CHAPTER
6
Triangles

CLASS 10TH

NCERT EXERCISE AND SOLUTIONS - MATHEMATICS

EXERCISE 6.1

1. **Fill in the blanks using correct word given in the brackets:-**

(i) **All circles are ______. (congruent, similar)**

Answer: Similar

(ii) **All squares are ______. (similar, congruent)**

Answer: Similar

(iii) **All ______triangles are similar. (isosceles, equilateral)**

Answer: Equilateral

(iv) **Two polygons of the same number of sides are similar, if**

(a) **their corresponding angles are _and**

(b) **their corresponding sides are ___. (equal, proportional)**

Answer: **(a)** Equal

(b) Proportional

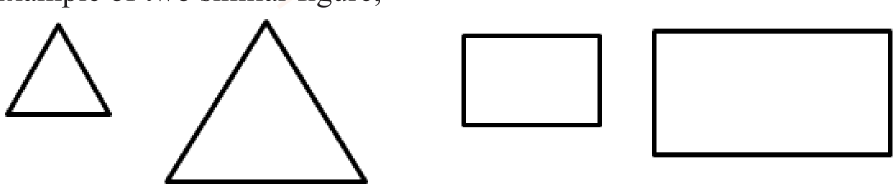
2. **Give two different examples of pair of**

(i) **Similar figures**

(ii) **Non-similar figures**

SOLUTION:

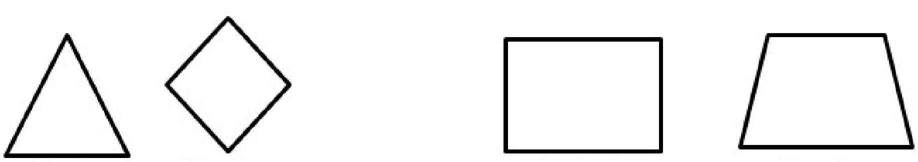
(i) Example of two similar figure;



Two Equilaterial Triangle

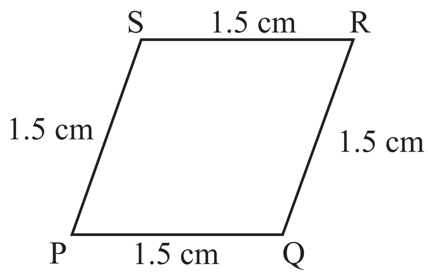
Two Rectangle

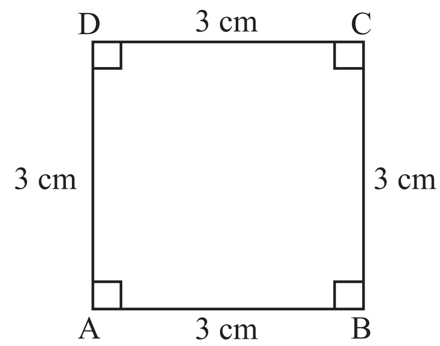
(ii) Example of two Non-similar figure;



Triangle Rhombus Rectangle Trapezium

3. State whether the following quadrilaterals are similar or not:



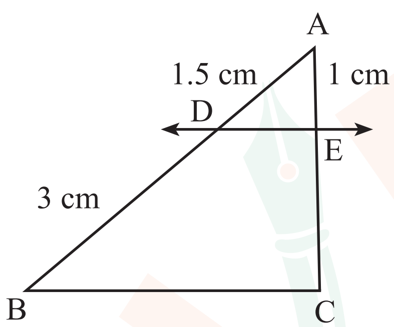


SOLUTION:

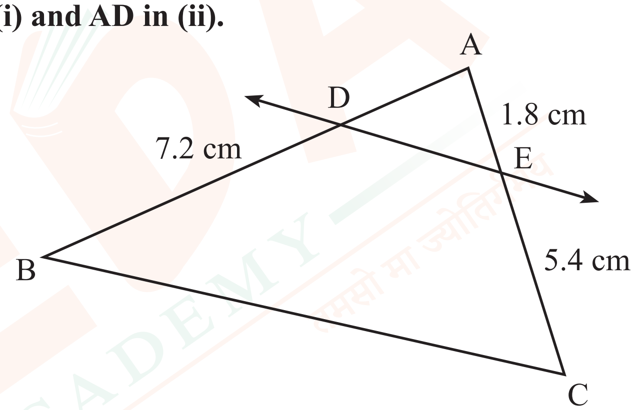
From the given two figures, we can see their corresponding angles are different or unequal. Therefore, they are not similar.

EXERCISE 6.2

1. In figure (i) and (ii), DE || BC. Find EC in (i) and AD in (ii).



(i)



(ii)

SOLUTION:

(i) Given, in Δ ABC, DE||BC

$\therefore$ AD/DB $= \frac{AE}{EC}$ [Using Basic proportionality theorem]

$\Rightarrow \frac{1.5}{3} = \frac{1}{EC}$

$\Rightarrow EC = \frac{3}{1.5}$

$EC = 3 \times \frac{10}{15} = 2$ cm

Hence, EC = 2 cm.

(ii) Given, in Δ ABC, DE||BC

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$ [Using Basic proportionality theorem]

Hence, AD = 2.4 cm.

2. E and F are points on the sides PQ and PR respectively of a ΔPQR. For each of the following cases, state whether EF || QR.

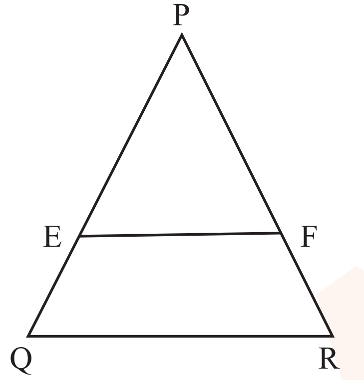
(i) **PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm and FR = 2.4 cm**

(ii) **PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm**

(iii) **PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.36 cm**

SOLUTION:

Given, in ΔPQR, E and F are two points on side PQ and PR, respectively. See the figure below;



(i) Given, PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm and FR = 2.4 cm

Therefore, by using Basic proportionality theorem, we get,

$\frac{PE}{EQ} = \frac{3.9}{3} = \frac{39}{30} = \frac{13}{10} = 1.3$

And $\frac{PF}{FR} = \frac{3.6}{2.4} = \frac{36}{24} = \frac{3}{2} = 1.5$

So, we get, $\frac{PE}{EQ} \neq \frac{PF}{FR}$

Hence, EF is not parallel to QR.

(ii) Given, PE = 4 cm, QE = 4.5 cm, PF = 8cm and RF = 9cm

Therefore, by using Basic proportionality theorem, we get,

$\frac{PE}{QE} = \frac{4}{4.5} = \frac{40}{45} = \frac{8}{9}$

And, $\frac{PF}{RF} = \frac{8}{9}$ So, we get here,

$\frac{PE}{QE} = \frac{PF}{RF}$

Hence, EF is parallel to QR.

(iii) Given, PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.36 cm

From the figure,

EQ = PQ – PE = 1.28 – 0.18 = 1.10 cm And, FR = PR – PF = 2.56 – 0.36 = 2.20 cm

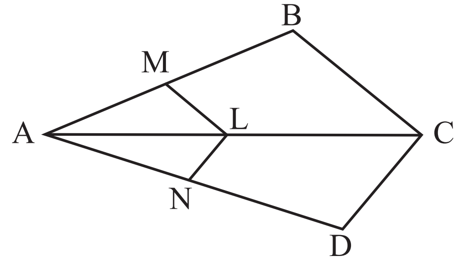
So, $\frac{PE}{EQ} = \frac{0.18}{1.10} = \frac{18}{110} = \frac{9}{55}$

And, $\frac{PF}{FR} = \frac{0.36}{2.20} = \frac{36}{220} = \frac{9}{55}$

So, we get here, $\frac{PE}{FQ} = \frac{PF}{FR}$

Hence, EF is parallel to QR.

3. In the figure, if LM || CB and LN || CD, prove that AM/AB = AN/AD



SOLUTION:

In the given figure, we can see, LM || CB,

By using basic proportionality theorem, we get,

$\frac{AM}{AB} = \frac{AL}{AC}$........(i)

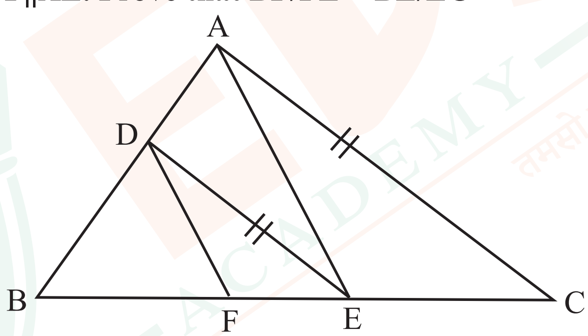
Similarly, given, LN || CD and using basic proportionality theorem,

$\therefore \frac{AN}{AD} = \frac{AL}{AC}$........(ii)

From equation (i) and (ii), we get, $\frac{AM}{AB} = \frac{AN}{AD}$

Hence, proved.

4. In the figure, DE||AC and DF||AE. Prove that BF/FE = BE/EC



SOLUTION:

In ΔABC, given as, DE || AC

Thus, by using Basic Proportionality Theorem, we get,

$\therefore \frac{BD}{DA} = \frac{BE}{EC}$...........(i)

In ΔBAE, given as, DF || AE

Thus, by using Basic Proportionality Theorem, we get,

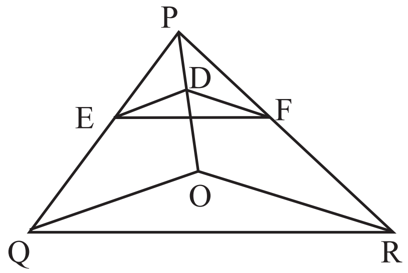
$\therefore \frac{BD}{DA} = \frac{BF}{FE}$...........(ii)

From equation (i) and (ii), we get

$\frac{BF}{FE} = \frac{BE}{EC}$

Hence, proved.

5. In the figure, DE||OQ and DF||OR, show that EF||QR.



SOLUTION:

Given,

In ΔPQO, DE || OQ

So by using Basic Proportionality Theorem,

$\frac{PD}{DO} = \frac{PE}{EQ}$..............................(i)

Again given, in ΔPOR, DF || OR,

So by using Basic Proportionality Theorem,

$\frac{PD}{DO} = \frac{PF}{FR}$............................(ii)

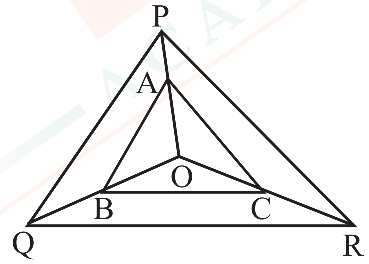
From equation (i) and (ii), we get,

$\frac{PE}{EQ} = \frac{PF}{FR}$

Therefore, by converse of Basic Proportionality Theorem,

In ΔPQR, EF || QR.

6. **In the figure, A, B and C are points on OP, OQ and OR respectively such that AB || PQ and AC || PR. Show that BC || QR.**



SOLUTION:

Given here,

In ΔOPQ, AB || PQ

By using Basic Proportionality Theorem,

$\frac{OA}{AP} = \frac{OB}{BQ}$..........(i)

Also given,

In ΔOPR, AC || PR

So, By using Basic Proportionality Theorem

$\therefore \frac{OA}{AP} = \frac{OC}{CR}$........(ii)

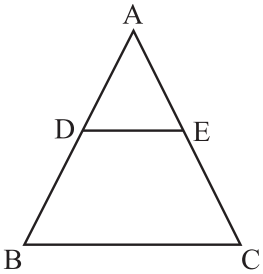
From equation (i) and (ii), we get,

$\frac{OB}{BQ} = \frac{OC}{CR}$

Therefore, by converse of Basic Proportionality Theorem,

In ΔOQR, BC || QR.

7. Using Basic proportionality theorem, prove that a line drawn through the mid-points of one side of a triangle is parallel to another side bisects the third side. (Recall that you have proved it in class IX)



SOLUTION:

Given, in ΔABC, D is the midpoint of AB such that AD = DB.

A line parallel to BC intersects AC at E as shown in above figure such that DE||BC. We have to prove that E is the mid point of AC.

Since, D is the mid-point of AB.

$\therefore AD = DB$

$\Rightarrow \frac{AD}{DB} = 1$..........(i)

In ΔABC, DE || BC,

By using Basic Proportionality Theorem,

Therefore, $\frac{AD}{DB} = \frac{AE}{EC}$

From equation (i), we can write,

$\Rightarrow 1 = \frac{AE}{EC}$

$\therefore AE = EC$

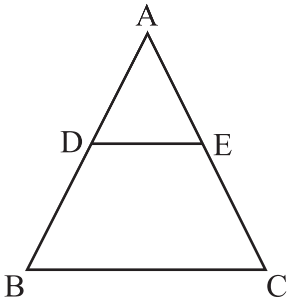
Hence, proved, E is the midpoint of AC.

8. Using Converse of basic proportionality theorem, prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side. (Recall that you have done it in Class IX).

SOLUTION:

Given, in ΔABC, D and E are the mid points of AB and AC, respectively, such that,

AD = BD and AE = EC.



We have to prove that: DE || BC.

Since, D is the midpoint of AB

$\therefore$ AD = DB

$\Rightarrow \frac{AD}{BD} = 1$........(i)

Also given, E is the mid-point of AC.

$\therefore$ AE = EC

$\Rightarrow \frac{AE}{EC} = 1$

From equation (i) and (ii), we get,

$\frac{AD}{BD} = \frac{AE}{EC}$

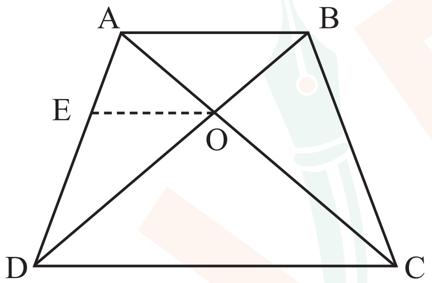
By converse of Basic Proportionality Theorem, DE || BC

Hence, proved.

9. **ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$.**

SOLUTION:

Given, ABCD is a trapezium where AB || DC and diagonals AC and BD intersect each other at O.



We have to prove, $\frac{AO}{BO} = \frac{CO}{DO}$

From the point O, draw a line EO touching AD at E, in such a way that, EO || DC || AB

In ΔADC, we have OE || DC

Therefore, by using Basic Proportionality Theorem $\frac{AE}{ED} = \frac{AO}{CO}$.......(i)

Now, In ΔABD, OE || AB

Therefore, by using Basic Proportionality Theorem $\frac{DE}{EA} = \frac{DO}{BO}$..........(ii)

From equation (i) and (ii), we get, $\frac{AO}{CO} = \frac{BO}{DO}$

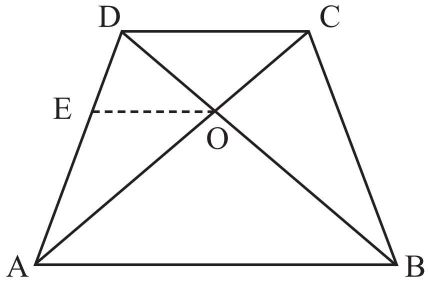
$\Rightarrow \frac{AO}{BO} = \frac{CO}{DO}$

Hence, proved.

10. **The diagonals of a quadrilateral ABCD intersect each other at the point O such that AO/BO = CO/DO. Show that ABCD is a trapezium.**

SOLUTION:

Given, Quadrilateral ABCD where AC and BD intersect each other at O such that, $\frac{AO}{BO} = \frac{CO}{DO}$.



We have to prove here, ABCD is a trapezium

From the point O, draw a line EO touching AD at E, in such a way that, EO || DC || AB

In ΔDAB, EO || AB

Therefore, by using Basic Proportionality Theorem

$\frac{DE}{EA} = \frac{DO}{OB}$........(i)

Also, given, $\frac{AO}{BO} = \frac{CO}{DO}$

$\Rightarrow \frac{AO}{CO} = \frac{BO}{DO}$

$\Rightarrow \frac{CO}{AO} = \frac{DO}{BO}$

$\Rightarrow \frac{DO}{OB} = \frac{CO}{AO}$.........(ii)

From equation (i) and (ii), we get $\frac{DE}{EA} = \frac{CO}{AO}$

Therefore, by using converse of Basic Proportionality Theorem, EO || DC also EO || AB

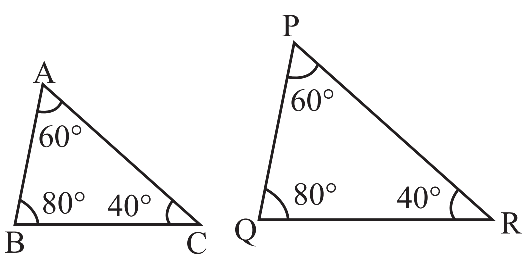
⇒ AB || DC.

Hence, quadrilateral ABCD is a trapezium with AB || CD.

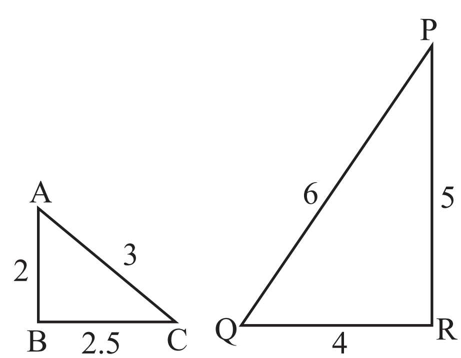
EXERCISE 6.3

1. **State which pairs of triangles in the figure are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form:**

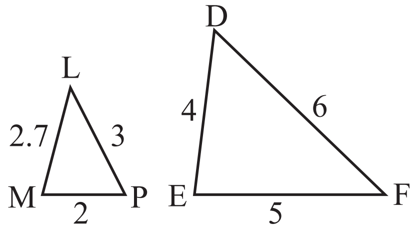
(i)



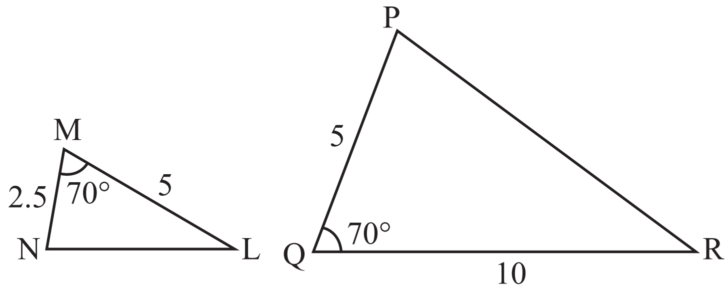
(ii)



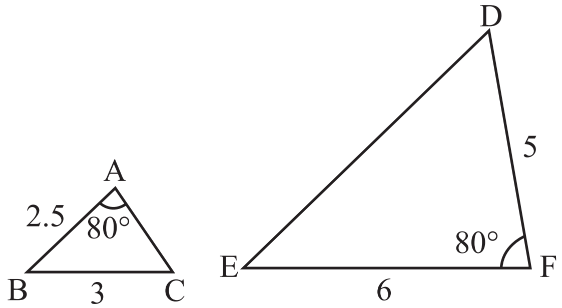
(iii)



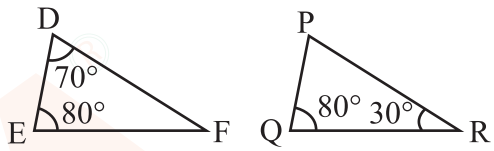
(iv)



(v)



(vi)



SOLUTION:

(i) Given, in ΔABC and ΔPQR,

$\angle A = \angle P = 60°$

$\angle B = \angle Q = 80°$

$\angle C = \angle R = 40°$

Therefore, by AAA similarity criterion,

$\therefore \Delta ABC \sim \Delta PQR$

(ii) Given, in ΔABC and ΔPQR,

$\frac{AB}{QR} = \frac{2}{4} = \frac{1}{2},$

$\frac{BC}{RP} = \frac{2.5}{5} = \frac{1}{2},$

$\frac{CA}{PQ} = \frac{3}{6} = \frac{1}{2}$

By SSS similarity criterion,

$\Delta ABC \sim \Delta QRP$

(iii) Given, in ΔLMP and ΔDEF,

LM = 2.7, MP = 2, LP = 3, EF = 5, DE = 4, DF = 6

$\frac{MP}{DE} = \frac{2}{4} = \frac{1}{2}$

$\frac{PL}{DF} = \frac{3}{6} = \frac{1}{2}$

$\frac{LM}{EF} = \frac{2.7}{5} = \frac{27}{50}$

Here, $\frac{MP}{DE} = \frac{PL}{DF} \neq \frac{LM}{EF}$

Therefore, ΔLMP and ΔDEF are not similar.

(iv) In ΔMNL and ΔQPR, it is given,

$\frac{MN}{QP} = \frac{ML}{QR} = \frac{1}{2}$

$\angle M = \angle Q = 70°$

Therefore, by SAS similarity criterion

∴ ΔMNL ~ ΔQPR

(v) In ΔABC and ΔDEF, given that,

AB = 2.5, BC = 3, ∠A = 80°, EF = 6, DF = 5, ∠F = 80°

Here, $\frac{AB}{DF} = \frac{2.5}{5} = \frac{1}{2}$

And, $\frac{BC}{EF} = \frac{3}{6} = \frac{1}{2}$

$\Rightarrow \angle B \neq \angle F$

Hence, ΔABC and ΔDEF are not similar.

(vi) In ΔDEF, by sum of angles of triangles, we know that,

$\angle D + \angle E + \angle F = 180°$

$\Rightarrow 70° + 80° + \angle F = 180°$

$\Rightarrow \angle F = 180° - 70° - 80°$

$\Rightarrow \angle F = 30°$

Similarly, In ΔPQR,

$\angle P + \angle Q + \angle R = 180$ (Sum of angles of Δ)

$\Rightarrow \angle P + 80° + 30° = 180°$

$\Rightarrow \angle P = 180° - 80° - 30°$

$\Rightarrow \angle P = 70°$

Now, comparing both the triangles, ΔDEF and ΔPQR, we have

$\angle D = \angle P = 70°$

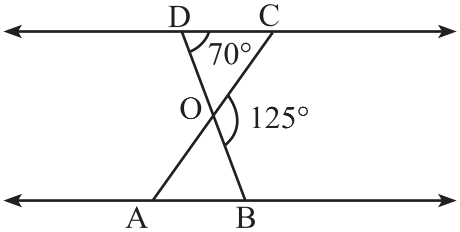
$\angle F = \angle Q = 80°$

$\angle F = \angle R = 30°$

Therefore, by AAA similarity criterion,

Hence, ΔDEF ~ ΔPQR

2. In figure, ΔODC ~ ΔOBA, ∠ BOC = 125° and ∠ CDO = 70°. Find ∠ DOC, ∠ DCO and ∠ OAB.



SOLUTION:

As we can see from the figure, DOB is a straight line. Therefore, ∠DOC + ∠ COB = 180°

$\Rightarrow \angle DOC = 180° - 125°$ (Given, $\angle BOC = 125°$)

$= 55°$

In ΔDOC, sum of the measures of the angles of a triangle is 180º

Therefore, $\angle DCO + \angle CDO + \angle DOC = 180°$

$\Rightarrow \angle DCO + 70° + 55° = 180°$(Given, $\angle CDO = 70°$)

$\Rightarrow \angle DCO = 55°$

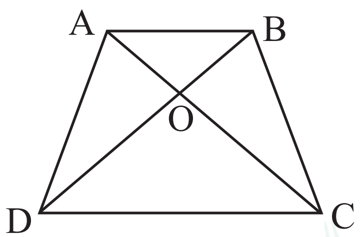
It is given that, $\Delta ODC \sim \Delta OBA$,

Hence, corresponding angles are equal in similar triangles

$\angle OAB = \angle OCD$

$\Rightarrow \angle OAB = 55°$

3. Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at the point O. Using a similarity criterion for two triangles, show that $\frac{OA}{OC} = \frac{OB}{OD}$



In ΔDOC and ΔBOA,

AB || CD, thus alternate interior angles will be equal,

$\therefore \angle CDO = \angle ABO$ Similarly,

$\angle DCO = \angle BAO$

Also, for the two triangles ΔDOC and ΔBOA, vertically opposite angles will be equal;

$\therefore \angle DOC = \angle BOA$

Hence, by AAA similarity criterion,

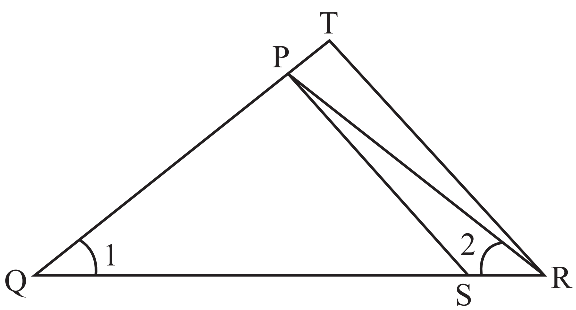
$\Delta DOC \sim \Delta BOA$

Thus, the corresponding sides are proportional. $\frac{DO}{BO} = \frac{OC}{OA}$

$\Rightarrow \frac{OA}{OC} = \frac{OB}{OD}$

Hence, proved.

4. In the fig. $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$. Show that $\Delta PQS \sim \Delta TQR$.



SOLUTION:

In ΔPQR,

$\angle PQR = \angle PRQ$

$\therefore PQ = PR$..........(i)

Given,

$\frac{QR}{QS} = \frac{QT}{PR}$ Using equation (i), we get $\frac{QR}{QS} = \frac{QT}{QP}$........(ii)

In ΔPQS and ΔTQR, by equation (ii),

$\frac{QR}{QS} = \frac{QT}{QP}$

$\angle Q = \angle Q$

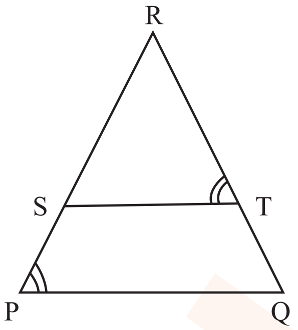
$\therefore \Delta PQS \sim \Delta TQR$ [By SAS similarity criterion]

5. S and T are point on sides PR and QR of ΔPQR such that $\angle P = \angle RTS$. Show that $\Delta RPQ \sim \Delta RTS$.

SOLUTION:

Given, S and T are point on sides PR and QR of ΔPQR

And $\angle P = \angle RTS$.



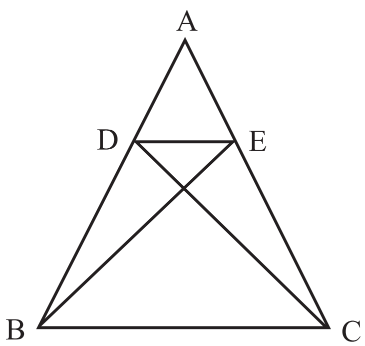
In ΔRPQ and ΔRTS,

$\angle RTS = \angle QPS$ (Given)

$\angle R = \angle R$ (Common angle)

$\therefore \Delta RPQ \sim \Delta RTS$ (AA similarity criterion)

6. In the figure, if $\Delta ABE \cong \Delta ACD$, show that $\Delta ADE \sim \Delta ABC$.



SOLUTION:

Given, $\Delta ABE \cong \Delta ACD$.

$\therefore$ AB = AC [By CPCT]................(i)

And, AD = AE [By CPCT]................(ii)

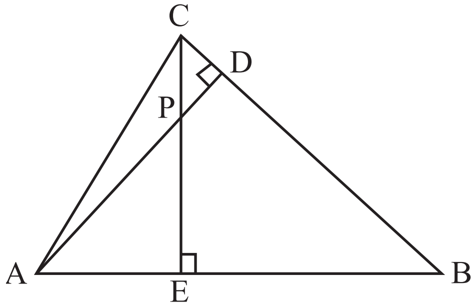
In ΔADE and ΔABC, dividing eq.(ii) by eq.(i),

$\frac{AD}{AB} = \frac{AE}{AC}$

$\angle A = \angle A$ [Common angle]

$\therefore \Delta ADE \sim \Delta ABC$ [SAS similarity criterion]

7. In the figure, altitudes AD and CE of ΔABC intersect each other at the point P. Show that:



(i) $\Delta AEP \sim \Delta CDP$

(ii) $\Delta ABD \sim \Delta CBE$

(iii) $\Delta AEP \sim \Delta ADB$

(iv) $\Delta PDC \sim \Delta BEC$

SOLUTION:

Given, altitudes AD and CE of ΔABC intersect each other at the point P.

(i) In ΔAEP and ΔCDP,

$\angle AEP = \angle CDP$ (90° each)

$\angle APE = \angle CPD$ (Vertically opposite angles)

Hence, by AA similarity criterion,

$\Delta AEP \sim \Delta CDP$

(ii) In ΔABD and ΔCBE,

$\angle ADB = \angle CEB$ (90° each)

$\angle ABD = \angle CBE$ (Common Angles)

Hence, by AA similarity criterion,

$\Delta ABD \sim \Delta CBE$

(iii) In ΔAEP and ΔADB,

$\angle AEP = \angle ADB$ (90° each)

$\angle PAE = \angle DAB$ (Common Angles)

Hence, by AA similarity criterion,

$\Delta AEP \sim \Delta ADB$

(iv) In ΔPDC and ΔBEC,

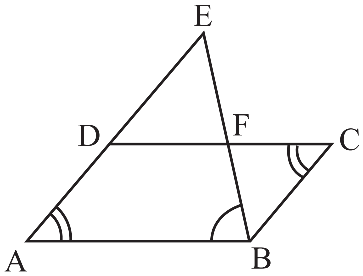
$\angle PDC = \angle BEC$ (90° each)

∠PCD = ∠BCE (Common angles)

Hence, by AA similarity criterion,

ΔPDC ~ ΔBEC

8. E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that ΔABE ~ ΔCFB.



SOLUTION:

Given, E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Consider the figure below,

In ΔABE and ΔCFB,

∠A = ∠C (Opposite angles of a parallelogram)

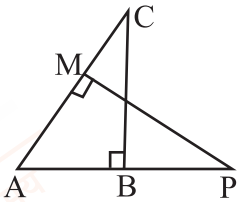
∠AEB = ∠CBF (Alternate interior angles as AE || BC)

∴ ΔABE ~ ΔCFB (AA similarity criterion)

9. In the figure, ABC and AMP are two right triangles, right angled at B and M, respectively, prove that:

(i) ΔABC ~ ΔAMP

(ii) CA/PA = BC/MP



SOLUTION:

Given, ABC and AMP are two right triangles, right angled at B and M, respectively.

(i) In ΔABC and ΔAMP, we have,

∠CAB = ∠MAP (common angles)

∠ABC = ∠AMP = 90° (each 90°)

∴ ΔABC ~ ΔAMP (AA similarity criterion)

(ii) As, ΔABC ~ ΔAMP (AA similarity criterion)

If two triangles are similar then the corresponding sides are always equal,

Hence, CA/PA = BC/MP

10. CD and GH are respectively the bisectors of ∠ACB and ∠EGF such that D and H lie on sides AB and FE of ΔABC and ΔEFG respectively. If ΔABC ~ ΔFEG, Show that:

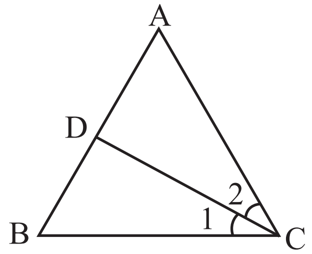
(i) $\frac{CD}{GH} = \frac{AC}{FG}$

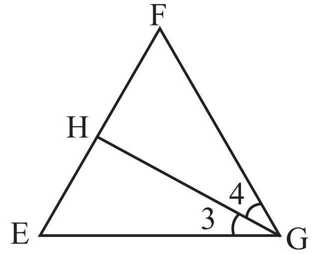
(ii) **ΔDCB ~ ΔHGE**

(iii) **ΔDCA ~ ΔHGF**

SOLUTION:

Given, CD and GH are respectively the bisectors of ∠ACB and ∠EGF such that D and H lie on sides AB and FE of ΔABC and ΔEFG, respectively.





(i) From the given condition,

$\Delta ABC \sim \Delta FEG$.

$\therefore \angle A = \angle F$, $\angle B = \angle E$, and $\angle ACB = \angle FGE$ Since, $\angle ACB = \angle FGE$

$\therefore \angle ACD = \angle FGH$ (Angle bisector) And, $\angle DCB = \angle HGE$ (Angle bisector)

In ΔACD and ΔFGH,

$\angle A = \angle F$

$\angle ACD = \angle FGH$

$\therefore \Delta ACD \sim \Delta FGH$ (AA similarity criterion)

$\Rightarrow \frac{CD}{GH} = \frac{AC}{FG}$

(ii) In ΔDCB and ΔHGE,

$\angle DCB = \angle HGE$ (Already proved)

$\angle B = \angle E$ (Already proved)

$\therefore \Delta DCB \sim \Delta HGE$ (AA similarity criterion)

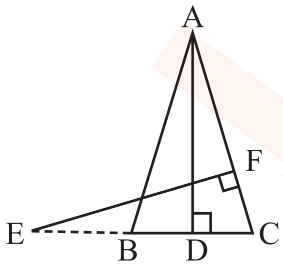
(iii) In ΔDCA and ΔHGF,

$\angle ACD = \angle FGH$ (Already proved)

$\angle A = \angle F$ (Already proved)

$\therefore \Delta DCA \sim \Delta HGF$ (AA similarity criterion)

11. In the following figure, E is a point on side CB produced of an isosceles triangle ABC with AB = AC. If AD $\perp$BC and EF $\perp$ AC, prove that $\Delta ABD \sim \Delta ECF$.



SOLUTION:

Given, ABC is an isosceles triangle.

$\therefore$ AB = AC

$\Rightarrow \angle ABD = \angle ECF$

In ΔABD and ΔECF,

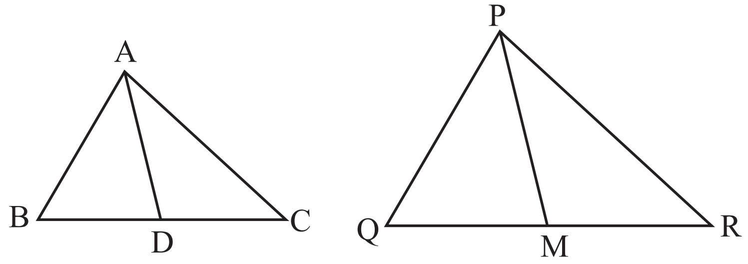
$\angle ADB = \angle EFC$ (Each 90°)

$\angle BAD = \angle CEF$ (Already proved)

$\therefore \Delta ABD \sim \Delta ECF$ (using AA similarity criterion)

12. Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of ΔPQR.

Show that $\Delta ABC \sim \Delta PQR$.



Given, ΔABC and ΔPQR, AB, AC and median AD of ΔABC are proportional to sides PQ, PR and median PM of ΔPQR

i.e. $\frac{AB}{PQ}=\frac{BC}{QR}=\frac{AD}{PM}$

We have to prove: ΔABC ~ ΔPQR

As we know here,

$\frac{AB}{PQ}=\frac{BC}{QR}=\frac{AD}{PM}$

$\frac{AB}{PQ}=\frac{\frac{1}{2}BC}{\frac{1}{2}QR}=\frac{AD}{PM}$

$\Rightarrow \frac{AB}{PQ}=\frac{BC}{QR}=\frac{AD}{PM}$ (D is the midpoint of BC. M is the midpoint of QR)

⇒ ΔABD ~ ΔPQM [SSS similarity criterion]

∴ ∠ABD = ∠PQM [Corresponding angles of two similar triangles are equal]

⇒ ∠ABC = ∠PQR

In ΔABC and ΔPQR

$\frac{AB}{PQ}=\frac{BC}{QR}$........(i)

∠ABC = ∠PQR....... (ii)

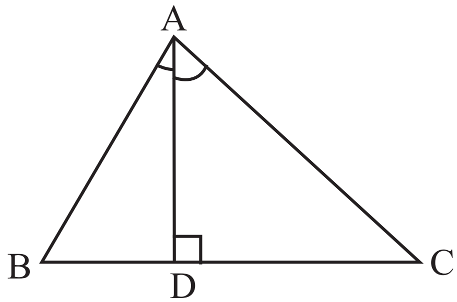
From equation (i) and (ii), we get,

ΔABC ~ ΔPQR [SAS similarity criterion]

13. D is a point on the side BC of a triangle ABC such that ∠ADC = ∠BAC. Show that CA^2 = CB.CD

SOLUTION:

Given, D is a point on the side BC of a triangle ABC such that ∠ADC = ∠BAC.



In ΔADC and ΔBAC,

∠ADC = ∠BAC (Already given)

∠ACD = ∠BCA (Common angles)

∴ ΔADC ~ ΔBAC (AA similarity criterion)

We know that corresponding sides of similar triangles are in proportion.

$\therefore \frac{CA}{CB} = \frac{CD}{CA}$

$\Rightarrow CA^2 = CB.CD.$

Hence, proved.

14. Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that ΔABC ~ ΔPQR.

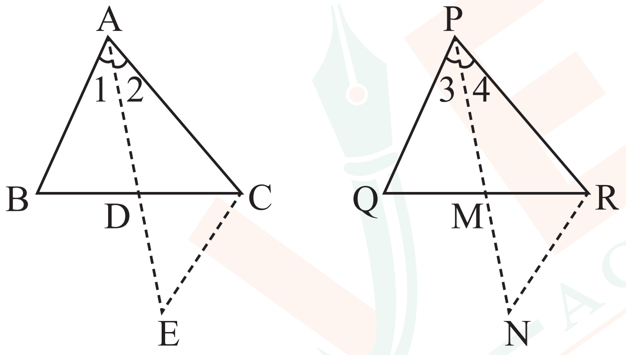
SOLUTION:

Given: Two triangles ΔABC and ΔPQR in which AD and PM are medians such that;

$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$

We have to prove, ΔABC ~ ΔPQR

Let us construct first: Produce AD to E so that AD = DE. Join CE, Similarly produce PM to N such that PM = MN, also Join RN.



In ΔABD and ΔCDE, we have

AD = DE [By Construction.]

BD = DC [Since, AP is the median]

and, ∠ADB = ∠CDE [Vertically opposite angles]

∴ ΔABD ≅ ΔCDE [SAS criterion of congruence]

⇒ AB = CE [By CPCT]...............(i)

Also, in ΔPQM and ΔMNR,

PM = MN [By Construction.]

QM = MR [Since, PM is the median]

and, ∠PMQ = ∠NMR [Vertically opposite angles]

∴ ΔPQM ≅ ΔMNR [SAS criterion of congruence]

⇒ PQ = RN [CPCT].................(ii)

Now, AB/PQ = AC/PR = AD/PM

From equation (i) and (ii),

$\Rightarrow \frac{CE}{RN} = \frac{AC}{PR} = \frac{AD}{PM}$

$\Rightarrow \frac{CE}{RN} = \frac{AC}{PR} = \frac{2AD}{2PM}$

$\Rightarrow \frac{CE}{RN} = \frac{AC}{PR} = \frac{AE}{PN}$ [Since 2AD = AE and 2PM = PN]

$\Rightarrow \Delta ACE \sim \Delta PRN$ [SSS similarity criterion] Therefore, $\angle 2 = \angle 4$

Similarly, $\angle 1 = \angle 3$

$\therefore \angle 1 + \angle 2 = \angle 3 + \angle 4$

$\Rightarrow \angle A = \angle P$..................(iii)

Now, in ΔABC and ΔPQR, we have $\frac{AB}{PQ} = \frac{AC}{PR}$ (Already given) From equation (iii),

$\angle A = \angle P$

$\therefore \Delta ABC \sim \Delta PQR$ [SAS similarity criterion]

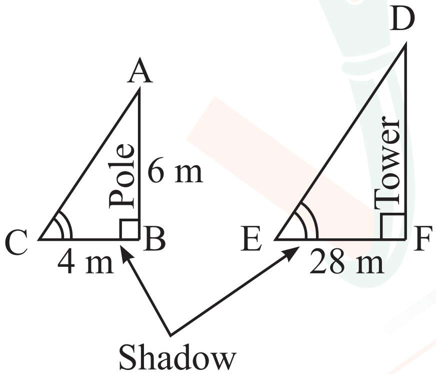
15. **A vertical pole of a length 6 m casts a shadow 4m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.**

SOLUTION:

Given, Length of the vertical pole = 6 m, Shadow of the pole = 4 m

Let Height of tower = h m

Length of shadow of the tower = 28 m



In ΔABC and ΔDEF,

$\angle C = \angle E$ (angular elevation of sum)

$\angle B = \angle F = 90°$

$\therefore \Delta ABC \sim \Delta DEF$ (AA similarity criterion)

$\therefore \frac{AB}{DF} = \frac{BC}{EF}$ (If two triangles are similar corresponding sides are proportional)

$\therefore \frac{6}{h} = \frac{4}{28}$

$\Rightarrow h = \frac{6 \times 28}{4}$

$\Rightarrow h = 6 \times 7$

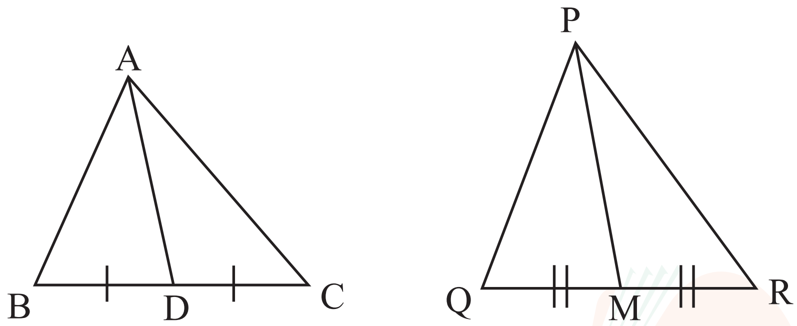
$\Rightarrow h = 42$ m

Hence, the height of the tower is 42 m.

16. If AD and PM are medians of triangles ABC and PQR, respectively where ΔABC ~ ΔPQR prove that $\frac{AB}{PQ} = \frac{AD}{PM}$.

SOLUTION:

Given, ΔABC ~ ΔPQR



We know that the corresponding sides of similar triangles are in proportion.

$\therefore \frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$.............(i)

Also, ∠A = ∠P, ∠B = ∠Q, ∠C = ∠R...........(ii)

Since AD and PM are medians, they will divide their opposite sides.

$\therefore BD = \frac{BC}{2}$ and $QM = \frac{QR}{2}$...........(iii)

From equations (i) and (iii), we get

$\frac{AB}{PQ} = \frac{BD}{QM}$..........(iv)

In ΔABD and ΔPQM,

From equation (ii), we have

∠B = ∠Q

From equation (iv), we have,

$\frac{AB}{PQ} = \frac{BD}{QM}$

∴ ΔABD ~ ΔPQM (SAS similarity criterion)

$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$