CHAPTER 5

Arithmetic Progression

CLASS 10TH

NCERT EXERCISE AND SOLUTIONS - MATHEMATICS

EXERCISE 5.1

1. In which of the following situations does the list of numbers involved make an arithmetic progression and why?

(i) The taxi fare after each km when the fare is ₹15 for the first km and ₹8 for each additional km.

(ii) The amount of air present in a cylinder when a vacuum pump removes $\frac{1}{4}$ of the air remaining in the cylinder at a time.

(iii) The cost of digging a well after every metre of digging, when it costs ₹150 for the first metre and rises by ₹50 for each subsequent metre.

(iv) The amount of money in the account every year, when ₹10,000 is deposited at compound interest at 8% per annum.

SOLUTION:

To find :Check whether the given word problem is Arithmetic Progression or not .

(i) Let t_n be the taxi fare for first .

Then $t_1 = a = 15, t_2 = 15 + 8 = 23,\ t_3 = 23 + 8 = 31$

So, the list will be as follows: 15, 23, 31,…

Here

$t_2 - t_1 = t_3 - t_2 = 8$

Thus, this situation forms an AP.

(ii) Let the first term be x units.

Then, $t_1 = a = x$

$$t_2 = x - \frac{1}{4}x = \frac{3}{4}x \text{ units}$$

$$t_3 = \frac{3}{4}x - \frac{1}{4}\left(\frac{3}{4}x\right) = \frac{9}{16}x \text{ units}$$

$$t_4 = \frac{9}{16}x - \frac{1}{4}\left(\frac{9}{16}x\right) = \frac{27}{64}x \text{ units}$$

The list of numbers is $x, \frac{3}{4}x, \frac{9}{16}x, \frac{27}{64}x, \ldots$

Since $t_2 - t_1 \neq t_3 - t_2$, therefore, it is not an AP.

(iii) First term a = ₹150.

Common difference for every subsequent metre is ₹ 50 .

$t_1 = a = 150$

$t_2 = a + d = 150 + 50 = 200$

$t_3 = a + 2d = 150 + 2 \times 50 = 250$

$t_4 = a + 3d = 150 + 150 = 300$

Since $t_2 - t_1 = t_3 - t_2 = 50$, therefore, it is an AP.

(iv) Let t_n be the amount of money in the nth year.

Then, $t_1 = a = 10,000$,

$$t_2 = 10,000 + 10,000 \times \frac{8}{100}$$

$$= 10,000 + 800 = 10,800$$

$$t_3 = 10,800 + 10,800 \times \frac{8}{100}$$

$$= 10,800 + 864 = 11,664$$

$$t_4 = 11,664 + 11,664 \times \frac{8}{100}$$

$$= 11,664 + 933.12 = 12597.12$$

The list is 10000,10800,11664,12597.12,…

Here $t_2 - t_1 \neq t_3 - t_2$, therefore, it is not an AP.

2. Write the first four terms of the A.P. when the first term a and the common difference d are given as follows:

(i) $a = 10, d = 10$ **(ii)** $a = -2, d = 0$ **(iii)** $a = 4, d = -3$

(iv) $a = -1, d = \frac{1}{2}$ **(v)** $a = -1.25, d = -0.25$

SOLUTION:

To find : First four terms of A.P.

(i) Given: $a = 10$, $d = 10$

$a_1 = 10$

$a_2 = 10 + 10 = 20$

$a_3 = 20 + 10 = 30$

$a_4 = 30 + 10 = 40$

Thus, the first four terms of the AP are 10, 20, 30, 40.

(ii) Given: $a = -2$, $d = 0$

The first four terms of the AP are –2, –2, –2, –2.

(iii) Given: $a = 4$ and $d = -3$,

$a_1 = 4$

$a_2 = a_1 + d = 4 - 3 = 1$

$a_3 = a_2 + d = 1 - 3 = -2$

$a_4 = a_3 + d = -2 - 3 = -5$

Thus, the first four terms of the AP are 4, 1, –2, –5.

(iv) Given: $a = -1$ and $d = \frac{1}{2}$

$a_1 = -1$

$a_2 = a_1 + d = \frac{-1}{1} + \frac{1}{2} = \frac{-1}{2}$

$a_3 = a_2 + d = \frac{-1}{2} + \frac{1}{2} = 0$

$a_4 = a_3 + d = 0 + \frac{1}{2} = \frac{1}{2}$

Thus, the first four terms of the AP are $-1, -\frac{1}{2}, 0, \frac{1}{2}$

(v) Given : $a = -1.25$, $d = -0.25$, $a_1 = -1.25$

$a_2 = a_1 + d = -1.25 - 0.25 = -1.50$

$a_3 = a_2 + d = -1.50 - 0.25 = -1.75$

$a_4 = a_3 + d = -1.75 - 0.25 = -2.0$

Thus, the first four terms of the AP are –1.25, –1.50, –1.75,–2.0

3. For the following A.P.s, write the first term and the common difference.

(i) **3, 1, – 1, – 3 …** **(ii)** **–5, – 1, 3, 7 …**

(iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3} \ldots$ **(iv)** **0.6, 1.7, 2.8, 3.9 …**

SOLUTION :

In this question A.P are given and we have to find the first term and common difference.

(i) $a = 3$ and $d = t_2 - t_1 = 1 - 3 = -2$.

(ii) $a = -5$ and $d = t_2 - t_1' = -1 - (-5) = 4$

(iii) $a = \frac{1}{3}$ and $= t_2 - t_1 = \frac{5}{3} - \frac{1}{3} = \frac{4}{3}$

(iv) $a = 0.6$ and $d = t_2 - t_1 = 1.7 - 0.6 = 1.1$

4. Which of the following are APs? If they form an A.P., find the common difference d and write three more terms.

(i) **2, 4, 8, 16 …** **(ii)** $2, \frac{5}{2}, 3, \frac{7}{2} \ldots$ **(iii)** **–1.2, –3.2, –5.2, –7.2 …**

(iv) **–10, – 6, – 2, 2 …** **(v)** $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}$ **(vi)** **0.2, 0.22, 0.222, 0.2222 ….**

(vii) **0, – 4, – 8, – 12 …** **(viii)** $\frac{-1}{2}, \frac{-1}{2}, \frac{-1}{2}, \frac{-1}{2} \ldots$ **(ix)** **1, 3, 9, 27 …**

(x) $a, 2a, 3a, 4a \ldots$ **(xi)** $a, a^2, a^3, a^4 \ldots$ **(xii)** $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32} \ldots$

(xiii) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12} \ldots$ **(xiv)** $1^2, 3^2, 5^2, 7^2 \ldots$ **(xv)** $1^2, 5^2, 7^2, 73 \ldots$

SOLUTION:

In this question, sequence is given and we have check the sequence is an A.P or not then if it is an A.P then we have to find common difference and three more terms of that A.P.

(i) 2, 4, 8, 16,……

$a_2 - a_1 = 4 - 2 = 2$

$a_3 - a_2 = 8 - 4 = 4$

$a_2 - a_1 \neq a_3 - a_2$

Thus, the given sequence is not an AP.

(ii) $2, \frac{5}{2}, 3, \frac{7}{2}, \ldots$

$a_2 - a_1 = \frac{5}{2} - \frac{2}{1} = \frac{1}{2}$

$a_3 - a_2 = \frac{3}{1} - \frac{5}{2} = \frac{1}{2}$

$a_2 - a_1 = a_3 - a_2$

Thus, the given sequence is an AP.

$a_1 = 2, d = \frac{1}{2}$

Next three terms are $a_5 = a_4 + d = \frac{7}{2} + \frac{1}{2} = 4$

$a_6 = a_5 + d = 4 + \frac{1}{2} = \frac{9}{2}, a_7 = a_6 + d = \frac{9}{2} + \frac{1}{2} = 5$

(iii) –1.2, –3.2, –5.2, –7.2,…

$a_2 - a_1 = -3.2 - (-1.2) = -3.2 + 1.2 = -2$

$a_3 - a_2 = -5.2 - (-3.2) = -5.2 + 3.2 = -2$

$a_3 - a_2 = a_2 - a_1$

Thus, the given sequence is an AP.

$a_1 = -1.2, d = -2$

$a_5 = a_4 + d = -7.2 + (-2) = -9.2$

$a_6 = a_5 + d = (-9.2) + (-2) = -11.2$

$a_7 = a_6 + d = (-11.2) + (-2) = -13.2$

(iv) –10, – 6, –2, 2,…

$a_2 - a_1 = -6 - (-10) = 4$

$a_3 - a_2 = -2 - (-6) = 4$

Thus. the given sequence is an AP.

$a_1 = -10, d = 4$

$a_5 = a_4 + d = 2 + 4 = 6, a_6 = a_5 + d = 6 + 4 = 10$

$a_7 = a_6 + d = 10 + 4 = 14$

(v) $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}, \ldots$

$a_2 - a_1 = 3+\sqrt{2}-3 = \sqrt{2}$

$a_3 - a_2 = 3+2\sqrt{2}-3-\sqrt{2} = \sqrt{2}$

Thus, the given' sequence is an AP.

$a_1 = 3, d = \sqrt{2}$

$a_5 = a_4 + d = 3+3\sqrt{2}+\sqrt{2} = 3+4\sqrt{2}$

$a_6 = a_5 + d = 3+4\sqrt{2}+\sqrt{2} = 3+5\sqrt{2}$

$a_7 = a_6 + d = 3+5\sqrt{2}+\sqrt{2} = 3+6\sqrt{2}$

(vi) 0.2, 0.22, 0.222, 0.2222,…

$a_2 - a_1 = 0.22 - 0.2 = 0.02$

$a_3 - a_2 = 0.222 - 0.22 = 0.002$

$a_3 - a_2 \neq a_2 - a_1$

Thus, the given sequence is not an AP.

(vii) 0, –4, –8, –12,……

$a_2 - a_1 = -4 - 0 = -4$

$a_3 - a_2 = -8 - (-4) = -4$

$a_3 - a_2 = a_2 - a_1$

Thus, the given sequence is an AP.

$a_1 = 0, d = -4$

$a_5 = a_4 + d = -12 + (-4) = -16$

$a_6 = a_5 + d = -16 - 4 = -20$

$a_7 = a_6 + d = -20 - 4 = -24$

(viii) $\frac{-1}{2}, \frac{-1}{2}, \frac{-1}{2}, \frac{-1}{2}, \ldots$

$a_2 - a_1 = \frac{-1}{2} - \left(\frac{-1}{2}\right) = \frac{-1}{2} + \frac{1}{2} = 0$

$a_3 - a_2 = \frac{-1}{2} - \left(\frac{-1}{2}\right) = 0$

$a_3 - a_2 = a_2 - a_1$

Thus, the given sequence is an AP.

$a_1 = \frac{-1}{2}, d = 0$

Next three terms are

$a_5 = a_4 + d = \frac{-1}{2}, a_6 = a_5 + d = \frac{-1}{2}, a_7 = a_6 + d = \frac{-1}{2}$

(ix) 1,3,9,27,….

$a_2 - a_1 = 3 - 1 = 2$

$a_3 - a_2 = 9 - 3 = 6$

$a_3 - a_2 \neq a_2 - a_1$

Thus, the given sequence is not an AP.

(x) $a, 2a, 3a, 4a \ldots$

$a_2 - a_1 = 2a - a = a$

$a_3 - a_2 = 3a - 2a = a$

$a_4 - a_3 = 4a - 3a = a$

Since, the common difference is same every time.

Therefore, $d = a$ and the given series forms a A.P.

Hence, next three terms are;

$a_5 = 4a + a = 5a$

$a_6 = 5a + a = 6a$

$a_7 = 6a + a = 7a$

(xi) $a, a^2, a^3, a^4 \ldots$

$a_2 - a_1 = a^2 - a = a(a-1)$

$a_3 - a_2 = a^3 - a^2 = a^2(a-1)$

$a_4 - a_3 = a^4 - a^3 = a^3(a-1)$

Since, the common difference is not same every time.

Therefore, the given series doesn't form a A.P.

(xii) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32} \ldots$

$a_2 - a_1 = \sqrt{8} - \sqrt{2} = 2\sqrt{2} - \sqrt{2} = \sqrt{2}$

$a_3 - a_2 = \sqrt{18} - \sqrt{8} = 3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$

$a_4 - a_3 = 4\sqrt{2} - 3\sqrt{2} = \sqrt{2}$

Since, the common difference is same every time.

Therefore, $d = \sqrt{2}$ and the given series forms a A.P.

Hence, next three terms are;

$a_5 = \sqrt{32} + \sqrt{2} = 4\sqrt{2} + \sqrt{2} = 5\sqrt{2} = \sqrt{50}$

$a_6 = 5\sqrt{2} + \sqrt{2} = 6\sqrt{2} = \sqrt{72}$

$a_7 = 6\sqrt{2} + \sqrt{2} = 7\sqrt{2} = \sqrt{98}$

(xiii) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12} \ldots$

$a_2 - a_1 = \sqrt{6} - \sqrt{3} = \sqrt{3} \times \sqrt{2} - \sqrt{3} = \sqrt{3}(\sqrt{2} - 1)$

$a_3 - a_2 = \sqrt{9} - \sqrt{6} = 3 - \sqrt{6} = \sqrt{3}(\sqrt{3} - \sqrt{2})$

$a_4 - a_3 = \sqrt{12} - \sqrt{9} = 2\sqrt{3} - \sqrt{3} \times \sqrt{3} = \sqrt{3}(2 - \sqrt{3})$

Since, the common difference is not same every time.

Therefore, the given series doesn't form a A.P.

(xiv) $1^2, 3^2, 5^2, 7^2 \ldots$

i.e., 1, 9, 25, 49 …..

$a_2 - a_1 = 9 - 1 = 8$

$a_3 - a_2 = 25 - 9 = 16$

$a_4 - a_3 = 49 - 25 = 24$

Since, the common difference is not same every time.

Therefore, the given series doesn't form a A.P.

(xv) $1^2, 5^2, 7^2, 73 \ldots$

i.e., 1, 25, 49, 73 …

$a_2 - a_1 = 25 - 1 = 24$

$a_3 - a_2 = 49 - 25 = 24$

$a_4 - a_3 = 73 - 49 = 24$

Since, the common difference is same every time.

Therefore, $d = 24$ and the given series forms a A.P.

Hence, next three terms are;

$a_5 = 73 + 24 = 97$

$a_6 = 97 + 24 = 121$

$a_7 = 121 + 24 = 145$

EXERCISE 5.2

1. Fill in the blanks in the following table, given that *a* is the first term, *d* the common difference and the a_n th term of the A.P.

	a	d	n	a_n
(i)	7	3	8	…….
(ii)	–18	……	10	0
(iii)	……	–3	18	–5
(iv)	–18.9	2.5	……	3.6
(v)	3.5	0	105	……

SOLUTION:

(i) Here $a = 7$, $d = 3$ and $n = 8$

$\therefore a_n = a + (n - 1)\, d$

$a_n = 7 + (8 - 1)\, 3 = 7 + 21 = 28$

(ii) Here $a = -18$, $n = 10$ and $a_n = 0$.

$\therefore a_n = a + (n - 1)d$

$\Rightarrow 0 = -18 + (10 - 1)d$

$\Rightarrow 0 = -18 + 9d$

$\Rightarrow d = 2$

(iii) Here $d = -3$, $n = 18$ and $a_n = -5$

$\therefore a_n = a + (n-1)d$

$\Rightarrow -5 = a + (18-1)(-3)$

$\Rightarrow a = 46$

(iv) Here, $a = -18.9$, $d = 2.5$ and $a_n = 3.6$

$a_n = a + (n-1)d$

$\therefore a_n = a + (n-1)d$

$\Rightarrow 3.6 = -18.9 + (n-1)2.5$

$\Rightarrow 22.5 = (n-1)2.5$

$$\Rightarrow (n-1) = \frac{22.5}{2.5}$$

$\Rightarrow n - 1 = 9$

$\Rightarrow n = 10$

(v) Here, $a = 3.5$, $d = 0$ and $n = 105$

$\therefore a_n = a + (n-1)d$

$= 3.5 + (105-1)\,0 = 3.5$

2. Choose the correct choice in the following and justify:

(i) 30^{th} term of the A.P: 10,7, 4, …, is

(A) **97** (B) **77** (C) **−77** (D) **−87**

(ii) 11^{th} term of the A.P: $-3, -\frac{1}{2}, 2, \ldots$, is

(A) **28** (B) **22** (C) **−38** (D) $-48\frac{1}{2}$

SOLUTION:

(i) 10,7,4,…,

$a = 10$, $d = 7 - 10 = -3$, $n = 30$

$a_n = a + (n-1)\,d$

$\Rightarrow a_{30} = a + (30-1)\,d = a + 29d = 10 + 29(-3) = 10 - 87 = -77$

Hence, correct option is (C).

(ii) $-3, -\frac{1}{2}, 2, \ldots$

$a = -3$, $n = 11$

$$d = -\frac{1}{2} - (-3) = -\frac{1}{2} + \frac{3}{1} = \frac{5}{2}$$

$a_n = a + (n-1)d \Rightarrow a_{11} = a + (11-1)d$

$$\Rightarrow a_{11} = a + 10d = -3 + 10 \times \frac{5}{2} = -3 + 25 = 22$$

Hence, correct option is (B).

3. In the following APs find the missing term in the boxes.

(i) 2, □, 26 **(ii)** □, 13, □, 3 **(iii)** 5, □,□, $9\frac{1}{2}$

(iv) –4, □,□,□,□ 6 **(v)** □, 38, □, □, □, –22

SOLUTION:

In this question A.Ps are given and we have to find the missing terms.

(i) Here $a = 2$ and $t_3 = 26$

Then, $t_3 = a + (3 - 1)d$

$\Rightarrow 26 = 2 + 2d$

$\Rightarrow d = 12$

$\therefore t_2 = t_3 - d = 26 - 12 = 14$

Hence, the complete sequence is 2, 14, 26.

(ii) Here $t_2 = 13$ and $t_4 = 3$

Then $t_2 = a + (2 - 1)d$

$\Rightarrow 13 = a + d$ .....(i)

$t_4 = a + (4 - 1)d$

$\Rightarrow 3 = a + 3d$.......(ii)

Subtracting equation (i) form equation (ii), we get: $d = -5$

Putting $d = -5$ in equation (i), we get:

$a = 13 + 5 = 18$

$\therefore t_3 = a + (3 - 1)d$

$= 18 + 2 \times (-5) = 18 - 10 = 8$

Hence, the complete sequence is 18, 13, 8, 3.

(iii) Here, $a = 5, t_4 = 9\frac{1}{2} = \frac{19}{2}$

Then, $t_4 = a + (4 - 1)d$

$\Rightarrow \frac{19}{2} = 5 + 3d \Rightarrow d = \frac{9}{6} = \frac{3}{2}$

$\therefore t_3 = t_4 - d$

$= \frac{19}{2} - \frac{3}{2} = \frac{16}{2} = 8$ and $t_2 = t_3 - d = 8 - \frac{3}{2} = \frac{16-3}{2} = \frac{13}{2} = 6\frac{1}{2}$

Hence, the complete sequence is $5, 6\frac{1}{2}, 8, 9\frac{1}{2}$

(iv) Here, $a = -4$ and $t_6 = 6$

Then, $t_6 = a + (6 - 1)d$

$\Rightarrow 6 = -4 + 5d$

$\Rightarrow d = 2$

$\therefore t_2 = a + d = -4 + 2 = -2$

$t_3 = a + 2d = -4 + 4 = 0$

$t_4 = a + 3d = -4 + 6 = 2$ and t_5 & $= a + 4d = -4 + 8 = 4$

Hence, the complete sequence is –4, –2, 0, 2, ,4, 6.

(v) Here, $t_2 = 38$ and $t_6 = -22$

Then $t_2 = a + (2 - 1)d$

$\Rightarrow 38 = a + d$ ….(i)

and $t_6 = a + 5d$

$\Rightarrow -22 = a + 5d$ ….(ii)

Subtracting equation (i) from equation (ii), we get: d = – 15

Putting d = – 15 in equation (i), we get:

a = 38 + 15 = 53

$\therefore t_3 = a + 2d = 53 + 2 \times (-15)$

$= 53 - 30 = 23$

$t_4 = a + 3d = 53 + 3 \times (-15)$

$= 53 - 45 = 8$

$t_5 = a + 4d = 53 - 60 = -7$

Hence, the complete sequence is 53, 38, 23, 8, –7, –22.

4. Which term of the A.P. 3, 8, 13, 18, … is 78?

SOLUTION:

Given: AP = 3, 8, 13, 18,....

To find: 78 is which term of following A.P.

$a = 3, d = 8 - 3 = 5$

$a_n = 78$

$a + (n - 1)d = 78$

$\Rightarrow 3 + (n - 1)5 = 78$

$\Rightarrow (n - 1)5 = 78 - 3$

$\Rightarrow (n - 1)5 = 75$

$\Rightarrow n - 1 = 15$

$\Rightarrow n = 15 + 1$

$\Rightarrow n = 16$

Hence, $a_{16} = 78$

5. Find the number of terms in each of the following A.P.

(i) **7, 13, 19, …, 205** **(ii)** **$18, 15\frac{1}{2}, 13, \ldots, -47$**

SOLUTION:

In this question, A.P is given and we have to find the number of terms in A.P

(i) Here, $a = 7$, $d = 13 - 7 = 6$ and $l = 205$

Applying the formula $l = a + (n - 1)d$, we get:

$\Rightarrow 205 = 7 + (n - 1) \times 6$

$$\Rightarrow (n-1) = \frac{198}{6} = 33$$

$\Rightarrow n = 33 + 1 = 34$

Hence, the number of terms in this AP is 34.

(ii) Here, $a = 18$,

$$d = 15\frac{1}{2} - 18 = \frac{31-36}{2} = -\frac{5}{2}$$

Applying the formula, $l = a + (n - 1)d$, we get:

$$\Rightarrow -47 = 18 + (n-1)\left(-\frac{5}{2}\right)$$

$$\Rightarrow (n-1) = \frac{65 \times 2}{5} = 26$$

$$\Rightarrow n = 26 + 1 = 27$$

Hence, the number of terms in this AP is 27.

6. Check whether –150 is a term of the A.P. 11, 8, 5, 2, …

SOLUTION:

In this question A.P is given and we have to check –150 is term of this A.P or not

Here, $a = 11$, $d = 8 - 11 = -3$, $a_n = -150$

$\Rightarrow a + (n - 1)d = a_n$

$\Rightarrow 11 + (n - 1)(-3) = -150$

$\Rightarrow (n - 1)(-3) = -150 - 11$

$\Rightarrow -3(n-1) = -161$

$$\Rightarrow n - 1 = \frac{-161}{-3}$$

$$\Rightarrow n = \frac{161}{3} + 1 = \frac{164}{3} = 54\frac{2}{3}$$

Which is not an integral number.

Hence, –150 is not a term of the AP.

7. Find the 31st term of an A.P. whose 11th term is 38 and the 16th term is 73.

SOLUTION:

Given: 11^{th} term = 38 and 16^{th} term = 73

To find: 31^{st} term

Here,

$t_{11} = a + 10d = 38$…….(i)

$t_{16} = a + 15d = 73$ ……..(ii)

Subtracting equation (i) from equation (ii), we get:

$5d = 35 \Rightarrow d = 7$

Substituting the value of $d = 7$ in equation (i), we get:

$a + 10 \times 7 = 38$

$\Rightarrow a = 38 - 70 = -32$

$\therefore t_{31} = a + 30d$

$= -32 + 30 \times 7$

$= -32 + 210 = 178$

Hence, the 31st term is 178.

8. An A.P. consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term.

SOLUTION:

Given: total number of terms = 50 ,3^{rd} term = 12 and last term = 106

To find: 29^{th} term

$a_{50} = 106$

$a_{50} = a + (50 - 1)d$

$\Rightarrow a + 49d = 106.....$ (i)

and $a_3 = 12$

$\Rightarrow a_3 = a + (3 - 1)d$

$\Rightarrow a + 2d = 12....$ (ii)

Subtracting eqn. (ii) from (i), we get

$a + 49d - a - 2d = 106 - 12$

$\Rightarrow 47d = 94$

$\Rightarrow d = \frac{94}{47} = 2$

Put $d = 2$ in eqn. (ii)

$a + 2d = 12$

$\Rightarrow a + 2 \times 2 = 12$

$\Rightarrow a + 4 = 12$

$\Rightarrow a = 12 - 4 = 8$

Now, $a_{29} = a + (29 - 1)$ d

$= a + 28\text{d} = 8 + 28 \times 2 = 8 + 56 = 64$

9. If the 3rd and the 9th terms of an A.P. are 4 and − 8, respectively. Which term of this A.P. is zero?

SOLUTION:

Given: 3^{rd} term = 4 and 9^{th} term = –8

To find: Which term is 0.

Here, $t_3 = 4$

$\Rightarrow a + 2d = 4$ …(i)

and $t_9 = -8$

$\Rightarrow a + 8d = -8$….(ii)

Subtracting equation (i) from equation (ii), we get:

$6d = -12 \Rightarrow d = -2$

From equation (i), $a + 2 \times (-2) = 4 \Rightarrow a = 8$

Now let t_n be zero.

Then $a + (n - 1)d = 0$

$\Rightarrow 8 + (n - 1)(-2) = 0$

$\Rightarrow n = 5$

Hence, 5th term of the given AP is zero.

10. If 17th term of an A.P. exceeds its 10th term by 7. Find the common difference.

SOLUTION:

Given: $a_{17} - a_{10} = 7$

To find: Common Difference

$\Rightarrow [a + (17 - 1) d] - [a + (10 - 1) d] = 7$

$\Rightarrow (a + 16d) - (a + 9d) = 7$

$\Rightarrow 7d = 7$

$\Rightarrow d = 1$

11. Which term of the A.P. 3, 15, 27, 39.... will be 132 more than its 54th term?

SOLUTION:

Here, $a = 3$, $d = 15 - 3 = 12$

$\therefore t_n = a + (n - 1)d$

$= 3 + (n - 1) 12$

$= 3 + 12n - 12$

$\Rightarrow t_n = 12n - 9$ and $t_{54} = a + 53d = 3 + 53 \times 12 = 3 + 636 = 639$

According to the question, we have:

$t_n = 132 + t_{54}$

$\Rightarrow 12n - 9 = 132 + 639$

$\Rightarrow 12n = 780$

$\Rightarrow n = 65$

Hence, 65th term is the required term.

12. Two APs have the same common difference. The difference between their 100th term is 100, what is the difference between their 1000th terms?

SOLUTION:

Let a and A be the first term of two APs and d be the common difference.

Given: $a_{100} - A_{100} = 100$

To find: $a_{1000} - A_{1000} = 1000$

$a_{100} - A_{100} = 100$

$\Rightarrow a + 99d - A - 99d = 100$

$\Rightarrow a - A = 100$.....(i)

$\Rightarrow a_{1000} - A_{1000} = a + 999d - A - 999d$

Using eqn. (i)

$\Rightarrow a_{1000} - A_{1000} = 100$

13. How many three digit numbers are divisible by 7?

SOLUTION:

The list of three-digit numbers divisible by 7 is 105, 112, 119, ..., 994

Here, $a = 105$, $d = 112 - 105 = 7$, $t_n = 994$

$\therefore t_n = a + (n - 1)d$

$\Rightarrow 994 = 105 + (n - 1) \times 7$

$\Rightarrow 889 = (n - 1)7$

$\Rightarrow 127 = n - 1$

$\Rightarrow n = 128$

Hence, there are 128 three-digit numbers divisible by 7.

14. How many multiples of 4 lie between 10 and 250?

SOLUTION:

The multiples of 4 between 10 and 250 be 12,16,20,24,…,248

"Here, $a = 12$, $d = 16 - 12 = 4$, $a_n = 248$

$a_n = a + (n - 1)d$

$\Rightarrow 248 = 12 + (n - 1)4$

$\Rightarrow 248 - 12 = (n - 1)4$

$\Rightarrow 236 = (n - 1)4$

$\Rightarrow 59 = n - 1$

$\Rightarrow n = 59 + 1 = 60$

15. For what value of *n*, are the nth terms of two APs 63, 65, 67, and 3, 10, 17, … equal?

SOLUTION:

The given APs are:

63, 65, 67,…..(i)

and 3, 10, 17,…(ii)

From AP (i), we have:

The first term, $a = 63$ and the common difference $d = 2$.

$\therefore t_n = 63 + (n - 1)2 = 2n + 61$

From AP (ii), we have:

The first term, $a = 3$ and the common difference $d = 7$.

$\therefore t_n = 3 + (n - 1)7 = 7n - 4$

According to the question, we have:

$7n - 4 = 2n + 61$

$\Rightarrow 5n = 65 \Rightarrow n = 13$

Hence, the required value of n is 13.

16. Determine the A.P. whose third term is 16 and the 7th term exceeds the 5th term by 12.

SOLUTION:

Given: $a_3 = 16$ and $a_7 = a_5 + 12$

To find: We have to find the A.P.

$\Rightarrow a + (3 - 1)\, d = 16$

$\Rightarrow a + 2d = 16$

and $a_7 - a_5 = 12$

$\Rightarrow [a + (7 - 1)d] - [a + (5 - 1)d] = 12$

$\Rightarrow a + 6d - a - 4d = 12$

$\Rightarrow 2d = 12$

$\Rightarrow d = 6$

Since $a + 2d = 16$

$\Rightarrow a + 2(6) = 16$

$\Rightarrow a + 12 = 16$

$\Rightarrow a = 16 - 12 = 4$

$a_1 = a = 4$

$a_2 = a_1 + d = a + d = 4 + 6 = 10$

$a_3 = a_2 + d = 10 + 6 = 16$

$a_4 = a_3 + d = 16 + 6 = 22$

Thus, the required AP is $a_1, a_2, a_3, a_4, \ldots$, i.e. 4, 10, 16, 22.

17. Find the 20th term from the last term of the A.P. 3, 8, 13, …, 253.

SOLUTION:

Given: A.P. = 3, 8, 13, …, 253.

To find : 20^{th} from Last.

On reversing the given AP, the new AP will be 253, 248, …, 13, 8, 3

Now, $a = 253$ and $d = 248 - 253 = -5$

$\therefore t_{20} = a + (n - 1)d = 253 + (20 - 1)(-5)$

$= 253 + 19 \times (-5) = 158$

Hence, the 20th term from the last term is 158.

18. The sum of 4th and 8th terms of an A.P. is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the A.P.

SOLUTION:

Given: $a_4 + a_8 = 24$ and $a_6 + a_{10} = 44$

To find: First three terms of A.P.

$a_4 + a_8 = 24$

$\Rightarrow a + (4 - 1)d + a + (8 - 1)d$

$\Rightarrow a + 3d + a + 7d = 24$

$\Rightarrow 2a + 10d = 24 \ldots$(i)

And $a_6 + a_{10} = 44$

$a + (6 - 1)d + a + (10 - 1)d = 44$

$\Rightarrow a + 5d + a + 9d = 44$

$2a + 14d = 44$……(ii)

Subtracting eqn (i) from (ii), we get

$$2a + 14d - 2a - 10d = 44 - 24 \Rightarrow 4d = 20 \Rightarrow d = \frac{20}{4} = 5$$

Put $d = 5$ in eqn. (i), we get

$2a + 10d = 24 \Rightarrow 2a + 10 \times 5 = 24$

$2a = 24 - 50$

$2a = -26$

$$a = \frac{-26}{2} = -13$$

$a_1 = a = -13$

$a_2 = a_1 + d = -13 + 5 = -8$

$a_3 = a_2 + d = -8 + 5 = -3$

Hence, the first three terms are, −13, −8, −3.

19. Subba Rao started work in 1995 at an annual salary of ₹5000 and received an increment of ₹200 each year. In which year did his income reach ₹7000?

SOLUTION:

Salary for the year 1995 = ₹5000

Salary for the year 1996 = ₹5000 + 200 = ₹5200

Salary for the year 1997 = ₹5200 + 200 = ₹5400

Thus, in the form of $t_1, t_2, t_3, \ldots, t_n$, we have:

5000, 5200, 5400,…,7000

It is an AP in which $a = 5000$ and $d = 200$

Let after n years his salary will be ₹7000.

Then $t_n = 7000$

$\Rightarrow a + (n - 1)d = 7000$

$\Rightarrow 5000 + (n - 1)\,200 = 7000$

$$\Rightarrow (n - 1) = \frac{2000}{200} = 10$$

$\Rightarrow n = 11$

Now to find year we consider AP: 1995, 1996, 1997....

Here $a = 1995$ and $d = 1$

$\therefore t_{11} = 1995 + (11 - 1) \times 1$

Hence, in the year 2005, his salary was ₹7000.

20. Ramkali saved ₹5 in the first week of a year and then increased her weekly saving by ₹1.75. If in the *n*th week, her weekly savings become ₹20.75, find *n*.

SOLUTION:

Given: a = ₹5, d = ₹1.75

a_n = ₹20.75

$a + (n - 1)d - 20.75$

$\Rightarrow 5 + (n - 1)1.75 = 20.75$

$\Rightarrow (n - 1) \times 1.75 = 20.75 - 5$

$\Rightarrow (n - 1)\ 1.75 = 15.75$

$\Rightarrow n - 1 = 9$

$\Rightarrow n = 9 + 1$

$\Rightarrow n = 10$

Hence, in 10th week Ramkali's saving will be ₹20.75.

EXERCISE 5.3

1. Find the sum of the following APs.

(i) 2, 7, 12 ,…., to 10 terms.

(ii) −37, −33, −29 ,…., to 12 terms

(iii) 0.6, 1.7, 2.8 ,…….., to 100 terms

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}$, …… , to 11 terms

SOLUTION:

In this question ,A.P is given and we have to find the sum upto given number of terms.

(i) Here, $a = 2$, $t_2 = 7$, $t_3 = 12$ and $n = 10$

$\therefore d = t_3 - t_2 = 12 - 7 = 5$

The required sum,

$$S_{10} = \frac{10}{2}[2 \times 2 + (10 - 1)5]$$

$$= 5 \times 49 = 245$$

(ii) Here, $a = -37$, $t_2 = -33$, $t_3 = -29$ and

$n = 12$

$\therefore d = t_3 - t_2 = -29 + 33 = 4$

The required sum, $S_{12} = \frac{12}{2}[2 \times (-37) + (12 - 1)4]$

$$= 6 \times (-30) = -180$$

(iii) Here, $a = 0.6$, $t_2 = 1.7$, $t_3 = 2.8$ and $n = 100$

$\therefore d = t_3 - t_2 = 2.8 - 1.7 = 1.1$

The required sum,

$$S_{100} = \frac{100}{2}[2 \times 0.6 + (100 - 1)1.1]$$

$= 50 \times 110.1 = 5505$

(iv) Here, $a=\frac{1}{15}, t_2=\frac{1}{12}, t_3=\frac{1}{10}$ and $n = 11$

$\therefore d=t_3-t_2=\frac{1}{10}-\frac{1}{12}=\frac{6-5}{60}=\frac{1}{60}$

∴ The required sum,

$S_{11}=\frac{11}{2}\left[\frac{2}{15}+(11-1)\frac{1}{60}\right]$

$=\frac{11}{2}\left[\frac{2}{15}+\frac{1}{6}\right]=\frac{11}{2}\left[\frac{4+5}{30}\right]$

$=\frac{11}{2}\times\frac{3}{10}=\frac{33}{20}$

2. Find the sums given below:

(i) $\mathbf{7+10\frac{1}{2}+14+.......+84}$

(ii) **34 + 32 + 30 + ……….. + 10**

(iii) **−5 + (−8) + (−11) + ………… + (−230)**

SOLUTION:

In this question ,A.P is given and we have to find the sum .

(i) $7+10\frac{1}{2}+14+\cdots\ldots+84$

Here,

$a=7, d=\frac{21}{2}-\frac{7}{1}=\frac{7}{2}, a_n=84$

$a_n=a+(n-1)d$

$\Rightarrow 84=7+(n-1)\frac{7}{2}\Rightarrow 84-7=(n-1)\frac{7}{2}$

$\Rightarrow 77\times\frac{2}{7}=n-1\Rightarrow 22+1=n\Rightarrow n=23$

$S_n=\frac{n}{2}[a+l][\because a_n=l]$

$S_{23}=\frac{23}{2}[7+84]=\frac{23}{2}\times 91=\frac{2093}{2}=1046\frac{1}{2}$

(ii) 34 + 32 + 30 + … + 10

Here,

$a = 34, d = 32 – 34 = –2, a_n=10$

$a_n = a + (n – 1)\, d \Rightarrow 10 = 34 + (n – 1)(–2)$

$\Rightarrow\ 10-34=(n-1)(-2)\Rightarrow\frac{-24}{-2}=n-1\Rightarrow n-1=12$

$n = 12 + 1 = 13$

$\Rightarrow S_{13} = \frac{13}{2}[34+10] = \frac{13}{2} \times 44 = 13 \times 22 = 286$

(iii) $-5 + (-8) + (-11) + \ldots\ldots + (-230)$

$a = -5, a_n = -230,$

$d = -8 - (-5) = -3$

$a_n = a + (n-1)d \Rightarrow -230 = -5 + (n-1)(-3)$

$\Rightarrow \frac{-225}{-3} = n - 1 \Rightarrow 75 \Rightarrow n = 76$

$S_{76} = \frac{76}{2}[-5 + (-230)]$

$= 38 \times (-235) = -8930$

3. In an AP

(i) **Given $a = 5$, $d = 3$, $a_n = 50$, find n and S_n.**

(ii) **Given $a = 7$, $a_{13} = 35$, find d and S_{13}.**

(iii) **Given $a_{12} = 37$, $d = 3$, find a and S_{12}.**

(iv) **Given $a_3 = 15$, $S_{10} = 125$, find d and a_{10}.**

(v) **Given $d = 5$, $S_9 = 75$, find a and a_9.**

(vi) **Given $a = 2$, $d = 8$, $S_n = 90$, find n and a_n.**

(vii) **Given $a = 8$, $a_n = 62$, $S_n = 210$, find n and d.**

(viii) **Given $a_n = 4$, $d = 2$, $S_n = -14$, find n and a.**

(ix) **Given $a = 3$, $n = 8$, $S_n = 192$, find d.**

(x) **Given $l = 28$, $S_n = 144$ and there are total 9 terms. Find a.**

SOLUTION:

(i) Given: $a = 5$, and $d = 3$ and $a_n = 50$

Applying the formula, $a_n = a + (n-1)d$, we get:

$a + (n-1)d = 50$

$\Rightarrow 5 + (n-1)3 = 50$

$\Rightarrow 3n = 48 \Rightarrow n = 16$

$\therefore S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_{16} = \frac{16}{2}[2 \times 5 + (16-1)3]$

$= 8(10 + 45) = 8 \times 55 = 440$

Hence, $n = 16$ and $S_n = 440$.

(ii) Given: $a = 7$ and $a_{13} = 35$

Applying the formula, $a_n = a + (n-1)d$, we get:

$a_{13} = a + (13-1)d$

$\Rightarrow 35 = 7 + 12d$

$\Rightarrow d = \frac{28}{12} = \frac{7}{3}$

$\therefore S_{13} = \frac{13}{2}[2a + 12d]$

$= \frac{13}{2}\left[2 \times 7 + 12 \times \frac{7}{3}\right]$

$= \frac{13}{2} \times 42 = 273.$

Hence $d = \frac{7}{3}$ and S_{13} =273.

(iii) Given: $a_{12} = 37$ and $d = 3$

$\therefore a_{12} = a + (12-1)d$

$\Rightarrow 37 = a + 11 \times 3$

$\Rightarrow a = 37 - 33 = 4$

$\therefore \quad S_{12} = \frac{12}{2}[2a + (12-1)d]$

$= 6\ [2 \times 4 + 11 \times 3]$

$= 6\ [8 + 33] = 6 \times 41 = 246$

Hence, a = 4 and $S_{12} = 246$.

(iv) Given: $a_3 = 15$ and $S_{10} = 125$.

$\therefore a_3 = a + (3 - 1)d$

$\Rightarrow 15 = a + 2d$.....(i)

Now, $S_{10} = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow 125 = 5\ [2a + 9d]$

$\Rightarrow 25 = 2a + 9d$.......(ii)

Multiplying equation (i) by 2 and then subtracting equation (ii) from resulted equation, we get: $d = -1$

Putting $d = -1$ in equation (i), we get:

$a + 2d = 15$

$\Rightarrow a - 2 = 15$

$\Rightarrow a = 17$

$\therefore a_{10} = a + (n-1)d$

$= 17 + 9 \times (-1) = 17 - 9 = 8$

Hence, $d = -1$ and $a_{10} = 8$.

(v) Given: $d = 5$ and $S_9 = 75$

$\therefore S_9 = \frac{n}{2}[2a + (9-1)d]$

$\Rightarrow 75 = \frac{9}{2}[2a + 40]$

$\Rightarrow -210 = 18a$

$\Rightarrow a = \frac{-210}{18} = \frac{-35}{3}$

Now, $a_9 = a + (9 - 1)d$

$= \frac{-35}{3} + 40 = \frac{85}{3}$

Hence, $a = \frac{-35}{3}$ and $a_9 = \frac{85}{3}$

(vi) Given: a = 2, d = 8 and S_n =90.

$\therefore S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow \frac{n}{2}[4 + (n-1)\times 8] = 90$

$\Rightarrow n\ [2 + (n - 1)\ 4] = 90$

$\Rightarrow 4n^2 - 2n - 90 = 0$

$\Rightarrow 2n^2 - n - 45 = 0$

$\Rightarrow 2n\ (n - 5) + 9\ (n - 5) = 0$

$\Rightarrow (n - 5)\ (2n + 9) = 0$

$\Rightarrow n = 5$ or $n = \frac{-9}{2}$

Since the number of terms cannot be negative, so $n = -\frac{9}{2}$ is rejected.

$\therefore n = 5$

Now $a_5 = 2 + 4 \times 8 = 34$

Hence, $n =$ **5** and $a_5 = 34$.

(vii) Given: $a = 8$, $a_n = 62 = l$ and $S_n = 210$

$\therefore \frac{n}{2}(a + l) = 210\left[\text{ Given } S_n = 210\right]$

$\Rightarrow \frac{n}{2}(8 + 62) = 210 \Rightarrow n = 6$

Now, $a_6 = 62$ [Given]

$\Rightarrow 8 + 5d = 62 \Rightarrow d = \frac{54}{5}$

Hence, $n = 6$ and $d = \frac{54}{5}$

(viii) Given: $a_n = 4$, d = 2 and $S_n = -14$

$\Rightarrow a + (n - 1)2 = 4 \Rightarrow a = 6 - 2n$…..(i)

Now $S_n = -14$

$\Rightarrow \frac{n}{2}[2a + (n-1)d] = -14$

$$\Rightarrow \frac{n}{2}[2(6-2n)+(n-1)2]=-14[\text{ From (i)}]$$

$$\Rightarrow \frac{n}{2}[10-2n]=-14$$

$\Rightarrow n(n-5) = 14$

$\Rightarrow n^2 - 5n - 14 = 0$

$\Rightarrow (n-7)(n+2) = 0$

$\Rightarrow n = 7, n = -2$

Since the number of terms cannot be negative, so $n = -2$ is rejected.

$\therefore$ Putting $n = 7$ in equation (i), we get: $a = 6 - 2 \times 7 = -8$

Hence, n = 7 and a = -8.

(ix) Given: $a = 3$, $n = 8$ and $S = 192$.

$\because S_8 = 192$

$$\frac{8}{2}(2\times3+7d)=192$$

$\Rightarrow 4 \times (6 + 7d) = 192$

$\Rightarrow 6 + 7d = 48$

$\Rightarrow 7d = 42$

$\Rightarrow d = 6$

(x) Given: $l = 28 = t_n$ and $S = 144$

Since $n = 9$, so $t_9 = 28$ and $S_9 = 144$.

$$\therefore \quad \frac{9}{2}(a+t_9)=144$$

$$\Rightarrow \quad \frac{9}{2}(a+28)=144$$

$$\Rightarrow \quad a+28=\frac{288}{9}$$

$\Rightarrow a = 4.$

4. How many terms of the AP. 9, 17, 25 … must be taken to give a sum of 636?

SOLUTION:

In this question A.P is given and we have to find the number of terms which make the total sum of 636.

Given: $a = 9$, $d = 17 - 9 = 8$, $S_n = 636$

$$S_n=\frac{n}{2}[2a+(n-1)d]\Rightarrow 636=\frac{n}{2}[2\times9+(n-1)8]$$

$\Rightarrow 636 \times 2 = n[18 + 8n - 8] \Rightarrow 636 \times 2 = n(10 + 8n)$

$$\Rightarrow 636\times2=2n(5+4n)\Rightarrow \frac{636\times2}{2}=5n+4n^2$$

$\Rightarrow 4n^2 + 5n - 636 = 0 \Rightarrow 4n^2 + 53n - 48n - 636 = 0$

$\Rightarrow n\,(4\,n + 53) - 12(4n + 53) = 0 \Rightarrow (4n + 53)(n - 12) = 0$

$\Rightarrow 4n + 53 = 0$ or $n - 12 = 0$

$n = 12$

5. The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

SOLUTION:

Given: $a = 5$, $l = t_n = 45$ (last term) and $S_n = 400$.

To find: Number of terms and common difference.

$$S_n = 400$$

$$\therefore \frac{n}{2}[a + l] = 400$$

$$\Rightarrow \frac{n}{2}[5 + 45] = 400$$

$$\Rightarrow n = \frac{400}{25} = 16$$

Now, $t_{16} = 45$

$\Rightarrow 5 + 15d = 45$

$$\Rightarrow d = \frac{40}{15} = \frac{8}{3}$$

Thus, $n = 16$ and $d = \frac{8}{3}$.

6. The first and the last term of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

SOLUTION:

Given: $a = 17$, $l = a_n = 350$ (last term) and common difference $d = 9$.

To find: Number of terms and their total sum .

Here,

$a = 17, a_n = 350, d = 9$

$a_n = a + (n - 1)\,d$

$\Rightarrow 350 = 17 + (n - 1)9$

$$\Rightarrow 350 - 17 = (n - 1)9 \Rightarrow \frac{333}{9} = n - 1 \Rightarrow 37 = n - 1 \Rightarrow n = 38$$

$$S_{38} = \frac{38}{2}[17 + 350]$$

$= 19 \times 367 = 6973$

7. Find the sum of first 22 terms of an AP in which $d = 7$ and 22nd term is 149.

SOLUTION:

Given: $d = 7$, $t_{22} = 149$, and $n = 22$

To find: Sum of first 22 terms.

$\therefore t_{22} = a + (22 - 1)d$

$\Rightarrow 149 = a + 21 \times 7$

$\Rightarrow 149 = a + 147 \Rightarrow a = 2$

The required sum,

$$S_{22} = \frac{22}{2}[2\times2+(22-1)7]$$

$$=11[4+147]=11\times151=1661$$

8. **Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.**

SOLUTION:

Given: $a_2 = 14$ and $a_3 = 18$

To find: Sum of First 51 terms.

$\Rightarrow a + d = 14$ ….(i)

and $a + 2d = 18$….(ii)

Substracting (i) and (ii), we get

Since,

$a + 2d - a - d = 18 - 14 \Rightarrow d = 4$

$a + d = 14 \Rightarrow a + 4 = 14$

$\Rightarrow a = 14 - 4 \Rightarrow a = 10$

So,

$$S_{51} = \frac{51}{2}[2\times10+(51-1)4]$$

$$S_{51} = \frac{51}{2}[20+200]$$

$$=\frac{51}{2}\times220=51\times110=5610$$

9. **If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first *n* terms.**

SOLUTION:

Given: $S_7 = 49$ and $S_{17} = 289$

To find: $S_n \Rightarrow S_7 = 49$

$$\Rightarrow \frac{7}{2}(2a+6d)=49$$

$$\Rightarrow a+3d=7\text{…. (i)}$$

$$\text{and } S_{17}=289$$

$$\Rightarrow \frac{17}{2}(2a+16d)=289$$

$$\Rightarrow a+8d=17\text{…. (ii)}$$

Subtracting equation (i) from equation (ii), we get: $d = 2$

Substituting the value of $d = 2$ in equation (i), we get: $a = 7 - 6 = 1$

Sum of terms,

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$= \frac{n}{2}[2\times 1+(n-1)\times 2] = n^2$$

Hence, the required sum is n^2.

10. Show that $a_1, a_2 \ldots, a_n, \ldots$ form an AP where a_n is defined as below

(i) $a_n = 3 + 4n$ (ii) $a_n = 9 - 5n$

Also find the sum of the first 15 terms in each case.

SOLUTION:

In this question nth term is given and we have to show that it is an A.P and also find the sum of first 15 terms.

(i) $a_n = 3 + 4n$

Putting $n = 1, 2, 3\ldots\ldots$

$a_1 = 3 + 4 \times 1 = 3 + 4 = 7$

$a_2 = 3 + 4 \times 2 = 3 + 8 = 11$

$d = =a_2 - a_1 = 11 - 7 = 4$

$a_3 = 3 + 4 \times 3 = 15$

$d = a_3 - a_2 = 15 - 11 = 4$

Thus, the sequence 7, 11, 15,…. is an AP.

$$S_{15} = \frac{15}{2}[2\times 7+(15-1)\times 4] = \frac{15}{2}[14+56] = \frac{15}{2}\times 70 = 525$$

(ii) $a_n = 9 - 5n$

Putting $n = 1, 2, 3,\ldots$

$a_1 = 9 - 5 \times 1 = 9 - 5 = 4$

$a_2 = 9 - 5 \times 2 = 9 - 10 = -1$ $d = a_2 - a_1 = -1 - 4 = -5$

$a_3 = 9 - 5 \times 3 = 9 - 15 = -6$

$d = a_3 - a_2 = -6 - (-1) = -6 + 5 = -5$

$-6 + 1 = -5$

Thus, the sequence 4, –1, –6,…, is an A.P.

$a = 4, d = -1-4 = -5$

$$S_{15} = \frac{15}{2}[2\times 4+(15-1)(-5)] = \frac{15}{2}[8-70]$$

$$= \frac{15}{2}\times(-62) = -465$$

11. If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (that is S_1)? What is the sum of first two terms? What is the second term? Similarly find the 3rd, the10th and the nth terms.

SOLUTION:

Given: $S_n = 4n - n^2$

To find: t_1, t_2, t_3, t_{10} and t_n

We have $S_n = 4n - n^2$..............(i)

Putting $n = 1$ in equation (i), we get:

$S_1 = 4 - 1 = 3$

In an AP, $S_1 = t_1 = a$. So a = 3.

Putting $n = 2$ in equation (i), we get:

$S_2 = 4 \times 2 - (2)^2 = 4$

$t_2 = S_2 - S_1 = 4 - 3 = 1$

Putting $n = 3$ in equation (i), we get:

$S_3 = 4 \times 3 - 3^2 = 12 - 9 = 3$

$t_3 = S_3 - S_2 = 3 - 4 = -1$

Third term is -1.

Putting $n = 9$ in equation (i), we get:

$S_9 = 4 \times 9 - 9^2 = 36 - 81 = 45$

Similarly, $S_{10} = 4 \times 10 - 10^2 = 40 - 100 = -60$

$t_{10} = S_{10} - S_9 = -60 - (-45) = -60 + 45 = -15$

Tenth term is –15 .

Here, common difference, $d = t_2 - t_1$

$= 1 - 3 = -2$

$\therefore$ n th term, $t_n = a + (n - 1)d$

$= 3 + (n - 1)(-2)$

$= 3 - 2n + 2 = 5 - 2n$

12. Find the sum of first 40 positive integers divisible by 6.

SOLUTION:

Let 6, 12, 18,……., 240 be divisible by 6

$a = 6, d = 12 - 6 = 6$

$a_{40} = 6 + (40 - 1) \times 6 = 240$

$$S_{40} = \frac{40}{2}[a + a_{40}] = 20[6 + 240] = 20 \times 246 = 4920$$

13. Find the sum of first 15 multiples of 8.

SOLUTION:

The list of multiples of 8 is 8, 16, 24....

It is an AP in which $a = 8$ and $d = 8$.

Applying the formula $S_n = \frac{n}{2}[2a + (n-1)d]$ we get:

$$S_{15} = \frac{15}{2}[2 \times 8 + (15-1)8]$$

$$= \frac{15}{2}[16 + 112] = \frac{15}{2} \times 128 = 960$$

Hence, the sum of the first 15 multiples of 8 is 960.

14. Find the sum of the odd numbers between 0 and 50.

SOLUTION:

Let odd numbers between 0 and 50 be 1,3,5,7,……..,49.

Here,

$a = 1$, $d = 3 - 1 = 2$

$a_n = 49$

$a_n = a + (n-1)d$

$\Rightarrow a + (n-1)d = 49$

We have,

$\Rightarrow 1 + (n-1)2 = 49$

$\Rightarrow (n-1)2 = 49 - 1$

$\Rightarrow (n-1) = \frac{48}{2}$

$\Rightarrow n - 1 = 24$

$\Rightarrow n = 24 + 1 = 25$

$$\therefore S_{25} = \frac{25}{2}[a + a_{25}] = \frac{25}{2}[1+49]$$

$$= \frac{25}{2} \times 50 = 25 \times 25 = 625$$

15. A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: ₹200 for the first day, ₹250 for the second day, ₹300 for the third day, etc., the penalty for each succeeding day being ₹50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days.

SOLUTION:

Penalty for the first day = ₹200

Penalty for the second day = ₹250

Penalty for the third day = ₹300

The list is ₹200, ₹250, ₹300,…

It is an AP in which a = 200, d = 50 and $n = 30$.

Penalty for 30 days $= \frac{30}{2}(2a + 29d)$

= 15 (2 × 200 + 29 × 50) = 15(400 + 1450) = ₹27,750

Hence, the penalty for 30 days is ₹27,750.

16. A sum of ₹700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is ₹20 less than its preceding prize, find the value of each of the prizes.

SOLUTION:

Let 1 st prize be of ₹ a

2nd prize be ₹ $(a - 20)$ and

3rd prize be ₹ $(a - 20 - 20)$ = ₹$(a - 40)$

Then seven prizes are

₹ a, ₹ $(a - 20)$, ₹ $(a - 40)$, ……..₹$(a - 120)$

and $S_7 = 700$ $a_1 = a$, d = ₹$(a - 20 - a)$ = –₹20)

We have,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_7 = \frac{7}{2}[2a + (7-1)d] \Rightarrow 700 = \frac{7}{2}[2a + 6d]$$

$$700 = \frac{7}{2}[2a + 6(-20)] \Rightarrow 700 \times \frac{2}{7} = 2a - 120$$

$$200 = 2a - 120 \Rightarrow 320 = 2a \Rightarrow a = \frac{320}{2} = 160$$

So, the seven prizes are ₹160, ₹140, ₹120, ₹100, ₹80, ₹60 and ₹40.

17. In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of class I will plant 1 tree, a section of class II will plant 2 trees and so on till class XII. There are three sections of each class. How many trees will be planted by the students?

SOLUTION:

Trees planted by 3 sections of class I = 3 × 1 = 3

Trees planted by 3 sections of class II = 3 × 2 = 6

Trees planted by 3 sections of class III = 3 × 3 = 9

Trees planted by 3 sections of class XII

= 3 × 12 = 36

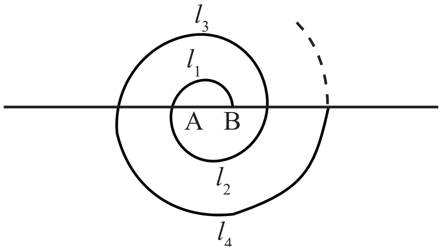
Thus, we have the list of trees planted 3, 6, 9,…36.

It is an AP in which $a = 3$, $d = 3$, $n = 12$ and $l = 36$.

∴ Total number of trees planted

$$= \frac{12}{2}(3 + 36) = 6 \times 39 = 234$$

18. A spiral is made up of successive semicircles, with centres alternately at A and B, starting with centre at A of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm, ……… as shown in figure. What is the total length of such a spiral made up of thirteen consecutive semicircles? (Take $\pi = \left(\frac{22}{7}\right)$)



SOLUTION:

We know,

Perimeter of a semi-circle = πr

Therefore,

$$P_1 = \pi(0.5) = \frac{\pi}{2} cm$$

$$P_2 = \pi(1) = \pi cm$$

$$P_3 = \pi(1.5) = \frac{3\pi}{2} cm$$

Where, P_1 P_2, P_3 are the lengths of the semi-circles.

Hence we got a series here, as,

$$\frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi, \ldots$$

$$\mathrm{P}_1 = \frac{\pi}{2} cm$$

$$P_2 = \pi\ cm$$

Common difference, $d = P_2 - P_1 = \pi - \frac{\pi}{2} = \frac{\pi}{2}$

First term $= \mathrm{P}_1 = a = \frac{\pi}{2} cm$

By the sum of n term formula, we know,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Therefore, Sum of the length of 13 consecutive circles is;

$$S_{13} = \frac{13}{2}\left[2\left(\frac{\pi}{2}\right) + (13-1)\frac{\pi}{2}\right]$$

$$= \frac{13}{2}[\pi + 6\pi]$$

$$= \frac{13}{2}(7\pi)$$

$$= \frac{13}{2} \times 7 \times \frac{22}{7}$$

= 143 cm

19. 200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on. In how many rows are the 200 logs placed and how many logs are in the top row?

SOLUTION:

Number of logs rowwise forms an AP as:

20, 19, 18, 17,…

Here, $a = 20$ and $d = -1$

Let the number of rows be n.

Then,

$S_n = 200$

$$\Rightarrow \frac{n}{2}[2\times 20+(n-1)\times(-1)] = 200$$

$$\Rightarrow \frac{n}{2}[41-n] = 200$$

$\Rightarrow 41n - n^2 = 400$

$\Rightarrow n^2 - 41n + 400 = 0$

$$\therefore n = \frac{41 \pm \sqrt{(-41)^2 - 4\times 1\times 400}}{2}$$

$$\therefore n = \frac{41 \pm \sqrt{1681-1600}}{2} = \frac{41\pm 9}{2}$$

$\Rightarrow n = 16$ or 25

Now

$a_n = a + (n - 1)\, d$

$a_{16} = 20 + (16 - 1)\,(-1)$

$a_{16} = 20 - 15$

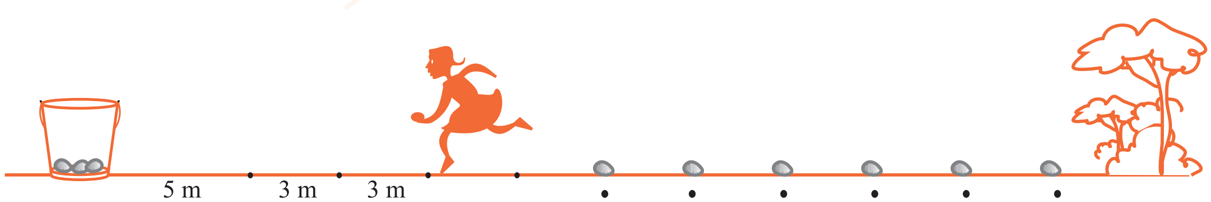
$a_{16} = 5$

and, $a_{25} = 20 + (25 - 1)\,(-1) = 20 - 24 = -4$

Rejecting the value because number of logs cannot be negative.

Therefore, 200 logs can be placed in 16 rows and the number of logs in the 16th row is 5.

20. In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato and other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line.



A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?

[Hint: to pick up the first potato and the second potato, the total distance (in metres) run by a competitor is $2 \times 5 + 2 \times (5 + 3)$]

SOLUTION:

The distances of potatoes from the bucket are 5, 8, 11, 14…, which is in the form of AP.

Given, the distance run by the competitor for collecting these potatoes are two times of the distance at which the potatoes have been kept.

Therefore, distances to be run w.r.t distances of potatoes, could be written as;

10, 16, 22, 28, 34,………

Hence, the first term, $a = 10$ and $d = 16 - 10 = 6$

By the formula of sum of terms, we know,

$$S_{10} = \frac{10}{2}[2(10) + (10-1)(6)]$$

= 5 [20 + 54]

= 5 (74)

= 370

Therefore, the competitor will run a total distance of 370 m.