CHAPTER 3

Pair of linear equations in two variables

CLASS 10TH

NCERT EXERCISE AND SOLUTIONS - MATHEMATICS

EXERCISE 3.1

1. Form the pair of linear equations in the following problems, and find their solutions graphically.

(i) 10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

(ii) 5 pencils and 7 pens together cost 50, whereas 7 pencils and 5 pens together cost 46. Find the cost of one pencil and that of one pen.

SOLUTION:

(i) Given: Total Number of students = 10

To find: Number of boys and girls.

Let the number of boys and girls be x and y respectively.

According to the question,

Number of girls = 4 more than number of boys

$\Rightarrow y = x + 4$……[i]

And, Total students = 10

$x + y = 10$ ……[ii]

Taking eq. [i], $y = x + 4$

Put $x = 0$

$y = 0 + 4 = 4$

Put $y = 0$

$0 = x + 4$ $x = -4$

x	0	–4
y	4	0

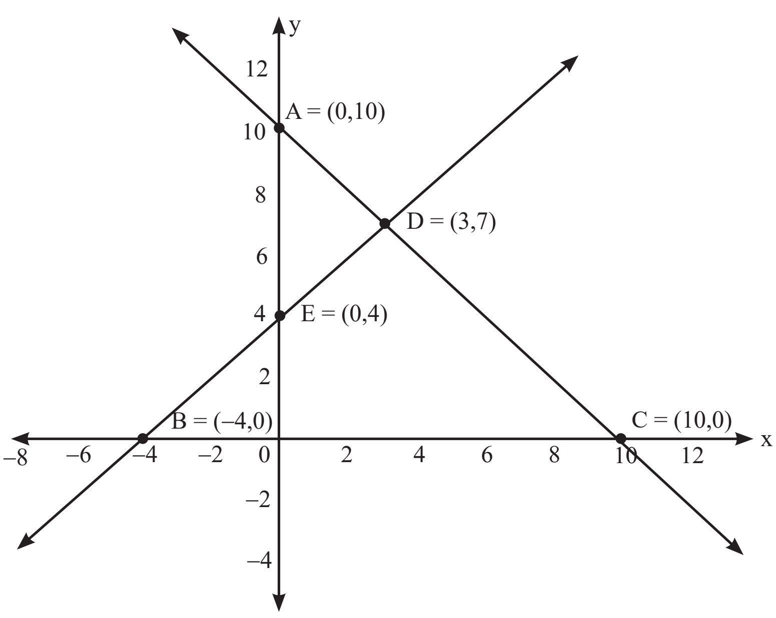
Now, Taking equation [ii], $x + y = 10$

Put $x = 0$

$0 + y = 10 \Rightarrow y = 10$

Put $y = 0$

$x + 0 = 10 \Rightarrow x = 10$



x	0	10
y	10	0

∴ Number of boys, x = 3

Number of girls, y = 7

(ii) Let 1 pencil costs ₹x and 1 pen costs ₹y.

According to the question, the algebraic expression cab be represented as;

5x + 7y = 50

7x + 5y = 46

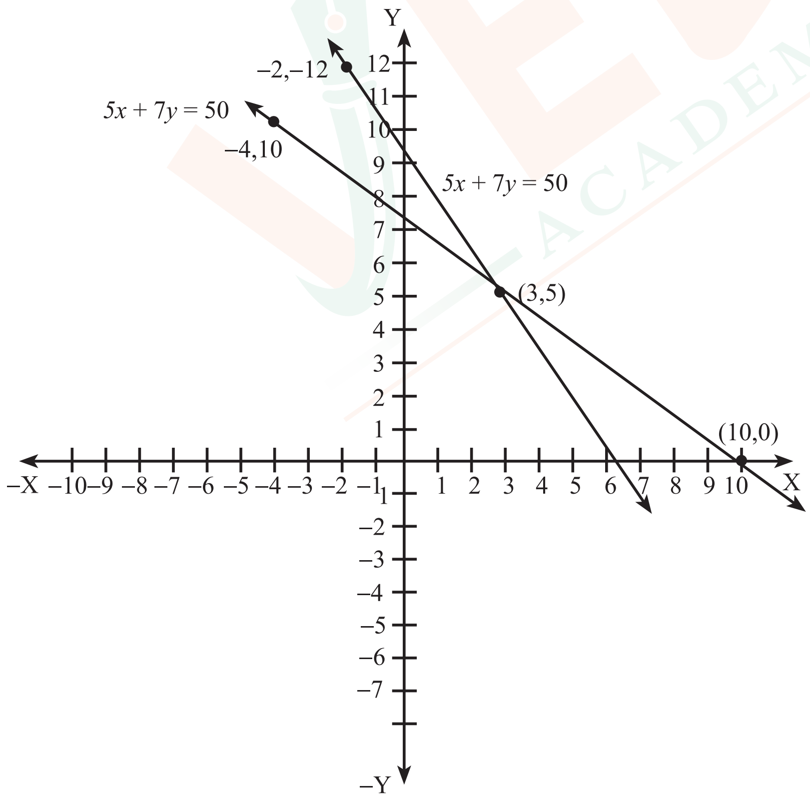
For, 5x + 7y =50 or $x = \frac{(50-7y)}{5}$ the solutions are;

x	3	10	–4
y	5	0	10

For 7x + 5y = 46 or $x = \frac{(46-5y)}{7}$, the solutions are;

x	8	3	–2
y	–2	5	12

Hence, the graphical representation is as follows;



From the graph, it is can be seen that the given lines cross each other at point (3,5).

So, the cost of a pencil is ₹3 and cost of a pen is ₹5.

2. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}, \frac{c_1}{c_2}$ find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincident:

(i) **5x – 4y + 8 = 0**
7x + 6y – 9 = 0

(ii) **9x + 3y + 12 = 0**
18x + 6y + 24 = 0

(iii) **6x – 3y + 10 = 0**
2x – y + 9 = 0

SOLUTION:

(i) 5x – 4y + 8 = 0;
7x + 6y – 9 = 0

$$\frac{a_1}{a_2} = \frac{5}{7}, \frac{b_1}{b_2} = \frac{-4}{6} = \frac{-2}{3}$$

$$\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

∴ Lines are Intersecting.

(ii) 9x + 3y + 12 = 0; 18x + 6y + 24 = 0

$$\frac{a_1}{a_2} = \frac{9}{18} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2} \text{ and } \frac{c_1}{c_2} = \frac{12}{24} = \frac{1}{2}$$

$$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

∴ Lines are Coincident.

(iii) 6x – 3y + 10 = 0; 2x – y + 9 = 0

$$\frac{a_1}{a_2} = \frac{6}{2} = 3, \frac{b_1}{b_2} = \frac{-3}{-1} = 3 \text{ and } \frac{c_1}{c_2} = \frac{10}{9}$$

$$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

∴ Lines are Parallel.

3. On comparing the ratio, $\frac{a_1}{a_2}, \frac{b_1}{b_2}, \frac{c_1}{c_2}$ find out whether the following pair of linear equations are consistent, or inconsistent.

(i) $3x + 2y = 5; \quad 2x - 3y = 7$

(ii) 2x – 3y = 8; 4x – 6y = 9

(iii) $\frac{3}{2}x + \frac{5}{3}y = 7$; 9x – 10y = 14

(iv) 5x – 3y = 11; – 10x + 6y = –22

(v) $\frac{4}{3}x + 2y = 8$; 2x + 3y = 12

SOLUTION:

(i) $3x + 2y = 5;\ 2x - 3y = 7$

$$\frac{a_1}{a_2} = \frac{3}{2},\ \frac{b_1}{b_2} = \frac{2}{-3} = \frac{-2}{3}$$

$$\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$\Rightarrow$ Lines are Intersecting

$\Rightarrow$ Consistent

(ii) $2x - 3y = 8;\ 4x - 6y = 9$

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2},\ \frac{b_1}{b_2} = \frac{-3}{-6} = \frac{1}{2} \text{ and } \frac{c_1}{c_2} = \frac{8}{9}$$

$$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$\Rightarrow$ Lines are Parallel

$\Rightarrow$ Inconsistent

(iii) $\frac{3}{2}x + \frac{5}{3}y = 7;\ 9x - 10y = 14$

$$\frac{a_1}{a_2} = \frac{\left(\frac{3}{2}\right)}{9} = \frac{3}{9\times 2} = \frac{1}{6},\ \frac{b_1}{b_2} = \frac{\left(\frac{5}{3}\right)}{-10} = \frac{5}{-10\times 3} = \frac{1}{-6} \text{ and } \frac{c_1}{c_2} = \frac{7}{14} = \frac{1}{2}$$

$$\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$\Rightarrow$ Lines are Intersecting

$\Rightarrow$ Consistent

(iv) $5x - 3y = 11;\ -10x + 6y = -22$

$$\frac{a_1}{a_2} = \frac{5}{-10} = \frac{1}{-2} = -\frac{1}{2},\ \frac{b_1}{b_2} = \frac{-3}{6} = -\frac{1}{2} \text{ and } \frac{c_1}{c_2} = \frac{-11}{22} = -\frac{1}{2}$$

$$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$\Rightarrow$ Lines are Coincident

$\Rightarrow$ Consistent

(v) $\frac{4}{3}x + 2y = 8;\ 2x + 3y = 12$

$$\frac{a_1}{a_2} = \frac{\left(\frac{4}{3}\right)}{2} = \frac{4}{2\times 3} = \frac{2}{3},\ \frac{b_1}{b_2} = \frac{2}{3} \text{ and } \frac{c_1}{c_2} = \frac{8}{12} = \frac{2}{3}$$

$$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$\Rightarrow$ Lines are Coincident

$\Rightarrow$ Consistent

4. Which of the following pairs of linear equations are consistent/inconsistent? If consistent, obtain the solution graphically:

(i) **x + y = 5, 2x + 2y = 10**

(ii) **x – y = 8, 3x – 3y = 16**

(iii) **2x + y – 6 = 0, 4x – 2y – 4 = 0**

(iv) **2x – 2y – 2 = 0, 4x – 4y – 5 = 0**

SOLUTION:

(i) Given, x + y = 5 and 2x + 2y =10

$$\frac{a_1}{a_2}=\frac{1}{2}, \frac{b_1}{b_2}=\frac{1}{2} \text{ and } \frac{c_1}{c_2}=\frac{5}{10}=\frac{1}{2}$$

$$\Rightarrow \frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

The equations are coincident and they have infinite number of possible solutions. So, the equations are consistent.

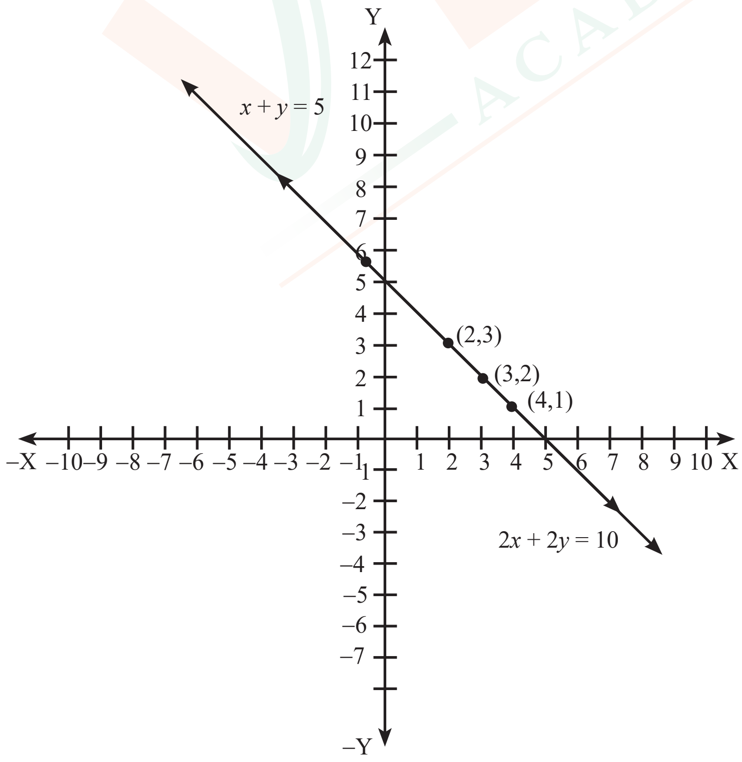
For, x + y = 5 or x = 5 – y

x	4	3	2
y	1	2	3

For $2x+2y=10$ or $x=\frac{(10-2y)}{2}$

x	4	3	2
y	1	2	3

So, the equations are represented in graphs as follows:



From the figure, we can see, that the lines are overlapping each other. Therefore, the equations have infinite possible solutions.

(ii) $x - y = 8, 3x - 3y = 16$

$\frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{-1}{-3} = \frac{1}{3}$ and $\frac{c_1}{c_2} = \frac{8}{16} = \frac{1}{2}$

$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow$ Lines are Parallel

$\Rightarrow$ Inconsistent

(iii) $2x + y - 6 = 0, 4x - 2y - 4 = 0$

$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{1}{-2} = \frac{-1}{2}$ and $\frac{c_1}{c_2} = \frac{-6}{-4} = \frac{3}{2}$

$\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\Rightarrow$ Lines are Intersecting

$\Rightarrow$ Consistent

$\Rightarrow$ Unique solution

Now,

Taking equation; $2x + y - 6 = 0$

$\Rightarrow 2x + y = 6$

Put $x = 0$

$2(0) + y = 6$

$y = 6$

Put $y = 0$

$2x + 0 = 6$

$2x = 6 \Rightarrow x = 3$

x	0	3
y	6	0

Taking equation; $4x - 2y = 4$

$2x - y = 2$

Put $x = 0$

$2(0) - y = 2$

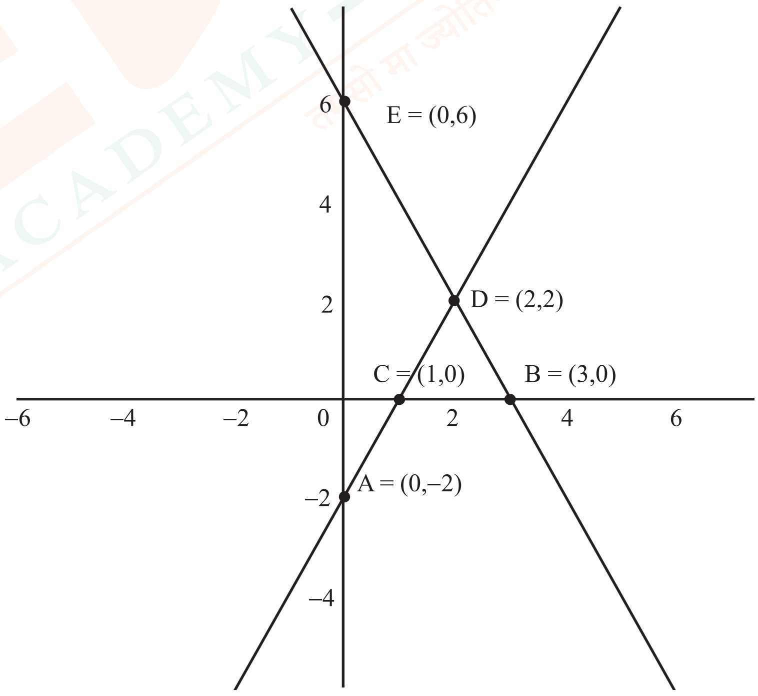
$y = -2$

Put $y = 0$

$2x - 0 = 2$

$2x = 2 \Rightarrow x = 1$

x	0	1
y	–2	0



$\therefore$ The solution of equation is $x = 2$ and $y = 2$.

(iv) $2x - 2y - 2 = 0, 4x - 4y - 5 = 0$

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{-2}{-4} = \frac{1}{2} \text{ and } \frac{c_1}{c_2} = \frac{-2}{-5} = \frac{2}{5}$$

$$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

⇒ Lines are Parallel

⇒ Inconsistent

5. Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.

SOLUTION:

Given: Half of Perimeter of a rectangular garden = 36 m

To find: Length and width of rectangular garden.

Let the length and width of the rectangular garden be x and y.

According to the question,

$x = y + 4$ ....[i]

and Half of perimeter of a rectangular garden = 36 m

$$\Rightarrow \frac{2(x+y)}{2} = 36$$

$\Rightarrow x + y = 36$ ....[ii]

Taking eq [i], $x = y + 4$

Put x = 0

0 = y + 4

y = – 4

Put y = 0

x = 0 + 4

x = 4

x	0	4
y	–4	0

Taking eq[ii], $x + y = 36$

Put x = 0

0 + y = 36

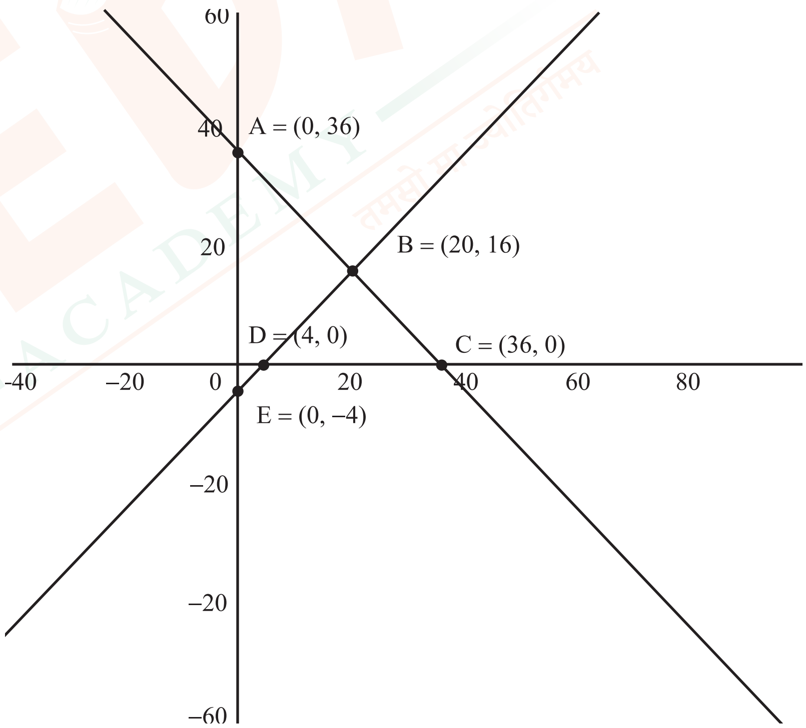
y = 36

Put y = 0

x + 0 = 36

x = 36

x	0	36
y	36	0



From the graph, intersection point is (20,16)

∴ Length of garden = x = 20 m

Width of garden = y = 16 m

6. **Given the linear equation 2x + 3y – 8 = 0, write another linear equation in two variables such that the geometrical representation of the pair so formed is:**
   (i) **Intersecting lines**
   (ii) **Parallel lines**
   (iii) **Coincident lines**

SOLUTION:

Given: linear equation 2x + 3y – 8 = 0

2x + 3y – 8 = 0

(i) For intersecting lines, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Equation: 3x + 2y – 7 = 0

(ii) For Parallel lines, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Equation: 2x + 3y – 12 = 0

(iii) For Coincident lines, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

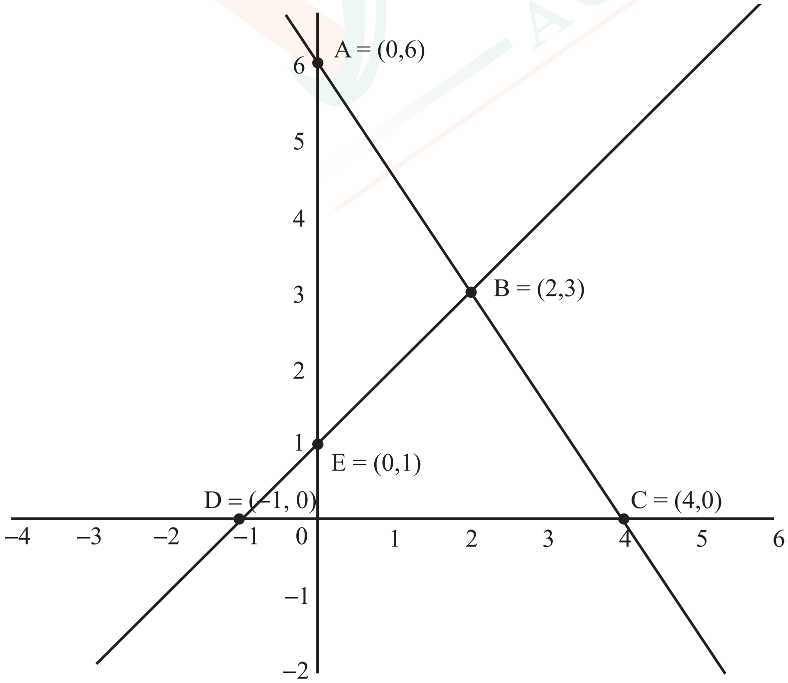
Equation: 4x + 6y – 16 = 0

7. **Draw the graphs of the equations x – y + 1 = 0 and 3x + 2y – 12 = 0. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis, and shade the triangular region.**

SOLUTION:

Given: Linear equations x – y + 1 = 0 and 3x + 2y – 12 = 0

To find: Coordinates of the vertices formed by these lines and x axis.



EXERCISE 3.2

1. Solve the following pair of linear equations by the substitution method

(i) $x + y = 14; x - y = 4$

(ii) $s - t = 3\ \frac{s}{3} + \frac{t}{2} = 6$

(iii) $3x - y = 3; 9x - 3y = 9$

(iv) $0.2x + 0.3y = 1.3; 0.4x + 0.5y = 2.3$

(v) $\sqrt{2}x + \sqrt{3}y = 0; \quad \sqrt{3}x - \sqrt{8}y = 0$

(vi) $\frac{3x}{2} - \frac{5y}{3} = -2; \quad \frac{x}{3} + \frac{y}{2} = \frac{13}{6}$

SOLUTION:

(i) $x + y = 14$ ……[i]; $x - y = 4$ …….[ii]

Taking equation [ii]

$x - y = 4$

$x = 4 + y$ …..[iii]

Put in [i]

$(4 + y) + y = 14$

$4 + 2y = 14$

$2y = 14 - 4$

$2y = 10$

$y = 5$

Put in [iii]

$x = 4 + 5 = 9$

(ii) $s - t = 3; \frac{s}{3} + \frac{t}{2} = 6$

Taking equation, $s - t = 3$

$s = t + 3$ …..[i]

Now, Taking equation,

$$\frac{s}{3} + \frac{t}{2} = 6$$

$$\Rightarrow \frac{2s + 3t}{6} = 6$$

$\Rightarrow 2s + 3t = 36$

$\Rightarrow 2(t + 3) + 3t = 36$ [From (i)]

$\Rightarrow 2t + 6 + 3t = 36$

$\Rightarrow 5t = 36 - 6$

$\Rightarrow 5t = 30$

$\Rightarrow t = \frac{30}{5}$

$\Rightarrow$ t = 6

Put in [i]

s = t + 3 = 6 + 3 = 9

(iii) $3x - y = 3$……[i]

$9x - 3y = 9$…..…….[ii]

From [i], we get

$x = \frac{3+y}{3}$

Now, substitute the value of x in the equation [ii]

$9\left(\frac{3+y}{3}\right) - 3y = 9$

$\Rightarrow$ 9 + 3y – 3y = 9

$\Rightarrow$ 9 = 9

Therefore, y has infinite values and since, $x = \frac{3+y}{3}$

So, x also has infinite values.

(iv) 0.2x + 0.3y = 1.3 …….[i]; 0.4x + 0.5y = 2.3 …….[ii]

Taking eq.[i] 0.2x + 0.3y = 1.3

Multiply both sides by 10

10(0.2x + 0.3y) = 10(1.3)

2x + 3y = 13

2x = 13 – 3y

$x = \frac{13-3y}{2}$ ……[iii]

Taking eq.[ii], 0.4x + 0.5y = 2.3

Multiply both sides by 10

10(0.4x + 0.5y) = 10(2.3)

4x + 5y = 23

$4\left(\frac{13-3y}{2}\right) + 5y = 23$ [From (iii)]

2(13 – 3y) + 5y = 23

26 – 6y + 5y = 23

–y = 23 – 26

–y = –3

y = 3

Put in [iii]

$x = \left(\frac{13-3(3)}{2}\right) = \frac{13-9}{2} = \frac{4}{2} = 2$

(v) $\sqrt{2}x+\sqrt{3}y=0;\quad \sqrt{3}x-\sqrt{8}y=0$

Taking equation, $\sqrt{3}x-\sqrt{8}y=0$

$\Rightarrow \sqrt{3}x=\sqrt{8}y$

$\Rightarrow x=\dfrac{\sqrt{8}}{\sqrt{3}}y$ ....[i]

Putting in equation, $\sqrt{2}x+\sqrt{3}y=0$

$\Rightarrow \sqrt{2}\left(\dfrac{\sqrt{8}}{\sqrt{3}}y\right)+\sqrt{3}y=0$ [From (i)]

$\Rightarrow \dfrac{\sqrt{16}}{\sqrt{3}}y+\sqrt{3}y=0$

$\Rightarrow \dfrac{4y+3y}{\sqrt{3}}=0$

$\Rightarrow 7y=0$

$\Rightarrow y=0$

Put in [i]

$x=\dfrac{\sqrt{8}}{\sqrt{3}}(0)=0$

(vi) $\dfrac{3x}{2}-\dfrac{5y}{3}=-2;\quad \dfrac{x}{3}+\dfrac{y}{2}=\dfrac{13}{6}$

Taking equation $\dfrac{3x}{2}-\dfrac{5y}{3}=-2;$

$\Rightarrow \dfrac{3(3x)-2(5y)}{6}=-2$

$\Rightarrow 9x-10y=-12$

$\Rightarrow 9x=10y-12$

$\Rightarrow x=\dfrac{10y-12}{9}$ ....(i)

Putting in equation $\dfrac{x}{3}+\dfrac{y}{2}=\dfrac{13}{6}$

$\Rightarrow \dfrac{10y-12}{27}+\dfrac{y}{2}=\dfrac{13}{6}$

$\Rightarrow \dfrac{2(10y-12)+27y}{54}=\dfrac{13}{6}$

$\Rightarrow 20y-24+27y=\dfrac{13}{6}\times 54$

$\Rightarrow 47y-24=13\times 9$

$\Rightarrow 47y=117+24$

$$\Rightarrow y = \frac{141}{47}$$

$\Rightarrow$ y = 3

Put in (i)

$$x = \frac{10(3)-12}{9} = \frac{30-12}{9} = \frac{18}{9} = 2$$

2. Solve 2x + 3y = 11 and 2x – 4y = –24 and hence find the value of 'm' for which y = mx + 3

SOLUTION:

2x + 3y = 11 ....(i)

2x – 4y = –24 ....(ii)

From equation (i), we get

$$x = \frac{11-3y}{2} \text{...... (iii)}$$

Substituting the value of x in equation (ii), we get

$$2\left(\frac{11-3y}{2}\right) - 4y = -24$$

11 – 3y – 4y = –24

–7y = –35

y = 5 ....(iv)

Putting the value of y in equation (iii), we get

$$x = \frac{11-3\times 5}{2} = -\frac{4}{2} = -2$$

Hence, x = –2, y = 5

Also,

y = mx + 3

5 = –2m + 3

–2m = 2 m = –1

Therefore the value of is –1.

3. Form the pair of linear equations for the following problems and find their solution by substitution method.

(i) The difference between two numbers is 26 and one number is three times the other. Find them.

(ii) The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them.

(iii) The coach of a cricket team buys 7 bats and 6 balls for ₹3800. Later, she buys 3 bats and 5 balls for ₹1750. Find the cost of each bat and each ball.

(iv) The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10 km, the charge paid is ₹105 and for a journey of 15 km, the charge paid is ₹155. What are the fixed charges and the charge per km? How much does a person have to pay for travelling a distance of 25 km?

(v) A fraction becomes $\frac{9}{11}$, if 2 is added to both the numerator and the denominator. If, 3 is added

to both the numerator and the denominator it becomes $\frac{5}{6}$ Find the fraction.

(vi) **Five years hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their present ages?**

SOLUTION:

(i) Given: Difference between two numbers = 26 and one number is 3 times the other.

To find :We have to find the numbers.

Let two numbers be x(larger number) any y(smaller number)

According to question,

$x = 3y$ …..[i]

$x - y = 26$ …..[ii]

Putting value of x from equation [i] in [ii]

$3y - y = 26$

$2y = 26$

$y = 13$

Put in [i]

$x = 3(13) = 39$

Therefore, two numbers are 39 and 13.

(ii) Given: larger of supplementary angles is greater than the smaller by 18 degrees.

To find :We have to find the supplementary angles.

Let two supplementary angles be x(larger) and y(smaller).

We know that the sum of supplementary angles is 180^o

$x + y = 180$ …..[i]

and $x - y = 18$ …..[ii]

$x = y + 18$

Put in [i]

$y + 18 + y = 180$

$2y + 18 = 180$

$2y = 180 - 18$

$2y = 162$

$y = 81$

Put in [ii]

$x - 81 = 18$

$x = 18 + 81 = 99$

Therefore, two supplementary angles are 81^o and 99^o

(iii) Given: Total cost of 7 Bats and 6 Balls = ₹3800

Total cost of 3 Bats and 5 Balls = ₹1750

To Find: Cost of one Bat and one Ball

Let the cost of a bat and a ball be x and y respectively.

According to question,

$7x + 6y = 3800$ ....[i]

$3x + 5y = 1750$ ....[ii]

$3x = 1750 - 5y$

$x = \frac{1750 - 5y}{3}$ ....[iii]

Put in [i]

$\Rightarrow 7\left(\frac{1750 - 5y}{3}\right) + 6y = 3800$

$\Rightarrow \frac{12250 - 35y}{3} + 6y = 3800$

$\Rightarrow \frac{12250 - 35y + 18y}{3} = 3800$

$\Rightarrow 12250 - 17y = 3800 \times 3$

$\Rightarrow 12250 - 17y = 11400$

$\Rightarrow -17y = 11400 - 12250$

$\Rightarrow -17y = -850$

$\Rightarrow y = \frac{-850}{-17}$

$y = 50$

Put in [iii]

$x = \frac{1750 - 5(50)}{3} = \frac{1750 - 250}{3} = \frac{1500}{3} = 500$

Therefore, cost of a bat is ₹500 and cost of a ball is ₹50.

(iv) Given: Total charge for 10 km distance is ₹105 and Total charge for 15 km distance is ₹155.

To Find: We have to find fixed charge and charge per km.

Let the fixed charge be ₹x and charge per kilometer is ₹y

According to the question,

$x + 10y = 105$ .....[i]

and $x + 15y = 155$ .....[ii]

$x = 155 - 15y$

Put value of x in [i]

$155 - 15y + 10y = 105$

$155 - 5y = 105$

$-5y = 105 - 155$

$-5y = -50$

$y = 10$

Put in [ii]

$x + 15 \times 10 = 155$

$x = 155 - 150$

$x = 5$

Therefore, the fixed charge is ₹5 and the charge per km is ₹10.

For 25 km total pay = 5 + 25 × 10 = ₹255

(v) To find: We have to find the Fraction.

Let the numerator be x and the denominator by y.

So fraction = $\frac{x}{y}$

According to question,

$$\frac{x+2}{y+2} = \frac{9}{11} \text{.........[i]}$$

and

$$\frac{x+3}{y+3} = \frac{5}{6} \text{.........[ii]}$$

Taking equation[i]

$$\frac{x+2}{y+2} = \frac{9}{11}$$

$\Rightarrow 11(x + 2) = 9(y + 2)$

$\Rightarrow 11x + 22 = 9y + 18$

$\Rightarrow 11x = 9y + 18 - 22$

$\Rightarrow 11x = 9y - 4$

$$\Rightarrow x = \frac{9y-4}{11} \text{ ....[iii]}$$

Taking equation[ii]

$$\frac{x+3}{y+3} = \frac{5}{6}$$

$\Rightarrow 6(x + 3) = 5(y + 3)$

$\Rightarrow 6x + 18 = 5y + 15$

$\Rightarrow 6x = 5y + 15 - 18)$

$$\Rightarrow 6\left(\frac{9y-4}{11}\right) = 5y - 3 \quad \text{[From iii]}$$

$\Rightarrow 6(9y - 4) = 11(5y - 3)$

$\Rightarrow 54y - 24 = 55y - 33$

$\Rightarrow 33 - 24 = 55y - 54y$

$\Rightarrow y = 9$

Put in equation[iii]

$$x = \frac{9(9)-4}{11} = \frac{81-4}{11} = \frac{77}{11} = 7$$

Therefore, the fraction is: $\frac{x}{y} = \frac{7}{9}$

(vi) To find : We have to find the present age of Jacob and his son.

Let the present age of Jacob and his son be x and y respectively.

Jacob's age after 5 years = x + 5

His son's age after 5 years = y + 5

According to question, x + 5 = 3(y + 5)

x + 5 = 3y + 15

x = 3y + 15 – 5

x = 3y + 10 .....[i]

Now, Jacob's age 5 years ago = x – 5

His son's age 5 years ago = y – 5

According to question, x – 5 = 7(y – 5)

x – 5 = 7y – 35

x = 7y – 35 + 5

3y + 10 = 7y – 30 [From i]

30 + 10 = 7y – 3y

40 = 4y

y = 10

Put in [i]

x = 3(10) + 10 = 30 +10 = 40

Therefore, Jacob's Present age is 40 years and his son's age is 10 years.

EXERCISE 3.3

1. Solve the following pair of linear equations by the elimination method and the substitution method:

(i) x + y = 5 and 2x – 3y = 4

(ii) 3x + 4y = 10 and 2x – 2y = 2

(iii) 3x – 5y – 4 = 0 and 9x = 2y + 7

(iv) $\frac{x}{2} + \frac{2y}{3} = -1$ and $x - \frac{y}{3} = 3$

(iv) $\frac{x}{2} + \frac{2y}{3} = -1$ and $x - \frac{y}{3} = 3$

SOLUTION:

(i) Solution Using Elimination Method

$x+y=5$ .....[i]

$2x-3y=4$ .....[ii]

eqn.[i]×3: $3x+3y=15$

$2x-3y=4$

$5x=19$

$x=\frac{19}{5}$

Put in [i]

$\frac{19}{5}+y=5$

$\Rightarrow y=5-\frac{19}{5}$

$\Rightarrow y=\frac{25-19}{5}=\frac{6}{5}$

Now, Solution Using Substitution Method

x + y = 5

$\Rightarrow$ y = 5 – x........[i]

And,

2x – 3y = 4............[ii]

$\Rightarrow$ 2x – 3 (5 – x) = 4

$\Rightarrow$ 2x – 15 + 3x = 4

$\Rightarrow$ 5x = 4 + 15

$\Rightarrow$ 5x = 19

$\Rightarrow x=\frac{19}{5}$

Put in [i] $\Rightarrow y=5-\frac{19}{5}=\frac{25-19}{5}=\frac{6}{5}$

(ii) Solution Using Elimination Method

$3x+4y=10$ .....[i]

$2x-2y=2$ .....[ii]

eqn.[ii]×2: $4x-4y=4$

$\Rightarrow 3x+4y=10$

$\Rightarrow 7x=14$

$\Rightarrow x=\frac{14}{7}=2$

Put in [ii]

2 (2) – 2y = 2

$\Rightarrow 4 - 2y = 2$

$\Rightarrow -2y = 2 - 4$

$\Rightarrow -2y = -2$

$\Rightarrow y = 1$

Now, Solution Using Substitution Method

$3x + 4y = 10$

$\Rightarrow 4y = 10 - 3x$

$\Rightarrow y = \left(\frac{10-3x}{4}\right)$ .....[i]

And,

$2x - 2y = 2$ .....[ii]

$\Rightarrow 2x - 2\left(\frac{10-3x}{4}\right) = 2$ [From i]

$\Rightarrow 2x - \left(\frac{20-6x}{4}\right) = 2$

$\Rightarrow \frac{8x - 20 + 6x}{4} = 2$

$\Rightarrow 14x - 20 = 8$

$\Rightarrow 14x = 8 + 20$

$\Rightarrow 14x = 28$

$\Rightarrow x = 2$

Put in [i]

$\Rightarrow y = \left(\frac{10-3(2)}{4}\right) = \frac{10-6}{4} = \frac{4}{4} = 1$

(iii) Solution Using Elimination Method

$3x - 5y - 4 = 0$ .....[i]

$9x = 2y + 7$

$\Rightarrow 9x - 2y - 7 = 0$ .....[ii]

eqn.[i]×3: $9x - 15y - 12 = 0$

$\Rightarrow 9x - 2y - 7 = 0$

Subtract (–) (+) (+)

$\Rightarrow -13y - 5 = 0$

$\Rightarrow -13y = 5$

$\Rightarrow y = -\frac{5}{13}$

Put in [i]

$3x - 5\left(-\frac{5}{13}\right) - 4 = 0$

$\Rightarrow 3x+\frac{25}{13}=4$

$\Rightarrow \quad 3x=4-\frac{25}{13}$

$\Rightarrow \quad 3x=\frac{52-25}{13}$

$\Rightarrow \quad x=\frac{27}{13\times 3}=\frac{9}{13}$

Now, Solution Using Substitution Method

$3x-5y-4=0$

$\Rightarrow 3x=5y+4$

$\Rightarrow x=\left(\frac{5y+4}{3}\right)$ .....[i]

And,

$9x=2y+7$ .....[ii]

$\Rightarrow 9\left(\frac{5y+4}{3}\right)=2y+7$ [From i]

$\Rightarrow 3(5y+4)=2y+7$

$\Rightarrow 15y+12=2y+7$

$\Rightarrow 15y-2y=7-12$

$\Rightarrow 13y=-5$

$\Rightarrow y=\frac{-5}{13}$

Put in [i]

$\Rightarrow x=\left(\frac{5\left(\frac{-5}{13}\right)+4}{3}\right)=\frac{\frac{-25}{13}+4}{3}=\frac{\frac{-25+52}{13}}{3}=\frac{27}{13\times 3}=\frac{9}{13}$

(iv) Solution Using Elimination Method.

$\frac{x}{2}+\frac{2y}{3}=-1$

$\frac{3x+4y}{6}=-1$

$\Rightarrow 3x + 4y = -6$ .....(i)

And,

$x-\frac{y}{3}=3$

$\Rightarrow \frac{3x-y}{3}=3$

$\Rightarrow 3x - y = 9 \ldots.(ii)$

Now, Subtracting equation (i) from (ii),we get

$$3x - y = 9$$
$$3x + 4y = -6$$
$$- \quad - \qquad +$$

$$\Rightarrow -5y = 15$$
$$\Rightarrow y = \frac{-15}{5} = -3$$

Put in [ii]

$$\Rightarrow 3x - y = 9$$
$$\Rightarrow 3x - (-3) = 9$$
$$\Rightarrow 3x + 3 = 9$$
$$\Rightarrow 3x = 9 - 3$$
$$\Rightarrow x = \frac{6}{3} = 2$$

Now, Solution Using Substitution Method

$$\frac{x}{2} + \frac{2y}{3} = -1$$
$$\frac{3x + 4y}{6} = -1$$
$$\Rightarrow 3x + 4y = -6 \ldots\ldots.(i)$$

And,

$$x - \frac{y}{3} = 3$$
$$\Rightarrow \frac{3x - y}{3} = 3$$
$$\Rightarrow 3x - y = 9$$
$$\Rightarrow -y = 9 - 3x$$
$$\Rightarrow y = 3x - 9 \ldots\ldots (ii)$$

Put in (i)

$$\Rightarrow 3x + 4(3x - 9) = -6$$
$$\Rightarrow 3x + 12x - 36 = -6$$
$$\Rightarrow 15x = 36 - 6$$
$$\Rightarrow 15x = 30$$
$$\Rightarrow x = \frac{30}{15} = 2$$

Put in (ii)

$$\Rightarrow y = 3 \times 2 - 9 = 6 - 9 = -3$$

2. Form the pair of linear equations in the following problems, and find their solutions (if they exist) by the elimination method:

(i) If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes $\frac{1}{2}$ if we only add 1 to the denominator. What is the fraction?

(ii) Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu?

(iii) The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.

(iv) Meena went to a bank to withdraw ₹2000. She asked the cashier to give her ₹50 and ₹100 notes only. Meena got 25 notes in all. Find how many notes of ₹50 and ₹100 she received.

(v) A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid ₹27 for a book kept for seven days, while Susy paid ₹21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

SOLUTION:

(i) To find: We have to find fraction.

Let the fraction be $\frac{a}{b}$

According to the given information,

$\frac{a+1}{b-1} = 1$

$\Rightarrow a - b = -2 \ldots$ (i)

$\frac{a}{b+1} = \frac{1}{2}$

$\Rightarrow 2a - b = 1 \ldots$ (ii)

When equation (i) is subtracted from equation (ii) we get,

a = 3

When is substituted in equation (i) we get,

3 – b = –2

–b = –5

b = 5

Fraction $= \frac{3}{5}$

(ii) To find: Present age of Nuri and Sonu.

Let the present age of Nuri and Sonu be x and y respectively.

Nuri's age before 5 years = x – 5

Sonu's age before 5 years = y – 5

According to question, x – 5 = 3(y – 5)

x – 5 = 3y – 15

x – 3y = – 15 + 5

x – 3y = –10 …..[i]

Now, Nuri's age after 10 years $= x + 10$

Sonu's age after 10 years $= y + 10$

According to question, $x + 10 = 2(y + 10)$

$$x + 10 = 2y + 20$$
$$x - 2y = 20 - 10$$
$$x - 2y = 10 \quad .....[ii]$$

Subtract eq[i] from eq[ii]

$$\begin{array}{l} \cancel{x} - 2y = 10 \\ \cancel{x} - 3y = -10 \\ (-) \quad (+) \quad (+) \\ \hline \Rightarrow \quad y = 20 \end{array}$$

Put in [ii]

$$x - 2(20) = 10$$
$$x - 40 = 10$$
$$x = 50$$

Therefore, Nuri's Present age is 50 years and Sonu's age is 20 years.

(iii) Given: Sum of the digits of a two-digit number is 9.

To find: We have to find the number.

Let the tens and ones place digits of a two digit number is x and y respectively.

So, the number is $10x + y$

According to the question,

$x + y = 9 \quad ......[i]$

and $9(10x + y) = 2(10y + x)$

$90x + 9y = 20y + 2x$

$90x - 2x + 9y - 20y = 0$

$88x - 11y = 0$

$11(8x - y) = 0$

$8x - y = 0 \quad .....[ii]$

Adding [i] and [ii]

$$\begin{array}{l} 8x - \cancel{y} = 0 \\ x + \cancel{y} = 9 \\ \hline 9x \quad = 9 \end{array}$$

$x = 1$

Put in [i]

$1 + y = 9$

$y = 8$

Therefore, the number is $10x + y = 10(1) + 8 = 18$

(iv) To Find: Number of notes of ₹50 and ₹100.

Let the number of ₹50 and ₹100 notes be x and y respectively.

According to the question,

$$x + y = 25 \quad \ldots\ldots[i]$$

and $$50x + 100y = 2000$$

$$50(x + 2y) = 50(40)$$

$$x + 2y = 40 \quad \ldots\ldots.[ii]$$

Subtract [i] from [ii]

$$\begin{array}{r} \cancel{x} + 2y = 40 \\ \cancel{x} + y = 25 \\ (-) \quad (-) \quad (-) \\ \hline y = 15 \end{array}$$

Put in [i]

$$x + 15 = 25$$

$$x = 10$$

Therefore, number of ₹50 notes is 10 and ₹100 is 15.

(v) To Find: Fixed charge and the charge for each extra day in a library.

Let the fixed charge for three days be ₹x and additional charge per day be ₹y

According to the question,

$$x + 4y = 27 \quad \ldots\ldots[i]$$

$$x + 2y = 21 \quad \ldots\ldots[ii]$$

Subtract [ii] from [i]

$$\begin{array}{r} \cancel{x} + 4y = 27 \\ \cancel{x} + 2y = 21 \\ (-) \quad (-) \quad (-) \\ \hline 2y = 6 \end{array}$$

$$y = 3$$

Put in [ii]

$$x + 2(3) = 21$$

$$x + 6 = 21$$

$$x = 15$$

Therefore, the fixed charge is ₹15 and the additional charge per km is ₹3