

(iv) The number of zeroes of p(x) is 2 because the graph intersects the x-axis at two points.

(v) The number of zeroes of p(x) is 4 because the graph intersects the x-axis at four points.

(vi) The number of zeroes of p(x) is 3 because the graph intersects the x-axis at three points.

EXERCISE 2.2

1. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

(i) $x^2 - 2x - 8$

(ii) $4s^2 - 4s + 1$

(iii) $6x^2 - 3 - 7x$

(iv) $4u^2 + 8u$

(v) $t^2 - 15$

(vi) $3x^2 - x - 4$

SOLUTION:

In this question quadratic polynomials are given, and we have to find the zeroes of quadratic polynomials.

1) i) $x^2 - 2x - 8 = 0$

$\Rightarrow x^2 - 4x + 2x - 8 = 0$

$\Rightarrow (x+2)(x-4) = 0$

$\Rightarrow (x+2) = 0;\ (x-4) = 0$

$\therefore x = -2, 4$

To Verify:

(a) $\alpha + \beta = -\frac{b}{a}$

$LHS: \quad \alpha + \beta = -2 + 4 = 2$

$RHS: \quad \frac{-b}{a} = -\frac{(-2)}{1} = 2$

LHS = RHS

(b) $\alpha\beta = \frac{c}{a}$

$LHS: \quad \alpha\beta = (-2)(4) = -8$

$RHS: \quad \frac{c}{a} = \frac{-8}{1} = -8$

LHS = RHS

ii) $4s^2 - 4s + 1 = 0$

$\Rightarrow 4s^2 - 2s - 2s + 1 = 0$

$\Rightarrow 2s(2s-1) - 1(2s-1) = 0$

$\Rightarrow (2s-1)(2s-1) = 0$

$\Rightarrow (2s-1) = 0;\ (2s-1) = 0$

$\therefore s = \frac{1}{2}, \frac{1}{2}$

To Verify:

(a) $\alpha+\beta=-\frac{b}{a}$

$LHS: \quad \alpha+\beta=\frac{1}{2}+\frac{1}{2}=\frac{2}{2}=1$

$RHS: \quad \frac{-b}{a}=-\frac{(-4)}{4}=1$

LHS = RHS

(b) $\alpha\beta=\frac{c}{a}$

$LHS: \quad \alpha\beta=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$

$RHS: \quad \frac{c}{a}=\frac{1}{4}$

LHS = RHS

iii) $6x^2-7x-3=0$

$\Rightarrow 6x^2-9x+2x-3=0$

$\Rightarrow 3x(2x-3)+1(2x-3)=0$

$\Rightarrow (2x-3)=0;\ (3x+1)=0$

$\therefore x=-\frac{1}{3},\frac{3}{2}$

To Verify:

(a) $\alpha+\beta=-\frac{b}{a}$

$LHS: \quad \alpha+\beta=-\frac{1}{3}+\frac{3}{2}=\frac{-2+9}{6}=\frac{7}{6}$

$RHS: \quad \frac{-b}{a}=-\frac{(-7)}{6}=\frac{7}{6}$

LHS = RHS

(b) $\alpha\beta=\frac{c}{a}$

$LHS: \quad \alpha\beta=-\frac{1}{3}\times\frac{3}{2}=-\frac{1}{2}$

$RHS: \quad \frac{c}{a}=\frac{-3}{6}=\frac{-1}{2}$

LHS = RHS

iv) $4u^2 + 8u = 0$

$\Rightarrow 4u(u+2) = 0$

$\Rightarrow u(u+2) = 0$

$\therefore u = 0, -2$

To Verify:

(a) $\alpha + \beta = -\frac{b}{a}$

$LHS: \quad \alpha + \beta = 0 - 2 = -2$

$RHS: \quad \frac{-b}{a} = -\frac{8}{4} = -2$

LHS = RHS

(b) $\alpha\beta = \frac{c}{a}$

$LHS: \quad \alpha\beta = (0)(-2) = 0$

$RHS: \quad \frac{c}{a} = \frac{0}{4} = 0$

LHS = RHS

v) $t^2 - 15 = 0$

$\Rightarrow (t)^2 - \left(\sqrt{15}\right)^2 = 0$

$\Rightarrow \left(t + \sqrt{15}\right)\left(t - \sqrt{15}\right) = 0$

$\therefore t = \sqrt{15}, -\sqrt{15}$

To Verify:

(a) $\alpha + \beta = -\frac{b}{a}$

$LHS: \quad \alpha + \beta = \sqrt{15} - \sqrt{15} = 0$

$RHS: \quad \frac{-b}{a} = -\frac{0}{1} = 0$

LHS = RHS

(b) $\alpha\beta = \frac{c}{a}$

$LHS: \quad \alpha\beta = \left(\sqrt{15}\right)\left(-\sqrt{15}\right) = -15$

$RHS: \quad \frac{c}{a} = \frac{-15}{1} = -15$

LHS = RHS

vi) $3x^2 - x - 4 = 0$

$\Rightarrow 3x^2 - 4x + 3x - 4 = 0$

$\Rightarrow x(3x-4) + 1(3x-4) = 0$

$\Rightarrow (3x-4) = 0;\ (x+1) = 0$

$x = \frac{4}{3}, -1$

To Verify:

(a) $\alpha + \beta = -\frac{b}{a}$

$LHS: \quad \alpha + \beta = \frac{4}{3} - 1 = \frac{1}{3}$

$RHS: \quad \frac{-b}{a} = -\frac{(-1)}{3} = \frac{1}{3}$

LHS = RHS

(b) $\alpha\beta = \frac{c}{a}$

$LHS: \quad \alpha\beta = \frac{4}{3}(-1) = -\frac{4}{3}$

$RHS: \quad \frac{c}{a} = \frac{-4}{3}$

LHS = RHS

2. Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes, respectively.

(i) $\frac{1}{4}, -1$ **(ii)** $\sqrt{2}, \frac{1}{3}$ **(iii)** $0, \sqrt{5}$

(iv) 1, 1 **(v)** $-\frac{1}{4}, \frac{1}{4}$ **(vi)** 4, 1

SOLUTION.

(i) Given that $\alpha + \beta = \frac{1}{4}$ *and* $\alpha\beta = -1$

Then we know that the quadratic polynomial equation is given by:

$p(x) = x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$= x^2 - \frac{1}{4}x + (-1) = 0$

$= x^2 - \frac{1}{4}x - 1 = 0$

$= 4x^2 - x - 4 = 0$

Thus, $4x^2 - x - 4$ is the quadratic polynomial.

(ii) Given that $\alpha + \beta = \sqrt{2}$ *and* $\alpha\beta = \frac{1}{3}$

Then we know that the quadratic polynomial equation is given by:

$p(x) = x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$= x^2 - \sqrt{2}\,x + \frac{1}{3} = 0$

$= 3x^2 - 3\sqrt{2}x + 1 = 0$

Thus, $3x^2 - 3\sqrt{2}x + 1$ is the quadratic polynomial.

(iii) Given that $\alpha + \beta = 0$ *and* $\alpha\beta = \sqrt{5}$

Then we know that the quadratic polynomial equation is given by:

$p(x) = x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$= x^2 - (0)\,x + \sqrt{5} = 0$

$= x^2 + \sqrt{5} = 0$

Thus, $x^2 + \sqrt{5}$ is the quadratic polynomial.

(iv) Given that $\alpha + \beta = 1$ *and* $\alpha\beta = 1$

Then we know that the quadratic polynomial equation is given by:

$p(x) = x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$= x^2 - (1)x + 1 = 0$

$= x^2 - x + 1 = 0$

Thus, $x^2 - x + 1$ is the quadratic polynomial.

(v) Given that $\alpha + \beta = -\frac{1}{4}$ *and* $\alpha\beta = \frac{1}{4}$

Then we know that the quadratic polynomial equation is given by:

$p(x) = x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$= x^2 - (-\frac{1}{4})x + \frac{1}{4} = 0$

$= 4x^2 + x + 1 = 0$

Thus, $4x^2 + x + 1$ is the quadratic polynomial.

(vi) Given that $\alpha + \beta = 4$ *and* $\alpha\beta = 1$

Then we know that the quadratic polynomial equation is given by:

$p(x) = x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$= x^2 - 4x + 1 = 0$

Thus, $x^2 - 4x + 1$is the quadratic polynomial.