CHAPTER 8

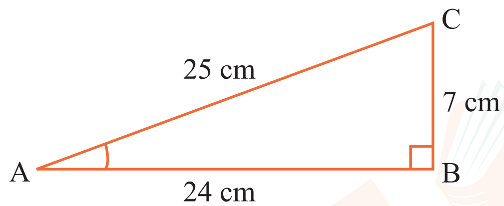
Introduction to Trigonometry

CLASS 10TH

NCERT EXERCISE AND SOLUTIONS - MATHEMATICS

EXERCISE 8.1

1. In ΔABC, right angled at B, AB = 24 cm, BC = 7 cm. Determine:



(i) sin A, cos A **(ii) sin C, cos C**

SOLUTION:

In Δ ABC, by Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2$

$= (24\text{ cm})^2 + (7\text{cm})^2$

$= (576 + 49)\text{cm}^2$

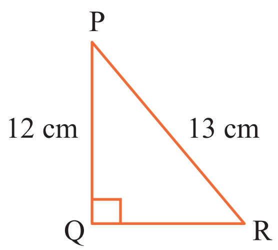
$= 625\text{ cm}^2$

$\Rightarrow AC = \sqrt{625} = 25\text{ cm}$

(i) $\sin A = \frac{BC}{AC} = \frac{7}{25}$ and $\cos A = \frac{AB}{AC} = \frac{24}{25}$

(ii) $\sin C = \frac{AB}{AC} = \frac{24}{25}$ and $\cos C = \frac{BC}{AC} = \frac{7}{25}$

2. In figure, find tan P – cot R.



SOLUTION:

In ΔPQR, by Pythagoras theorem, we have

$QR^2 = PR^2 - PQ^2$

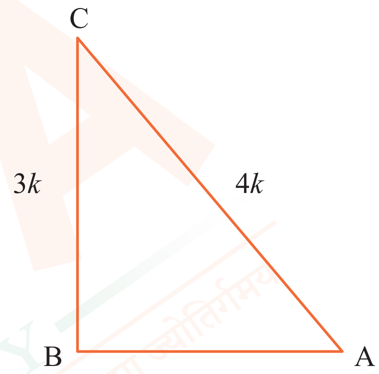
$= (13)^2 - (12)^2$

$= 169 - 144 = 25 \Rightarrow QR = \sqrt{25} = 5\,\text{cm}$

Hence, $\tan P - \cot R = \dfrac{QR}{PQ} - \dfrac{QR}{PQ} = \dfrac{5}{12} - \dfrac{5}{12} = 0$

3. If $\sin A = \dfrac{3}{4}$, calculate cos A and tan A.

SOLUTION:



Given that: $\sin A = \dfrac{3}{4}$

Let $\sin A = \dfrac{3k}{4k}$, where k is a real number.

In ΔABC, by Pythagoras theorem, we have

$AB^2 = AC^2 - BC^2$

$= (4k)^2 - (3k)^2$

$= 16k^2 - 9k^2$

$= 7k^2$

$\Rightarrow AB = \sqrt{7k^2} = \sqrt{7}k$

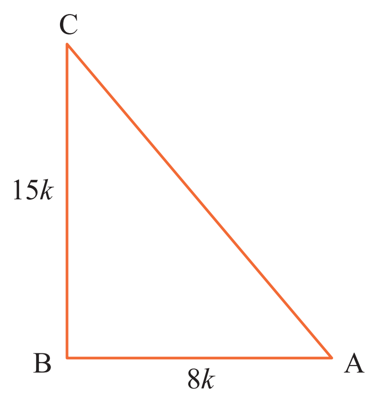
Hence,

$\cos A = \dfrac{AB}{AC} = \dfrac{\sqrt{7}k}{4k} = \dfrac{\sqrt{7}}{4}$

and

$\tan\ A = \dfrac{BC}{AB} = \dfrac{3k}{\sqrt{7}k} = \dfrac{3}{\sqrt{7}}$

4. Given 15 cot A = 8, find sin A and sec A.



SOKUTION:

Given that: 15 cot A = 8

$\Rightarrow \cot A = \frac{8}{15}$

Let $\cot A = \frac{8k}{15k'}$, where k is a real number.

In ΔABC, by Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2$

$= (8k)^2 + (15k)^2$

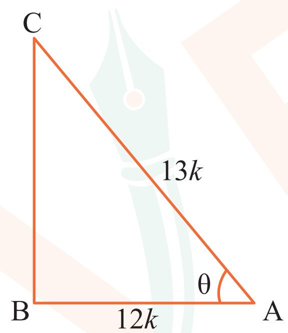
$= 64k^2 + 225k^2$

$= 289k^2$

$\Rightarrow AC = \sqrt{289k^2} = 17k$

Hence, $\sin A = \frac{BC}{AC} = \frac{15k}{17k} = \frac{15}{17}$ and $\sec A = \frac{AC}{AB} = \frac{17k}{8k} = \frac{17}{8}$

5. Given $\sec\theta = \frac{13}{12}$, calculate all other trigonometric ratios.



SOLUTION:

Given that $\sec\theta = \frac{13}{12}$

Let $\sec\theta = \frac{13k}{12k}$, where k is a real number,

In ΔABC, by Pythagoras theorem, we have

$BC^2 = AC^2 - AB^2$

$= (13k)^2 - (12k)^2$

$= 169k^2 - 144k^2$

$= 25k^2$

$\Rightarrow BC = \sqrt{25k^2} = 5k$

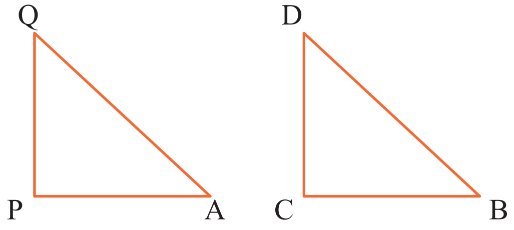
Hence, $\sin\theta = \frac{BC}{AC} = \frac{5k}{13k} = \frac{5}{13}$

$$\cos\theta = \frac{AB}{AC} = \frac{12k}{13k} = \frac{12}{13}$$

$$\tan\theta = \frac{BC}{AB} = \frac{5k}{12k} = \frac{5}{12}$$

$$\text{cosec}\,\theta = \frac{AC}{BC} = \frac{13k}{5k} = \frac{13}{5} \text{ and } \cot\theta = \frac{AB}{BC} = \frac{12k}{5k} = \frac{12}{5}$$

6. If ∠A and ∠B are acute angles such that cos A = cos B, then show that ∠A = ∠B.



SOLUTION:

Given that: $\cos A = \cos B$

$\cos A = \cos B$

$$\Rightarrow \frac{AP}{AQ} = \frac{BC}{BD}$$

$$\Rightarrow \frac{AP}{BC} = \frac{AQ}{BD}$$

Let $\frac{AP}{BC} = \frac{AQ}{BD} = k$

Therefore, $AP = k(BC)$ and $AQ = k(BD)$

Now, in $\Delta\, APQ$ and $\Delta\, BCD$

$$\frac{PQ}{CD} = \frac{\sqrt{AQ^2 - AP^2}}{\sqrt{BD^2 - BC^2}} = \frac{\sqrt{(k \cdot BD)^2 - (k \cdot BC)^2}}{\sqrt{BD^2 - BC^2}} = \frac{k\sqrt{BD^2 - BC^2}}{\sqrt{BD^2 - BC^2}} = k$$

From the equation (i) and (ii), we get

$$\frac{AP}{BC} = \frac{AQ}{BD} = \frac{PQ}{CD}$$

So, $\triangle APQ \sim \triangle BCD$

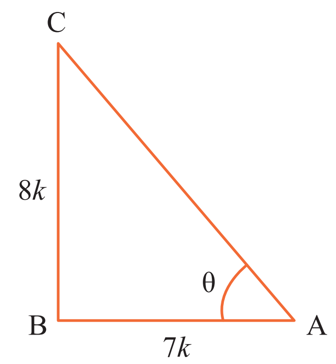
[SSS similarity criteria]

Hence, $\angle A = \angle B$

7. If $\cot\theta = \frac{7}{8}$, evaluate:

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$ (ii) $\cot^2\theta$

SOLUTION:



Given that: $\cot\theta = \frac{7}{8}$

Let $\cot\theta = \frac{7k}{8k}$, where k is a real number.

In ΔABC, by Pythagoras theorem, we have

$$AC^2 = BC^2 + AB^2 = (8k)^2 + (7k)^2 = 64k^2 + 49k^2 = 113k^2$$

$$\Rightarrow AC = \sqrt{113k^2} = \sqrt{113}k$$

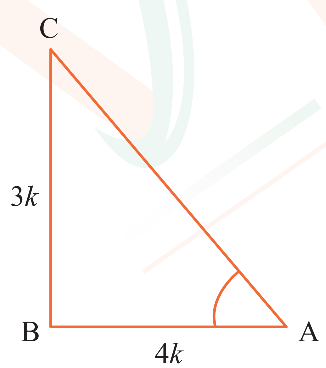
(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

$$= \frac{\left(1+\frac{8}{\sqrt{113}}\right)\left(1-\frac{8}{\sqrt{113}}\right)}{\left(1+\frac{7}{\sqrt{113}}\right)\left(1-\frac{7}{\sqrt{113}}\right)} = \frac{1-\left(\frac{8}{\sqrt{113}}\right)^2}{1-\left(\frac{7}{\sqrt{113}}\right)^2} = \frac{1-\frac{64}{113}}{1-\frac{49}{113}} = \frac{\frac{113-64}{113}}{\frac{113-49}{113}} = \frac{49}{64}$$

(ii) $\cot^2\theta$

$$= (\cot\theta)^2 = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$$

8. If 3 cot A = 4, check whether $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not.



SOLUTION:

Given that: $3\cot A = 4$

$$\Rightarrow \cot A = \frac{4}{3}$$

Let $\cot A = \frac{4k}{3k}$, where k is a real number.

In ΔABC, by Pythagoras theorem, we have

$$AC^2 = BC^2 + AB^2 = (3k)^2 + (4k)^2 = 9k^2 + 16k^2 = 25k^2$$

$\Rightarrow AC = \sqrt{25k^2} = 5k$, therefore,

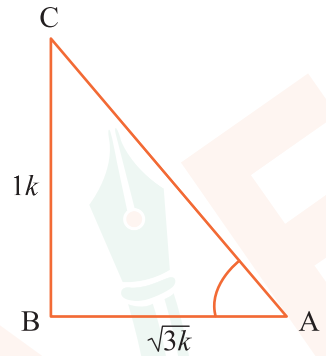
$$\frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2} = \frac{1-\frac{9}{16}}{1+\frac{9}{16}} = \frac{\frac{16-9}{16}}{\frac{16+9}{16}} = \frac{7}{25}$$

and

$$\cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \frac{16-9}{25} = \frac{7}{25}$$

Hence, $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$

9. In triangle ABC , right angled at B , if tanA = $\frac{1}{\sqrt{3}}$, find the value of:



(i) sinA cosC + cosA sinC **(ii) cosA cosC – sinA sinC**

SOLUTION:

Given that: $\tan A = \frac{1}{\sqrt{3}}$

Let $\tan A = \frac{1k}{\sqrt{3}k}$, where k is a real number.

In ΔABC, by Pythagoras theorem, we have

$$AC^2 = BC^2 + AB^2$$

$$= (1k)^2 + (\sqrt{3}k)^2$$

$$= k^2 + 3k^2$$

$$= 4k^2$$

$$\Rightarrow AC = \sqrt{4k^2} = 2k$$

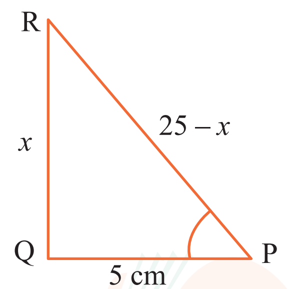
(i) sinA cosC + cosA sin C

$$= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} = \frac{1}{4} + \frac{3}{4} = \frac{1+3}{4} = \frac{4}{4} = 1$$

(ii) cosA cosC – sinA sinC

$$= \frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$$

10. In ΔPQR, right-angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of sinP, cosP and tanP.



SOLUTION:

Given that: In ΔPQR, angle Q is right angled.

Let QR = x, therefore, PR $= 25 - x$

In ΔPQR, by Pythagoras theorem, we have

$$PR^2 = PQ^2 + QR^2$$

$$\Rightarrow (25 - x)^2 = (5)^2 + (x)^2$$

$$\Rightarrow 625 + x^2 - 50x = 25 + x^2$$

$$\Rightarrow 625 - 50x = 25$$

$$\Rightarrow 50x = 600 \Rightarrow x = 12$$

$$\Rightarrow QR = 12$$

Therefore, PR $= 25 - 12 = 13$

Now, $\sin P = \frac{QR}{PR} = \frac{12}{13}$,

$\cos P = \frac{PQ}{PR} = \frac{5}{13}$

and $\tan P = \frac{QR}{PQ} = \frac{12}{5}$

11. State whether the following are true or false. Justify your answer.

(i) The value of tanA is always less than 1.

(ii) secA $= \frac{12}{5}$ for some value of angle A.

(iii) cosA is the abbreviation used for the cosecant of angle A .

(iv) cotA is the product of cot and A .

(v) $\sin\theta = \frac{4}{3}$ for some angle θ.

SOLUTION:

(i) False,

Because, $\tan 60° = \sqrt{3} > 1$

(ii) True,

Because, $\sec A = \frac{\text{Hypotenuse}}{\text{Adjecent side of angle } A}$ and we know that hypotenuse is always greater than adjacent side.

(iii) False,

Because, cos$A$ is used for cosine of angle A .

(iv) False,

Because, cot$A$ is used for cotangent of angle A.

(v) False,

Because, $\sin\theta = \frac{\text{Opposite side of angle } A}{\text{Hypotenuse}}$, we know that hypotenuse is always greater than opposite side.

EXERCISE 8.2

1. **Evaluate the following:**

(i) sin 60° cos 30°+sin 30° cos 60°

(ii) $2\tan^2 45° + \cos^2 30° - \sin^2 60°$

(iii) $\frac{\cos 45^o}{\sec 30^o + \text{cosec} 30^o}$

(iv) $\frac{\sin 30^o + \tan 45^o - \text{cosec} 60^o}{\sec 30^o + \cos 60^o + \cot 45^o}$

(v) $\frac{5\cos^2 60^o + 4\sec^2 30^o - \tan^2 45^o}{\sin^2 30^o + \cos^2 30^o}$

SOLUTION:

(i) sin 60° cos 30°+sin 30° cos 60°

Putting the value of each trigonometric ratios, we get

$$\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)+\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)=\frac{3}{4}+\frac{1}{4}=\frac{4}{4}=1$$

(ii) $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

Putting the value of each trigonometric ratios, we get

$$2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 = 2 + \frac{3}{4} - \frac{3}{4} = 2$$

(iii) $\dfrac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$

Putting the value of each trigonometric ratios, we get

$$\frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}} + 2} = \frac{\frac{1}{\sqrt{2}}}{\frac{2+2\sqrt{3}}{\sqrt{3}}} = \frac{\sqrt{3}}{\sqrt{2}\left(2+2\sqrt{3}\right)} = \frac{\sqrt{3}}{2\sqrt{2}+2\sqrt{6}}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}+2\sqrt{6}} \times \frac{2\sqrt{2}-2\sqrt{6}}{2\sqrt{2}-2\sqrt{6}} = \frac{2\sqrt{6}-2\sqrt{18}}{(2\sqrt{2})^2-(2\sqrt{6})^2} = \frac{2\sqrt{6}-6\sqrt{2}}{8-24} = \frac{-2\left(3\sqrt{2}-\sqrt{6}\right)}{-16}$$

$$= \frac{3\sqrt{2}-\sqrt{6}}{8}$$

(iv) $\dfrac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$

Putting the value of each trigonometric ratios, we get

$$\frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1} = \frac{\frac{\sqrt{3}+2\sqrt{3}-4}{2\sqrt{3}}}{\frac{4+\sqrt{3}+2\sqrt{3}}{2\sqrt{3}}} = \frac{3\sqrt{3}-4}{3\sqrt{3}+4}$$

$$= \frac{3\sqrt{3}-4}{3\sqrt{3}+4} \times \frac{3\sqrt{3}-4}{3\sqrt{3}-4} = \frac{27-12\sqrt{3}-12\sqrt{3}+16}{(3\sqrt{3})^2-4^2} = \frac{43-24\sqrt{3}}{27-16} = \frac{43-24\sqrt{3}}{11}$$

(v) $\dfrac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$

Putting the value of each trigonometric ratios, we get

$$\frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \frac{\frac{5}{4} + \frac{16}{3} - 1}{\frac{1}{4} + \frac{3}{4}} = \frac{\frac{15+64-12}{12}}{\frac{4}{4}} = \frac{67}{12}$$

2. Choose the correct option and justify your choice:

(i) $\frac{2\tan 30^\circ}{1+\tan^2 30^\circ} =$

(A) sin60°

(B) cos60°

(C) tan60°

(D) sin30°

(ii) $\frac{1-\tan^2 45^\circ}{1+\tan^2 45^\circ} =$

(A) tan90°

(B) 1

(C) sin45°

(D) 0

(iii) $\sin 2A = 2\sin A$ **is true when A =**

(A) 0°

(B) 30°

(C) 45°

(D) 60°

(iv) $\frac{2\tan 30^\circ}{1-\tan^2 30^\circ} =$

(A) cos60°

(B) sin60°

(C) tan60°

(D) sin30°

SOLUTION:

(i) $\frac{2\tan 30^\circ}{1+\tan^2 30^\circ}$

Putting the value of each trigonometric ratios, we get

$$\frac{2\left(\frac{1}{\sqrt{3}}\right)}{1+\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}} = \frac{6}{4\sqrt{3}} = \frac{\sqrt{3}}{2}$$

We know that $\sin 60^\circ = \frac{\sqrt{3}}{2}$, hence the option (A) is correct.

(ii) $\dfrac{1-\tan^2 45^\circ}{1+\tan^2 45^\circ}$

Putting the value of each trigonometric ratios, we get

$$\frac{1-(1)^2}{1+(1)^2}=\frac{1-1}{1+1}=\frac{0}{2}=0$$

Hence, the option (D) is correct.

(iii) $\sin 2A = 2\sin A$

We know that $\sin 0 = 0$, hence, the option (A) is correct.

(iv) $\dfrac{2\tan 30^\circ}{1-\tan^2 30^\circ}$

Putting the value of each trigonometric ratios, we get

$$\frac{2\left(\frac{1}{\sqrt{3}}\right)}{1-\left(\frac{1}{\sqrt{3}}\right)^2}=\frac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}}=\frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}}=\frac{3}{\sqrt{3}}=\sqrt{3}$$

We know that $\tan 60^\circ = \sqrt{3}$, hence, the option (c) is correct.

3. If $\tan(A+B)=\sqrt{3}$ and $\tan(A-B)=\frac{1}{\sqrt{3}}, 0^\circ < A+B \geq 90^\circ, A > B$, find A and B.

SOLUTION:

Given that: $\tan(A+B)=\sqrt{3}$

$\Rightarrow \tan(A+B)=\tan 60^\circ \quad \left[\because \tan 60^\circ = \sqrt{3}\right]$

$\Rightarrow A+B=60^\circ \quad \ldots$ (i)

Given that: $\tan(A-B)=\frac{1}{\sqrt{3}}$

$\Rightarrow \tan(A-B)=\tan 30^\circ \quad \left[\because \tan 30^\circ = \frac{1}{\sqrt{3}}\right]$

$\Rightarrow A-B=30^\circ \quad \ldots$ (ii)

Solving the equations (i) and (ii), we get

$2A=90^\circ \Rightarrow A=45^\circ$

From equation (1), we get

$45^\circ + B = 60^\circ \Rightarrow B=15^\circ$

Hence, $A=45^\circ$ and $B=15^\circ$

4. **State whether the following are true or false. Justify your answer.**

(i) sin $(A+B)$ = sin$A$ + sin$B$

(ii) The value of sinθ increases as θ increases.

(iii) The value of cosθ increases as θ increases.

(iv) sinθ = cosθ for all values of θ.

(v) cot$A$ is not defined for A= 0°.

SOLUTION:

(i) False,

Let $A=30^\circ$ and $B=60^\circ$

Therefore, $LHS=\sin\left(A+B\right)=\sin\left(30^\circ+60^\circ\right)=\sin 90^\circ=1$ and

$$\text{RHS}=\sin A+\sin B=\sin 30^\circ+\sin 60^\circ=\frac{1}{2}+\frac{\sqrt{3}}{2}=\frac{1+\sqrt{3}}{2}\neq 1$$

Hence, $\sin\left(A+B\right)\neq\sin A+\sin B$

(ii) True,

As we know that $\sin 0^\circ=0, \sin 30^\circ=\frac{1}{2}, \sin 45^\circ=\frac{1}{\sqrt{2}}, \sin 60^\circ=\frac{\sqrt{3}}{2}$ and $\sin 90^\circ=1$. Hence, for the increasing values of $\theta, \sin\theta$ is also increasing.

(iii) False,

As we know that $\cos 0^\circ=1, \cos 30^\circ=\frac{\sqrt{3}}{2}, \cos 45^\circ=\frac{1}{\sqrt{2}}, \cos 60^\circ=\frac{1}{2}$ and $\cos 90^\circ=0$ Hence, for the increasing values of $\theta, \cos\theta$ is decreasing.

(iv) False, $\because \cos 30^\circ=\frac{\sqrt{3}}{2}$, but $\sin 30^\circ=\frac{1}{2}$.

(v) True, $\because \tan 0^\circ=0$, we know that $\cot 0^\circ=\frac{1}{\tan 0^\circ}=\frac{1}{0}$, which is not defined.

EXERCISE 8.3

1. Express the trigonometric ratios sinA, secA and tanA in terms of cotA.

SOLUTION:

(i) sinA

$$=\sqrt{\sin^2 A}=\sqrt{\frac{1}{cosec^2 A}} \quad \left[\because \sin A=\frac{1}{cosecA}\right]$$

$$=\sqrt{\frac{1}{1+\cot^2 A}} \quad \left[\because cosec^2 A=1+\cot^2 A\right]$$

$$=\frac{1}{\sqrt{1+\cot^2 A}}$$

(ii) secA

$$=\sqrt{\sec^2 A}$$

$$=\sqrt{1+\tan^2 A} \quad \left[\because \sec^2 A=1+\tan^2 A\right]$$

$$=\sqrt{1+\frac{1}{\cot^2 A}} \quad \left[\because \tan A=\frac{1}{\cot A}\right]$$

$$=\sqrt{\frac{\cot^2 A+1}{\cot^2 A}}$$

$$=\frac{\sqrt{\cot^2 A+1}}{\cot A}$$

(iii) $\tan A=\frac{1}{\cot A}$

$$\left[\because \tan A=\frac{1}{\cot A}\right]$$

2. Write all the other trigonometric ratios of ∠A in terms of secA.

SOLUTION:

(i) sinA

$$=\sqrt{\sin^2 A}$$

$$=\sqrt{1-\cos^2 A}$$

$$\left[\because \sin^2 A=1-\cos^2 A\right]$$

$$\left[\because \cos A = \frac{1}{\sec A}\right]$$

$$= \sqrt{1 - \frac{1}{\sec^2 A}}$$

$$= \sqrt{\frac{\sec^2 A - 1}{\sec^2 A}} = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

(ii) cosA

$$\cos A = \frac{1}{\sec A}$$

$$\left[\because \cos A = \frac{1}{\sec A}\right]$$

(iii) $\tan A = \sqrt{\tan^2 A}$

$$= \sqrt{\sec^2 A - 1}$$

(iv) $\operatorname{cosec} A = \sqrt{cosec^2 A}$

$$= \sqrt{1 + \cot^2 A} \qquad [\because \operatorname{cosec}^2 A = 1 + \cot^2 A]$$

$$= \sqrt{1 + \frac{1}{\tan^2 A}} \qquad [\because \cot A = \frac{1}{\tan A}]$$

$$= \sqrt{1 + \frac{1}{\sec^2 A - 1}} \qquad [\because \sec^2 A = 1 + \tan^2 A]$$

$$= \sqrt{\frac{\sec^2 A - 1 + 1}{\sec^2 A - 1}} = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$$

(v) $\cot A = \sqrt{\cot^2 A}$

$$= \sqrt{\frac{1}{\tan^2 A}} \qquad \left[\because \cot A = \frac{1}{\tan A}\right]$$

$$= \sqrt{\frac{1}{\sec^2 A - 1}} \qquad \left[\because \sec^2 A = 1 + \tan^2 A\right]$$

$$= \frac{1}{\sqrt{\sec^2 A - 1}}$$

3. Choose the correct option. Justify your choice.

(i) $9\sec^2A - 9\tan^2A =$

(A) 1

(B) 9

(C) 8

(D) 0

(ii) $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \text{cosec}\theta) =$

(A) 0

(B) 1

(C) 2

(D) –1

(iii) $(\sec A + \tan A)(1 - \sin A) =$

(A) secA

(B) sinA

(C) cosecA

(D) cosA

(iv) $\dfrac{1+\tan^2A}{1+\cot^2A} =$

(A) $\sec^2A$

(B) –1

(C) $\cot^2A$

(D) $\tan^2A$

SOLUTION:

(i) $9\sec^2A - 9\tan^2A$

$9\sec^2A - 9(\sec^2A - 1)$ [$\because 1 + \tan^2A = \sec^2A$]

$= 9\sec^2A - 9\sec^2A + 9$

$= 9$

Hence, the option (B) is correct.

(ii) $(1+\tan\theta+\sec\theta)(1+\cot\theta-\text{cosec}\theta)$

$$=1+\cot\theta-cosec\theta+\tan\theta+\tan\theta\cot\theta-\tan\theta cosec\theta+\sec\theta+\sec\theta\cot\theta-\sec\theta cosec\theta$$

$$=1+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}+\frac{\sin\theta}{\cos\theta}+1-\frac{\sin\theta}{\cos\theta}\times\frac{1}{\sin\theta}+\frac{1}{\cos\theta}+\frac{1}{\cos\theta}\times\frac{\cos\theta}{\sin\theta}-\frac{1}{\cos\theta}\times\frac{1}{\sin\theta}$$

$$=1+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}+\frac{\sin\theta}{\cos\theta}+1-\frac{1}{\cos\theta}+\frac{1}{\cos\theta}+\frac{1}{\sin\theta}-\frac{1}{\sin\theta\cos\theta}$$

$$= 1 + \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta} + 1 - \frac{1}{\sin\theta\cos\theta}$$

$$= 2 + \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta} - \frac{1}{\sin\theta\cos\theta}$$

$$= 2 + \frac{\cos^2\theta + \sin^2\theta - 1}{\sin\theta\cos\theta} \qquad [\because \cos^2\theta + \sin^2\theta = 1]$$

$$= 2 + \frac{1-1}{\sin\theta\cos\theta}$$

$$= 2 + 0 = 2$$

Hence, the option (C) is correct.

(iii) (sec A+tan A)(1 – sin A)

$$= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)(1 - \sin A)$$

$$= \left(\frac{1 + \sin A}{\cos A}\right)(1 - \sin A)$$

$$= \frac{1 - \sin^2 A}{\cos A}$$

$$= \frac{\cos^2 A}{\cos A} = \cos A$$

Hence, the option (D) is correct.

(iv) $\frac{1 + \tan^2 A}{1 + \cot^2 A}$

$$= \frac{\sec^2 A}{cosec^2 A}\left[\because cosec^2 A = 1 + \cot^2 A, \sec^2 A = 1 + \tan^2 A\right]$$

$$= \frac{\left(\frac{1}{\cos^2 A}\right)}{\left(\frac{1}{\sin^2 A}\right)} = \frac{1}{\cos^2 A} \times \frac{\sin^2 A}{1} = \tan^2 A$$

Hence, the option (D) is correct.

4. Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

(i) $(cosec\theta - \cot\theta)^2 = \frac{1 - \cos\theta}{1 + \cos\theta}$

(ii) $\frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = 2\sec A$

(iii) $\frac{\tan\theta}{1 - \cot\theta} + \frac{\cot\theta}{1 - \tan\theta} = 1 + \sec\theta cosec\theta$

[Hint: Write the expression in terms of sinθ and cosθ]

(iv) $\frac{1+\sec A}{\sec A}=\frac{\sin^2 A}{1-\cos A}$

[Hint: Simplify LHS and RHS separately]

(v) $\frac{\cos A-\sin A+1}{\cos A+\sin A-1}=cosecA+\cot A$, using the identity $cosec^2 A=1+\cot^2 A$

(vi) $\sqrt{\frac{1+\sin A}{1-\sin A}}=\sec A+\tan A$

(vii) $\frac{\sin\theta-2\sin^3\theta}{2\cos^3\theta-\cos\theta}=\tan\theta$

(viii) $(\sin A+cosecA)^2+(\cos A+\sec A)^2=7+\tan^2 A+\cot^2 A$

(ix) $(cosecA-\sin A)(\sec A-\cos A)=\frac{1}{\tan A+\cot A}$

[Hint: Simplify LHS and RHS separately]

(x) $\frac{1+\tan^2 A}{1+\cot^2 A}=\left(\frac{1-\tan A}{1-\cot A}\right)^2=\tan^2 A$

SOLUTION:

(i) $(coscec\theta-\cot\theta)^2=\frac{1-\cos\theta}{1+\cos\theta}$

$\text{LHS}=(coscec\theta-\cot\theta)^2$

$=\left(\frac{1}{\sin\theta}-\frac{\cos\theta}{\sin\theta}\right)^2=\left(\frac{1-\cos\theta}{\sin\theta}\right)^2$

$=\frac{(1-\cos\theta)^2}{\sin^2\theta}$

$=\frac{(1-\cos\theta)^2}{1-\cos^2\theta}\left[\because \sin^2\theta=1-\cos^2\theta\right]$

$=\frac{(1-\cos\theta)(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$

$=\frac{1-\cos\theta}{1+\cos\theta}=RHS$

(ii) $\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}=2\sec A$

$LHS=\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}$

$=\frac{\cos^2 A+(1+\sin A)^2}{(1+\sin A)\cos A}$

$$= \frac{\cos^2 A + 1 + \sin^2 A + 2\sin A}{(1+\sin A)\cos A}$$

$$= \frac{1+1+2\sin A}{(1+\sin A)\cos A}$$

$$= \frac{2+2\sin A}{(1+\sin A)\cos A}$$

$$= \frac{2(1+\sin A)}{(1+\sin A)\cos A} = \frac{2}{\cos A} = 2\sec A = RHS$$

(iii) $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta cosec\theta$

$$\text{LHS} = \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$$

$$= \left[\frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}}\right] + \left[\frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}\right]$$

$$= \left[\frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}}\right] + \left[\frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}\right]$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} + \frac{\cos^2\theta}{\sin\theta(\cos\theta-\sin\theta)}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} - \frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)} [\because (\cos\theta-\sin\theta) = -(\sin\theta-\cos\theta)]$$

$$= \frac{\sin^3\theta-\cos^3\theta}{\cos\theta\sin\theta(\sin\theta-\cos\theta)} \left[\because a^3-b^3 = (a-b)\left(a^2+b^2+ab\right)\right]$$

$$= \frac{(\sin\theta-\cos\theta)\left(\sin^2\theta+\cos^2\theta+\cos\theta\sin\theta\right)}{\cos\theta\sin\theta(\sin\theta-\cos\theta)} \left[\because \sin^2\theta+\cos^2\theta = 1\right]$$

$$= \frac{(1+\cos\theta\sin\theta)}{\cos\theta\sin\theta}$$

$$= \frac{1}{\cos\theta\sin\theta} + \frac{\cos\theta\sin\theta}{\cos\theta\sin\theta} = \sec\theta cosec\theta + 1 = \text{ RHS}$$

(iv) $\frac{1+\sec A}{\sec A} = \frac{\sin^2 A}{1-\cos A}$

$$\text{LHS} = \frac{1+\sec A}{\sec A}$$

$$= \frac{1+\frac{1}{\cos A}}{\frac{1}{\cos A}}$$

$$= \frac{\frac{\cos A+1}{\cos A}}{\frac{1}{\cos A}}$$

$$= \frac{1+\cos A}{1}$$

$$= \frac{1+\cos A}{1} \times \frac{1-\cos A}{1-\cos A}$$

$$= \frac{1-\cos^2 A}{1-\cos A}$$

$$= \frac{\sin^2 A}{1-\cos A}$$

$$= RHS$$

(v) $\frac{\cos A-\sin A+1}{\cos A+\sin A-1} = cosecA+\cot A$

$$\text{LHS} = \frac{\cos A-\sin A+1}{\cos A+\sin A-1}$$

$$= \frac{\cot A-1+cosecA}{\cot A+1-cosecA} \quad \text{[Dividing Numerator and Denominator by} \sin A]$$

$$= \frac{\cot A+cosecA-(1)}{\cot A+1-cosecA}$$

$$= \frac{\cot A+cosecA-\left(cosec^2 A-\cot^2 A\right)}{\cot A+1-\cos ecA} \left[\because cosec^2 A-\cot^2 A=1\right]$$

$$= \frac{\cot A+cosecA-(cosecA+\cot A)(cosecA-\cot A)}{\cot A+1-cosecA}$$

$$= \frac{(\cot A+cosecA)(1-cosecA+\cot A)}{1-cosecA+\cot A}$$

$$= \cot A+cosecA = RHS$$

(vi) $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A+\tan A$

$$\text{LHS} = \sqrt{\frac{1+\sin A}{1-\sin A}}$$

$$= \sqrt{\frac{1+\sin A}{1-\sin A} \times \frac{1+\sin A}{1+\sin A}} = \sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}}$$

$$=\sqrt{\frac{(1+\sin A)^2}{\cos^2 A}}\left[\because 1-\sin^2 A=\cos^2 A\right]$$

$$=\frac{1+\sin A}{\cos A}$$

$$=\frac{1}{\cos A}+\frac{\sin A}{\cos A}=\sec A+\tan A=RHS$$

(vii) $\dfrac{\sin\theta-2\sin^3\theta}{2\cos^3\theta-\cos\theta}=\tan\theta$

$$\text{LHS}=\frac{\sin\theta-2\sin^3\theta}{2\cos^3\theta-\cos\theta}$$

$$=\frac{\sin\theta\left(1-2\sin^2\theta\right)}{\cos\theta\left(2\cos^2\theta-1\right)}$$

$$=\frac{\sin\theta\left(1-2\sin^2\theta\right)}{\cos\theta\left[2\left(1-\sin^2\theta\right)-1\right]}\left[\because \cos^2\theta=1-\sin^2\theta\right]$$

$$=\frac{\sin\theta\left(1-2\sin^2\theta\right)}{\cos\theta\left(2-2\sin^2\theta-1\right)}=\frac{\sin\theta\left(1-2\sin^2\theta\right)}{\cos\theta\left(1-2\sin^2\theta\right)}$$

$$=\frac{\sin\theta}{\cos\theta}=\tan\theta=\text{ RHS}$$

(viii) $(\sin A+cosecA)^2+(\cos A+\sec A)^2=7+\tan^2 A+\cot^2 A$

$$\text{LHS}=(\sin A+cosecA)^2+(\cos A+\sec A)^2$$

$$=\sin^2 A+cosec^2A+2\sin A\,cosecA+\cos^2 A+\sec^2 A+2\cos A\sec A$$

$$=\left(\sin^2 A+\cos^2 A\right)+cosec^2A+2+\sec^2 A+2$$

$$[\because \cos A\sec A=1, \sin A\,cosecA=1]$$

$$=1+\left(1+\cot^2 A\right)+2+\left(1+\tan^2 A\right)+2\left[\because cosec^2A=1+\cot^2 A, \sec^2 A=1+\tan^2 A\right]$$

$$=7+\tan^2 A+\cot^2 A=\text{ RHS}$$

(ix) $(cosecA-\sin A)(\sec A-\cos A)=\dfrac{1}{\tan A+\cot A}$

$$\text{LHS}=(cosecA-\sin A)(\sec A-\cos A)$$

$$=\left(\frac{1}{\sin A}-\sin A\right)\left(\frac{1}{\cos A}-\cos A\right)=\left(\frac{1-\sin^2 A}{\sin A}\right)\left(\frac{1-\cos^2 A}{\cos A}\right)=\left(\frac{\cos^2 A}{\sin A}\right)\left(\frac{\sin^2 A}{\cos A}\right)$$

$$=\sin A\cos A$$

$$=RHS=\frac{1}{\tan A+\cot A}$$

$$= \frac{1}{\left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right)} = \frac{1}{\left(\frac{\sin^2 A + \cos^2 A}{\cos A \sin A}\right)} = \frac{1}{\left(\frac{1}{\cos A \sin A}\right)}$$

$$= \cos A \sin A \ldots \text{ (ii)}$$

From the equation (i) and (ii), we get, LHS = RHS

(x) $\frac{1+\tan^2 A}{1+\cot^2 A} = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$

$$\text{LHS} = \frac{1+\tan^2 A}{1+\cot^2 A}$$

$$= \frac{\sec^2 A}{cosec^2 A}$$

$$= \frac{\left(\frac{1}{\cos^2 A}\right)}{\left(\frac{1}{\sin^2 A}\right)} = \frac{1}{\cos^2 A} \times \frac{\sin^2 A}{1} = \tan^2 A = \text{ RHS}$$

$$\text{Now, } \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \left(\frac{1-\frac{\sin A}{\cos A}}{1-\frac{\cos A}{\sin A}}\right)^2 = \left(\frac{\frac{\cos A - \sin A}{\cos A}}{\frac{\sin A - \cos A}{\sin A}}\right)^2 = \left(\frac{\cos A - \sin A}{\cos A} \times \frac{\sin A}{\sin A - \cos A}\right)^2$$

$$= \left(-\frac{\sin A - \cos A}{\cos A} \times \frac{\sin A}{\sin A - \cos A}\right)^2 = \left(-\frac{\sin A}{\cos A}\right)^2 = \tan^2 A = \text{ RHS}$$