CHAPTER 4

Quadratic Equations

CLASS 10TH

NCERT EXERCISE AND SOLUTIONS - MATHEMATICS

EXERCISE 4.1

1. Check whether the following are quadratic equations:

(i) $(x + 1)^2 = 2(x - 3)$

(ii) $x^2 - 2x = (-2)(3 - x)$

(iii) $(x - 2)(x + 1) = (x - 1)(x + 3)$

(iv) $(x - 3)(2x + 1) = x(x + 5)$

(v) $(2x - 1)(x - 3) = (x + 5)(x - 1)$

(vi) $x^2 + 3x + 1 = (x - 2)^2$

(vii) $(x + 2)^3 = 2x(x^2 - 1)$

(viii) $x^3 - 4x^2 - x + 1 = (x - 2)^3$

SOLUTION:

In this question equation is given and we have to check it whether it is quadratic or not.

(i) Given: $(x + 1)^2 = 2(x-3)$

$\Rightarrow x^2 + 1 + 2x = 2x - 6$

$\Rightarrow x^2 + 1 + 2x - 2x + 6 = 0$

$\Rightarrow x^2 + 7 = 0$

As the highest power of x is 2 , so the given equation is quadratic.

(ii) Given: $x^2 - 2x = (-2)(3-x)$

$\Rightarrow x^2-2x = -6 + 2x$

$\Rightarrow x^2 - 4x + 6=0$

As the highest power of x is 2 , so the given equation is quadratic.

(iii) Given: $(x - 2)(x + 1) = (x - 1)(x + 3)$

$\Rightarrow x^2 - 2x + x - 2 = x^2 - x + 3x - 3$

$\Rightarrow x^2 - x - 2$

$\Rightarrow 3x - 1 = 0$

As the highest power of x is 1, so the given equation is not quadratic.

(iv) Given: $(x-3)(2x+1) = x(x+5)$

$\Rightarrow 2x^2 - 6x + x - 3 = x^2 + 5x$

$\Rightarrow x^2 - 10x - 3 = 0$

As the highest power of x is 2, so the given equation is quadratic.

(v) Given: $(2x-1)(x-3) = (x+5)(x-1)$

$\Rightarrow 2x^2 - 6x - x + 3$

$\Rightarrow x^2 - 11x + 8 = 0$

As the highest power of x is 2, so the given equation is quadratic.

(vi) Given: $x^2 + 3x + 1 = (x-2)^2$

$\Rightarrow x^2 + 3x + 1 =$

$\Rightarrow 7x - 3 = 0$

As the highest power of x is 1, so the given equation is not quadratic.

(vii) Given: $(x+2)^3 = 2x(x^2-1)$

$\Rightarrow x^3 + 8 + 6x^2 + 12x = 2x^3 - 2x$

$\Rightarrow x^3 - 6x^2 - 14x - 8 = 0$

As the highest power of x is 3, so the given equation is not quadratic.

(viii) Given $x^3 - 4x^2 - x + 1 = (x-2)^3$

$\Rightarrow x^3 - 4x^2 - x + 1 = x^3 - 6x^2 + 12x - 8$

$\Rightarrow 2x^2 - 13x + 9 = 0$

As the highest power of x is 2, so the given equation is quadratic.

2. Represent the following situations in the form of quadratic equations:

(i) The area of a rectangular plot is 528 m². The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.

(ii) The product of two consecutive positive integers is 306 . We need to find the integers.

(iii) Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.

(iv) A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train.

SOLUTION:

Given : Area of a rectangular plot = 528 m²

(i) Let breadth of the rectangular plot = x m

Then, length of the plot = $(2x+1)$m

Area of a rectangular plot = $l \times b$,

$\Rightarrow 528 = (2x+1)x$

$\Rightarrow 528 = 2x^2 + x$

$\Rightarrow 2x^2 + x - 528 = 0$

Which is the required quadratic equation.

(ii) Given: Product of two consecutive positive integers = 306

Let the two consecutive integers be x and $x + 1$

Then, $x(x + 1) = 306$

$\Rightarrow x^2 + x - 306 = 0$

Which is the required quadratic equation.

(iii) Given: Rohan's mother is 26 years more than Rohan and their product of ages 3 years from present =360

Let the present age of Rohan = x years

Rohan's mother's present age = $(x + 26)$ years

After 3 years, Rohan's age = $(x + 3)$ years

After 3 years, Rohan's mother's age = $(x + 26 + 3)$ years

According to question,

$(x + 3)(x + 29) = 360$

$\Rightarrow x^2 + 29x + 3x + 87 - 360 = 0$

$\Rightarrow x^2 + 32x - 273 = 0$

Which is the required quadratic equation.

(iv) Given: Distance travelled by train = 480km

Let speed of the train = x km/h

Total distance to be covered = 480 km

$$\text{Time} = \frac{\text{distance}}{\text{speed}} = \frac{480}{x}$$

Decreased speed of the train = $(x - 8)$km/h

Now,

$$\text{Time} = \frac{480}{x-8}$$

According to question,

$$\frac{480}{x-8} - \frac{480}{x} = 3 \Rightarrow 480\left[\frac{1}{x-8} - \frac{1}{x}\right] = 3$$

$$\Rightarrow 480\left[\frac{x-x+8}{x(x-8)}\right] = 3 \Rightarrow 480 \times 8 = 3x(x-8)$$

$\Rightarrow 3840 = 3x^2 - 24x \Rightarrow 3x^2 - 24x - 3840 = 0$

$\Rightarrow x^2 - 8x - 1280 = 0$

Which is the required quadratic equation.

EXERCISE 4.2

1. Find the roots of the following quadratic equations by factorisation:

(i) $x^2 - 3x - 10 = 0$

(ii) $2x^2 + x - 6 = 0$

(iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

(iv) $2x^2 - x + \frac{1}{8} = 0$

(v) $100x^2 - 20x + 1 = 0$

SOLUTION:

(i) Given: $x^2 - 3x - 10 = 0$

$x^2 - 5x + 2x - 10 = 0$

$x(x - 5) + 2(x - 5) = 0$

$(x - 5)(x + 2) = 0$

Either $x - 5 = 0$ or $x + 2 = 0$

$\Rightarrow x = 5$ or $x = -2$

Hence, the roots are 5 and –2.

(ii) Given: $2x^2 + x - 6 = 0$

$\Rightarrow 2x^2 + 4x - 3x - 6 = 0$

$\Rightarrow 2x(x + 2) - 3(x + 2) = 0$

$\Rightarrow (x + 2)(2x - 3) = 0$

Either $x + 2 = 0$ or $2x - 3 = 0$

Hence, the roots are –2 and $\frac{3}{2}$.

(iii) Given: $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$\Rightarrow \sqrt{2}x^2 + 5x + 2x + 5\sqrt{2} = 0$

$\Rightarrow x(\sqrt{2}x + 5) + \sqrt{2}(\sqrt{2}x + 5) = 0$

$\Rightarrow (\sqrt{2}x + 5)(x + \sqrt{2}) = 0$

Either $\sqrt{2}x + 5 = 0$ or $x + \sqrt{2} = 0$

$\Rightarrow x = -\frac{5}{\sqrt{2}}$ or $x = -\sqrt{2}$

Hence, the roots are $-\frac{5}{\sqrt{2}}$ and $-\sqrt{2}$..

(iv) Given: $2x^2 - x + \frac{1}{8} = 0$

$\Rightarrow 16x^2 - 8x + 1 = 0$

$\Rightarrow 16x^2 - 4x - 4x + 1 = 0$

$\Rightarrow 4x(4x - 1) - 1(4x - 1) = 0$

$\Rightarrow (4x - 1)(4x - 1) = 0$

Either $4x - 1 = 0$ or $4x - 1 = 0$

$\Rightarrow x = \frac{1}{4}$ or $x = \frac{1}{4}$

Hence, the roots are $\frac{1}{4}$ and $\frac{1}{4}$.

(v) Given: $100x^2 - 20x + 1 = 0$

$\Rightarrow 100x^2 - 10x - 10x + 1 = 0$

$\Rightarrow 10x(10x - 1) - 1(10x - 1) = 0$

$\Rightarrow (10x - 1)(10x - 1) = 0$

Either $10x - 1 = 0$ or $10x - 1 = 0$

Hence, the roots are $\frac{1}{10}$ and $\frac{1}{10}$

2. Solve the problems given in Example 1.

(i) John and Jivanti together have 45 marbles. Both of them lost 5 marbles each and the product of the number of marbles they now have is 124. We would like to find out how many marbles they had to start with.

SOLUTION:

(i) Given : Number of marbles John and Jivanti together = 45

Let the number of marbles John had be x

Then, the number of marbles Jivanti had = $45 - x$

The number of marbles left with John, when he lost 5 marbles = $x - 5$

The number of marbles left with Jivanti, when she lost 5 marbles = $45 - x - 5 = 40 - x$

According to question,

$(x - 5)(40 - x) = 124$

$\Rightarrow x^2 - 45x + 324 = 0$

$\Rightarrow x^2 - 36x - 9x + 324 = 0$

$\Rightarrow x(x - 36) - 9(x - 36) = 0$

$\Rightarrow (x - 36)(x - 9) = 0$

$\Rightarrow x - 36 = 0; x - 9 = 0$

$\Rightarrow x = 36, 9$

So, If John's marbles = 36

Then, Jaivanti's marbles = 45 – 36 = 9

And if John's marbles = 9

Then, Jivanti's marbles = 45 – 9 = 36

Number of marbles they had to start with 9 and 36 or 36 and 9.

(ii) A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was ₹750. We would like to find out the number of toys produced on that day.

SOLUTION:

Given: Total cost of production = ₹750

Let the number of toys produced in a day be x.

Then, cost of production of each toy on that day = ₹$(55 - x)$

Total cost of production on that day = $x(55 - x)$

According to question,

$\Rightarrow x(55 - x) = 750$

$\Rightarrow x^2 - 55x + 750 = 0$

$\Rightarrow x^2 - 25x - 30x + 750 = 0$

$\Rightarrow x(x - 25) - 30(x - 25) = 0$

$\Rightarrow (x - 25)(x - 30) = 0$

$\Rightarrow x - 25 = 0; x - 30 = 0$

$\Rightarrow x = 25, 30$

Number of toys produced on that day was 25 or 30.

3. Find two numbers whose sum is 27 and product is 182.

SOLUTION:

Given: sum of two numbers = 27 and product of two numbers = 182.

Let the first number be x, then another number will be $27 - x$.

According to the questions, we have:

$\Rightarrow x(27 - x) = 182$

$\Rightarrow x^2 - 27x + 182 = 0$

$\Rightarrow x^2 - 13x - 14x + 182 = 0$

$\Rightarrow x(x - 13) - 14(x - 13) = 0$

$\Rightarrow (x - 13)(x - 14) = 0$

$x - 13 = 0; x - 14 = 0$

$\Rightarrow x = 13, 14$

So, if first number = 13,

then second number = 27 – 13 = 14

And if first number = 14,

then second number = 27 – 14 = 13

Hence, the numbers are 13 and 14.

4. Find two consecutive positive integers, the sum of whose squares is 365.

SOLUTION:

Given: Sum of squares of two consecutive positive integers = 365

Let the two consecutive integers be x and $(x + 1)$.

According to question,

$x^2 + (x + 1)^2 = 365 \Rightarrow x^2 + x^2 + 2x + 1 = 365$

$\Rightarrow 2x^2 + 2x - 364 = 0 \Rightarrow x^2 + x - 182 = 0$

$\Rightarrow x^2 + 14x - 13x - 182 = 0 \Rightarrow x(x + 14) - 13(x + 14) = 0$

$\Rightarrow (x - 13)(x + 14) = 0 \Rightarrow x = 13, -14$ (-14 is rejected because it is negative integer)

Hence, the two consecutive positive integers are 13 and 13+1=14.

5. The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm , find the other two sides.

SOLUTION:

Given : Hypotenuse of right triangle =13

To find : Other two sides of right triangle.

Let the base of the right triangle be x cm.

Then, the altitude of right triangle = $(x - 7)$ cm

Now, Using Pythagoras' theorem

$\text{Base}^2 + \text{Altitude}^2 = \text{Hypotenuse}^2$

$\therefore x^2 + (x - 7)^2 = 13^2$

$\Rightarrow x^2 + x^2 + 49 - 14x = 169$

$\Rightarrow 2x^2 - 14x - 120 = 0$

$\Rightarrow x^2 - 7x - 60 = 0$

$\Rightarrow x^2 - 12x + 5x - 60 = 0$

$\Rightarrow x(x - 12) + 5(x - 12) = 0$

$\Rightarrow (x - 12)(x + 5) = 0$

$x - 12 = 0; x + 5 = 0,$

$\Rightarrow x = 12, -5$

Since sides cannot be negative, $x = 12$.

Therefore, the base of the given triangle = 12 cm,

and the altitude of this triangle = (12 – 7) cm = 5 cm.

6. A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was ₹90, find the number of articles produced and the cost of each article.

SOLUTION:

Given: Total cost of production = ₹90

To find: Number of articles produced and Cost of each article.

Let total number of pottery articles produced in a day = x

Cost of production of each article = ₹ $\frac{90}{x}$

According to question,

$$2x + 3 = \frac{90}{x}$$

$\Rightarrow x(2x + 3) = 90 \Rightarrow 2x^2 + 3x = 90$

$\Rightarrow 2x^2 + 3x - 90 = 0$

$\Rightarrow 2x^2 + 15x - 12x - 90 = 0$

$\Rightarrow x\,(2x + 15) - 6\,(2x + 15) = 0$

$\Rightarrow (2x + 15)\,(x - 6) = 0$

$\Rightarrow 2x = -15$ or $x - 6 = 0$

$\Rightarrow x = -\frac{15}{2}$ or $x = 6\left(-\frac{15}{2} \text{ is rejected}\right)$

$\therefore$ Number of articles produced per day = 6

Cost of production of each article $= \frac{90}{6}$ = ₹15

EXERCISE 4.3

1. Find the nature of the roots of the following quadratic equations. If the real roots exist, find them:

(i) $2x^2 - 3x + 5 = 0$

(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$

(iii) $2x^2 - 6x + 3 = 0$

SOLUTION:

In this question, quadratic equation is given so, first we have to check the nature of the roots and if real roots are exist then we have to find it.

(i) $2x^2 - 3x + 5 = 0$

Here, $a = 2$, $b = -3$ and $c = 5$

Now, Discriminant $= b^2 - 4ac = (-3)^2 - 4\,(2)\,(5) = 9 - 40 = -31$

$b^2 - 4ac < 0$

Therefore, no real roots.

(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$

Here, $a = 3$, $b = -4\sqrt{3}$ and $c = 4$

Now, Discriminant $= b^2 - 4ac$

$= (-4\sqrt{3})^2 - 4(3)(4) = 48 - 48 = 0$

$b^2 - 4ac = 0$,

Therefore, roots are real and equal.

Hence, $x = \frac{-b}{2a} = \frac{-(-4\sqrt{3})}{2\times 3} = \frac{4\sqrt{3}}{6} = \frac{2\sqrt{3}}{3} = \frac{2}{\sqrt{3}}$

Therefore, the roots are $\frac{2}{\sqrt{3}}$ and $\frac{2}{\sqrt{3}}$.

(iii) $2x^2 - 6x + 3 = 0$

Here, $a = 2$, $b = -6$, $c = 3$

Now, Discriminant $= b^2 - 4ac = (-6)^2 - 4\,(2)\,(3) = 36 - 24 = 12$

$b^2 - 4ac > 0$,

Therefore, roots are real and distinct.

Now,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(3)}}{2(2)}$$

$$= \frac{6 \pm \sqrt{36 - 24}}{4}$$

$$= \frac{6 \pm \sqrt{12}}{4}$$

$$= \frac{6 \pm 2\sqrt{3}}{4}$$

$$= \frac{3 \pm \sqrt{3}}{2}$$

$$= \frac{3 + \sqrt{3}}{2}, \frac{3 - \sqrt{3}}{2}$$

2. Find the values of k for each of the following quadratic equations so that they have two equal roots.

(i) $2x^2 + kx + 3 = 0$

(ii) $kx(x - 2) + 6 = 0$

SOLUTION:

In this question quadratic equation is given having equal roots and we have to find the value of k.

(i) $2x^2 + kx + 3 = 0$

Here, $a = 2$, $b = k$ and $c = 3$

If roots are equal then, Discriminant = 0

i.e., $b^2 - 4ac = 0$

$(k)^2 - 4(2)(3) = 0$

$k^2 - 24 = 0$

$k^2 = 24$

$k = \pm\sqrt{24} = \pm 2\sqrt{6}$

(ii) $kx(x - 2) + 6 = 0$

or $kx^2 - 2kx + 6 = 0$

Here, $a = k$, $b = -2k$ and $c = 6$

If roots are equal then, Discriminant = 0

i.e., $b^2 - 4ac = 0$

$(-2k)^2 - 4k(6) = 0$

$4k^2 - 24k = 0$

$4k(k - 6) = 0$

$(k - 6) = 0$, $4k = 0 \Rightarrow k = 0$ (not posible)

$k = 6$

3. Is it possible to design a rectangular mango grove whose length is twice its breadth and the area is 800 m²? If so, find its length and breadth.

SOLUTION:

Given: Area of rectangular mango grove = 800

Let the breadth of the mango grove be x.

Then, the length of the mango grove will be $2x$.

Area of the mango grove = $(2x)(x) = 2x^2$

$2x^2 = 800$

$x^2 = \frac{800}{2} = 400$

$x^2 - 400 = 0$

Here, $a = 1$, $b = 0$, $c = -400$

Now, Discriminant = $b^2 - 4ac = (0)^2 - 4 \times (1) \times (-400) = 1600$

$b^2 - 4ac > 0$

Thus, the equation will have real roots.

Hence, it is possible to make the mango grove with given dimensions.

Now,

$x^2 - 400 = 0$

$x^2 = 400$

$x = \pm 20$

So, the breadth of grove = 20 m

And length = $2x = 2(20) = 40$ m

4. Is the following situation possible? If so, determine their present ages. The sum of the ages of the two friends is 20 years. Four years ago, the product of their age in years was 48.

SOLUTION:

Given: The sum of ages of two friends = 20years and after four year their product of ages = 48

Let the age of one friend be x years.

Then, the age of the other friend = $(20 - x)$ years.

According to question,

Four years ago,

Age of First friend = $(x - 4)$ years

Age of Second friend = $(20 - x - 4) = (16 - x)$ years

It is given that four years ago, the product of their ages was 48.

i.e., $(x - 4)(16 - x) = 48$

$16x - x^2 - 64 + 4x = 48$

$x^2 - 20x - 112 = 0$

Here, $a = 1$, $b = -20$ and $c = 112$

Discriminant = $b^2 - 4ac = (-20)^2 - 4 \times 112 = 400 - 448 = -48$

$b^2 - 4ac < 0$

Therefore, no real roots.

So, the given situation is not possible.

5. Is it possible to design a rectangular park with a perimeter of 80 and an area of 400 m^2? If so, find its length and breadth.

SOLUTION:

Given :Perimeter of rectangular park = 80 m and Area of rectangular park = 400 m^2

Let the length and breadth of the park be x and y.

Perimeter of the rectangular park = $2(x + y) = 80$

So, $x + y = 40$

$y = 40 - x$

Area of the rectangular park = $(x) \times (y) = 400$

$x(40 - x) = 400$

$40x - x^2 = 400$

$x^2 - 40x + 400 = 0$

Here, $a = 1$, $b = -40$, $c = 400$

Discriminant = $b^2 - 4ac = (-40)^2 - 4 \times 400 = 1600 - 1600 = 0$

$b^2 - 4ac = 0$

Therefore, this equation has equal real roots.

Hence, the given situation is possible.

Now,

$$x = \frac{-b}{2a} = \frac{-(-40)}{2(1)} = \frac{40}{2} = 20$$

therefore, the length of the rectangular park is 20 m

And the breadth of the park, $b = 40 - l = 40 - 20 = 20$ m.