CHAPTER 9

Some Applications of Trigonometry

CLASS 10TH

NCERT EXERCISE AND SOLUTIONS - MATHEMATICS

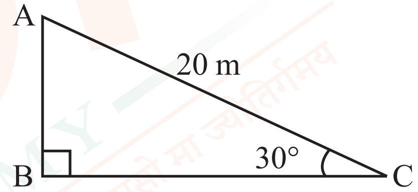
EXERCISE 9.1

1. **A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is 30°**

SOLUTION:

Given: length of rope = 20 m and angle of elevation from point on ground with top of pole = 30°.

To find: height of pole.



$$\frac{AB}{AC} = \sin 30^\circ$$

$$\frac{AB}{20} = \frac{1}{2}$$

AB = 10

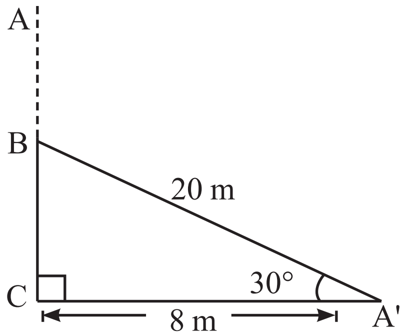
Thus, the height of the pole is 10 m.

2. **A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.**

SOLUTION:

Given = Distance between the foot of the tree to the point where the broken tree touches the ground is 8 m and angle of elevation made by the broken tree with the ground = 30°

To find: Height of the tree.



Let AC be the original tree and $A'B$ be the broken part which makes an angle of 30° with the ground.

From figure In $\Delta A'BC$

$$\frac{BC}{A'C} = \tan 30^\circ$$

$$\frac{BC}{8} = \frac{1}{\sqrt{3}}$$

$$BC = \frac{8}{\sqrt{3}}$$

$$\frac{A'C}{A'B} = \cos 30^\circ$$

$$\frac{8}{A'B} = \frac{\sqrt{3}}{2}$$

$$A'B = \frac{16}{\sqrt{3}}$$

Height of tree $= A'B + BC = \frac{16}{\sqrt{3}} + \frac{8}{\sqrt{3}} = \frac{24}{\sqrt{3}} = 8\sqrt{3}$ m

Hence, the height of the tree was $8\sqrt{3}$ m

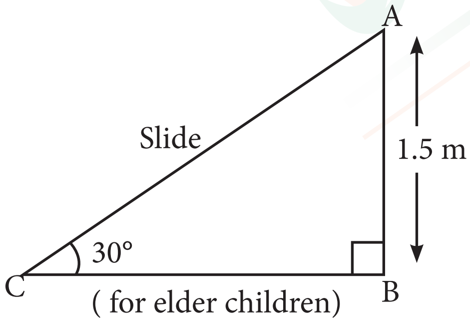
3. **A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a slide whose top is at a height of 1.5 m, and is inclined at an angle of 30°to the ground, whereas for elder children, she wants to have a steep slide at a height of 3 m, and inclined at an angle of 60° to the ground. What should be the length of the slide in each case?**

SOLUTION:

Given: Height of first slide = 1.5m, Height of second slide = 3m, angle of elevation from point on ground with top of first slide = 30° and angle of elevation from point on ground with top of second slide = 60°

To find: Length of slides

In the two figures, AC and PR are the slides for younger and elder children respectively

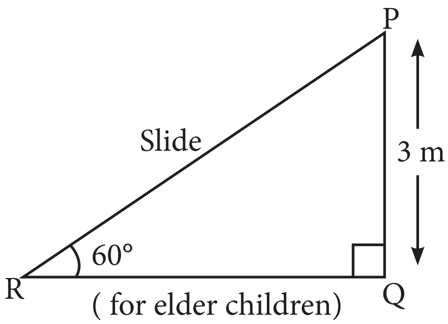


From figure In Δ ABC

$$\frac{AB}{AC} = \sin 30^\circ$$

$$\frac{1.5}{AC} = \frac{1}{2}$$

$AC = 3$ m



From figure In ΔPQR

$$\frac{PQ}{PR} = \sin 60$$

$$\frac{3}{PR} = \frac{\sqrt{3}}{2}$$

$$PR = \frac{6}{\sqrt{3}} = 2\sqrt{3}\text{m}$$

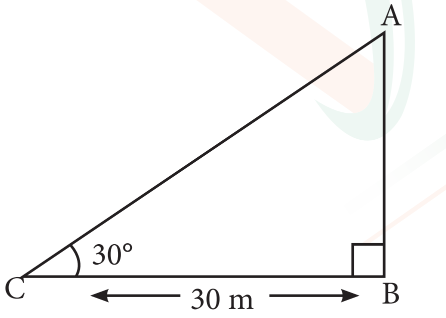
Thus, the lengths of the two slides were 3 m and $2\sqrt{3}$ m.

4. **The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower, is 30°. Find the height of the tower.**

SOLUTION:

Given-Distance between tower and point on ground = 30m and angle of elevation from point on ground to top of tower = 30°

To find: Height of tower.



Let be the tower.

From figure In ΔABC

$$\frac{AB}{BC} = \tan 30^{\circ}$$

$$\frac{AB}{30} = \frac{1}{\sqrt{3}}$$

$$AB = \frac{30}{\sqrt{3}} = 10\sqrt{3}\text{m}$$

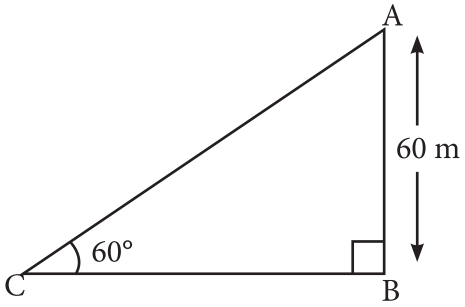
Thus, the height of tower is $10\sqrt{3}$m

5. **A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60° Find the length of the string, assuming that there is no slack in the string.**

SOLUTION:

Given: Distance of kite above the ground = 60m and angle of elevation made from a point on ground with kite = 60°

To find: length of the string



Let A be the position of the kite and the string is tied to point C on ground.

From figure In $\triangle ABC$,

$$\frac{AB}{AC} = \sin 60^{\circ}$$

$$\frac{60}{AC} = \frac{\sqrt{3}}{2}$$

$$AC = \frac{120}{\sqrt{3}} = 40\sqrt{3}\text{m}$$

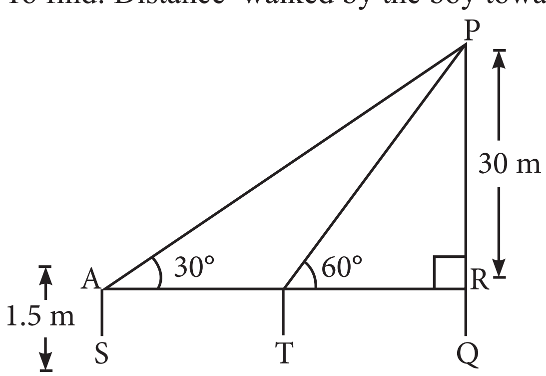
Thus, the length of the string is $40\sqrt{3}$ m.

6. **A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.**

SOLUTION:

Given: height of boy = 1.5m ,height of building = 30m and angle of elevation from boy to the top of the building = 30° and angle of elevation from boy to the top of the same building after some time = 60°

To find: Distance walked by the boy towards the building.



Let the initial position of the boy be at S. He walks towards building and reached at point T.

In the figure, PQ is the building of height 30 m.

AS = BT = RQ = 1.5 m

PR = PQ – RQ = 30 m – 1 .5m = 28.5 m

From figure In ΔPAR,

$$\frac{PR}{AR} = \tan 30^\circ$$

$$\frac{28.5}{AR} = \frac{1}{\sqrt{3}}$$

$$AR = 28.5\sqrt{3}$$

From figure In $\triangle PRB$,

$$\frac{PR}{BR} = \tan 60^\circ$$

$$\frac{28.5}{BR} = \sqrt{3}$$

$$BR = \frac{28.5}{\sqrt{3}} = 9.5\sqrt{3}$$

$$ST = AB = AR - BR = 28.5\sqrt{3} - 9.5\sqrt{3} = 19\sqrt{3}$$

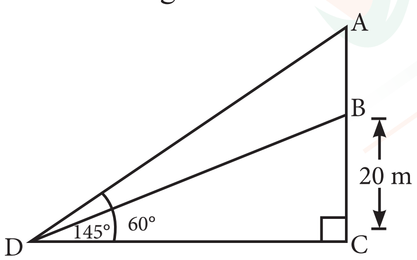
Thus, the distance which the boy walked towards the building is $19\sqrt{3}$ m.

7. **From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60°respectively. Find the height of the tower.**

SOLUTION:

Given: Height of building = 20m, angle of elevation of bottom of transmission tower from point on ground = 45°and the angle of elevation of top of transmission tower from same point on ground = 60°

To find: Height of the tower.



Let BC be the building, AB be the transmission tower, and D be the point on ground from where elevation angles are to be measured.

From figure In ΔBCD.

$$\frac{BC}{CD} = \tan 45^\circ$$

$$\frac{20}{CD} = 1$$

CD = 20 m ………..(i)

In ΔACD

$$\frac{AC}{CD} = \tan 60^\circ$$

$$\frac{AB+BC}{CD} = \sqrt{3}$$

$$\frac{AB+20}{20} = \sqrt{3} \qquad \text{From (i)]}$$

$$AB = 20\sqrt{3} - 20$$

$$= 20(\sqrt{3} - 1)$$

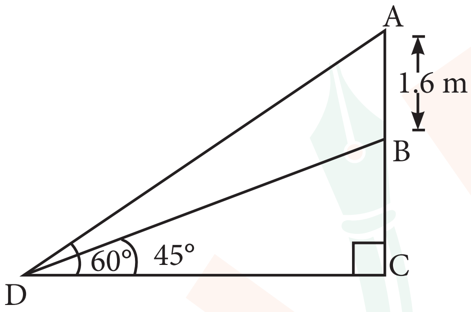
Thus, the height of the tower is $20(\sqrt{3}-1)$ m.

8. **A statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 7° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.**

SOLUTION:

Given: Height of statue = 1.6m, angle of elevation of top of statue from point on ground = 60° and angle of elevation of top of pedestal from same point on ground = 45°

To find: Height of the Pedestal.



Let AB be the statue, BC be the pedestal and D be the point on ground from where elevation angles are to be measured.

From figure In ΔBCD

$$\frac{BC}{CD} = \tan 45^\circ$$

$$\frac{BC}{CD} = 1$$

$$BC = CD \ldots \text{(i)}$$

From figure In ΔACD

$$\frac{AB+BC}{CD} = \tan 60^\circ$$

$$\frac{AB+BC}{BC} = \sqrt{3} \qquad [\text{From}(i).$$

$$1.6 + BC = BC\sqrt{3}$$

$$BC(\sqrt{3}-1) = 1.6$$

$$BC = \frac{(1.6)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$$

$$= \frac{1.6(\sqrt{3}+1)}{2} = 0.8(\sqrt{3}+1)$$

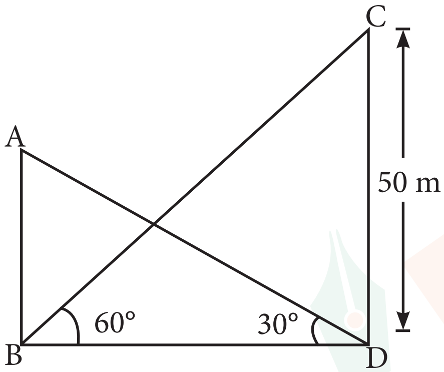
Thus, the height of pedestal is $0.8(\sqrt{3}+1)\text{m}$.

9. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building.

SOLUTION:

Given: Height of tower = 50 m, angle of elevation of top of building from foot of tower = 30° and angle of elevation of top of tower from foot of building = 60°.

To find: Height of the Building.



Let AB be the building and CD be the tower.

From figure In $\triangle CDB$,

$$\frac{CD}{BD} = \tan 60^{\circ}$$

$$\frac{50}{BD} = \sqrt{3}$$

$$BD = \frac{50}{\sqrt{3}}$$

From figure In $\triangle ABD$,

$$\frac{AB}{BD} = \tan 30^{\circ}$$

$$AB = \frac{50}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{50}{3} = 16\frac{2}{3}$$

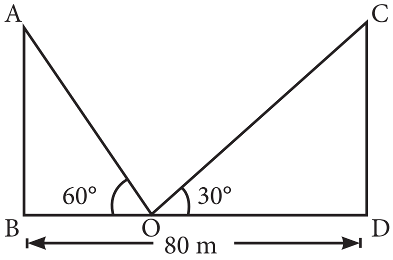
Thus, the height of the building is $16\frac{2}{3}$ m.

10. Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60°and 30°, respectively. Find the height of the poles and the distances of the point from the poles.

SOLUTION:

Given: Distance between two poles = 80m, the angle of elevation of the top of first pole from point on road = 60°and the angle of elevation of the top of second pole from same point on road = 30°

To find: Height of the Poles and the distances of the point from the poles



Let AB and CD be the poles and O is the point on the road.

From figure In $\triangle ABO$,

$$\frac{AB}{BO} = \tan 60^\circ$$

$$\frac{AB}{BO} = \sqrt{3}$$

$$BO = \frac{AB}{\sqrt{3}} \quad \ldots\ldots(i)$$

From figure In $\triangle CDO$,

$$\frac{CD}{DO} = \tan 30^\circ$$

$$\frac{CD}{80 - BO} = \frac{1}{\sqrt{3}}$$

$$CD\sqrt{3} = 80 - BO$$

$$CD\sqrt{3} = 80 - \frac{AB}{\sqrt{3}} \quad \ldots\ldots\ldots\ldots\ldots\text{[From (i)]}$$

$$CD\sqrt{3} + \frac{AB}{\sqrt{3}} = 80$$

$$CD\left[\sqrt{3} + \frac{1}{\sqrt{3}}\right] = 80 \qquad \text{(Since, } AB = CD\text{)}$$

$$CD\left(\frac{3+1}{\sqrt{3}}\right) = 80$$

$$CD = 20\sqrt{3}$$

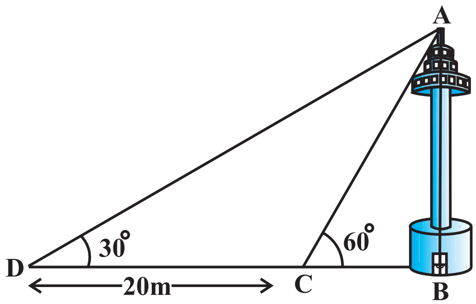
$$BO = \frac{AB}{\sqrt{3}} = \frac{CD}{\sqrt{3}} = \frac{20\sqrt{3}}{\sqrt{3}} = 20\text{m}$$

$DO = BD – BO = 80$ m – 20 m = 60 m

Thus, the height of the poles is $20\sqrt{3}$ m and the point between the poles is 20 m and 60 m far from these poles.

11. A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°. From another point 20 m away from this point on the line joing this point to the foot of the tower, the angle of elevation of the top of the

tower is 30° (see Fig. 9.12). Find the height of the tower and the width of the canal.



SOLUTION:

Given: Distance between two points = 20m, the angle of elevation of the top of tower from first point = 60° and the angle of elevation of the top of tower from second point in same line as first point = 30°

To find: Height of the tower and the width of the canal.

From figure In ΔABC,

$$\frac{AB}{BC} = \tan 60^\circ$$

$$\frac{AB}{BC} = \sqrt{3}$$

$$BC = \frac{AB}{\sqrt{3}} \text{.........(i)}$$

From figure In ΔABD,

$$\frac{AB}{BD} = \tan 30^\circ$$

$$\frac{AB}{BC + CD} = \frac{1}{\sqrt{3}}$$

$$\frac{AB}{\frac{AB}{\sqrt{3}} + 20} = \frac{1}{\sqrt{3}}. \quad \text{.......(Using (i)}$$

$$\frac{AB\sqrt{3}}{AB + 20\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$3AB = AB + 20\sqrt{3}$$

$$2AB = 20\sqrt{3}$$

$$AB = 10\sqrt{3}$$

$$\Rightarrow BC = \frac{AB}{\sqrt{3}} = \frac{10\sqrt{3}}{\sqrt{3}} = 10$$

Thus, the height of the tower is $10\sqrt{3}$ m and width of canal is 10 m.

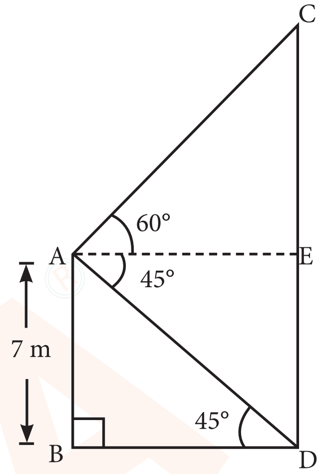
12. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

SOLUTION:

Given: Height of building = 7m, angle of elevation of the top of a cable tower from top of building = 60°and the angle of depression of foot of a cable tower from top of building = 45°.

To find: Height of tower.

Let AB be a building and CD be a cable tower.



From figure, In $\triangle ABD$,

$$\frac{AB}{BD} = \tan 45^\circ$$

$$\frac{7\text{m}}{BD} = 1$$

$BD = 7\text{m}$

From figure In $\triangle ACE$,

$AE = BD = 7$

$$\frac{CE}{AE} = \tan 60^\circ$$

$$\frac{CE}{7\text{m}} = \sqrt{3}$$

$CE = 7\sqrt{3}\text{m}$

$CD = CE + ED = (7\sqrt{3} + 7)\text{m} = 7(\sqrt{3} + 1)\text{ m}$

Thus, the height of the cable tower is $7(\sqrt{3} + 1)$ m.

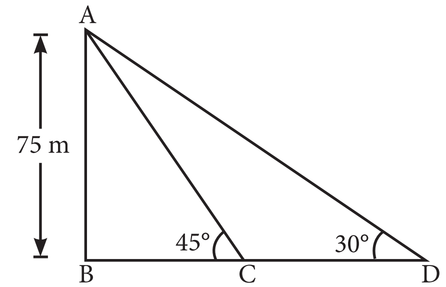
13. **As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.**

SOLUTION:

Given: Height of lighthouse = 75m, angle of depression of first ship from top of lighthouse = 30° and angle of depression of second ship from top of lighthouse = 45°

To find: Distance between the two ships.

Let AB be the lighthouse and the two ships be at point C and D respectively.



From figure In ΔABC,

$$\frac{AB}{BC} = \tan 45^\circ$$

$$\frac{75\text{m}}{BC} = 1$$

$BC = 75$ m

From figure In ΔABD,

$$\frac{AB}{BD} = \tan 30^\circ$$

$$\frac{75\text{m}}{BC + CD} = \frac{1}{\sqrt{3}}$$

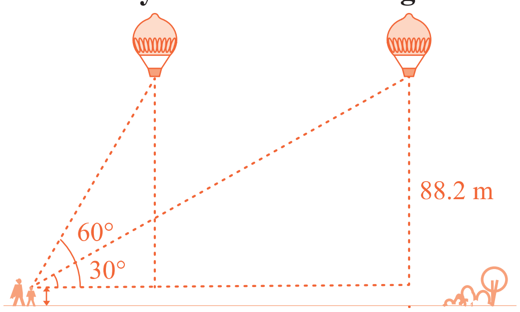
$$\frac{75\text{m}}{75\text{m}+CD}=\frac{1}{\sqrt{3}}$$

$$75\sqrt{3}\text{m}=75\text{m}+CD$$

$$CD=75(\sqrt{3}-1)\text{m}$$

Thus, the distance between the two ships is $75(\sqrt{3}-1)\text{m}$

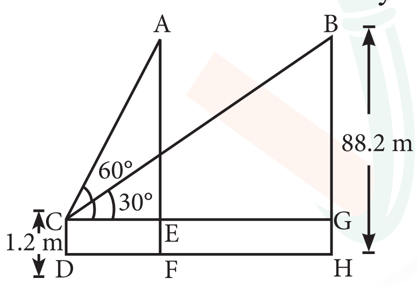
14. A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After some time, the angle of elevation reduces to 30° (see Fig. 9.13). Find the distance travelled by the balloon during the interval.



SOLUTION:

Given-Height of girl = 1.2m ,Height of Ballon from ground = 88.2m, angle of elevation of the balloon from the eyes of the girl = 60°and angle of elevation of that balloon from the eyes of the girl after some time = 30°

To find: Distance travelled by the balloon during the interval.



Let A be the initial position of the balloon and the position changes to B after some time and CD is the girl.

From figure In $\triangle ACE$,

$$\frac{AE}{CE}=\tan 60^\circ$$

$$\frac{AF-EF}{CE}=\tan 60^\circ$$

$$\frac{88.2-1.2}{CE}=\sqrt{3}$$

$$\frac{87}{CE}=\sqrt{3}$$

From figure In $\triangle BCG$

$$\frac{BG}{CG} = \tan 30^\circ$$

$$\frac{88.2 - 1.2}{CG} = \frac{1}{\sqrt{3}}$$

$$87\sqrt{3} = CG$$

Distance travelled by balloon = EG = CG - CE

$$= 87\sqrt{3} - 29\sqrt{3} = 58\sqrt{3}\text{m}$$

15. A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point.

SOLUTION:

Given: angle of depression of car from top of tower = 30° and the angle of depression of same car from top of tower after some time = 60°.

To find: Time taken by car to reach the foot of tower.

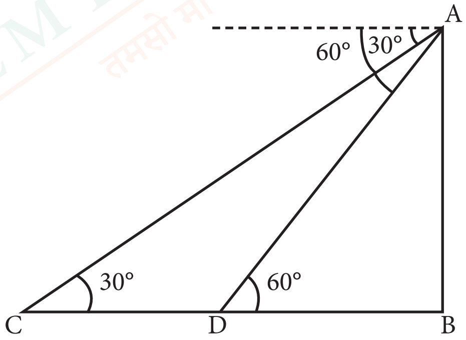
Let AB be the tower. C is the original position of the car which changes to D after six seconds.

From figure In $\triangle ADB$

$$\frac{AB}{DB} = \tan 60^\circ$$

$$\frac{AB}{DB} = \sqrt{3}$$

$$DB = \frac{AB}{\sqrt{3}} \ldots\ldots. \ldots..(i)$$



From figure In $\triangle ABC$

$$\frac{AB}{BC} = \tan 30^\circ$$

$$\frac{AB}{BD + DC} = \frac{1}{\sqrt{3}}$$

$$AB\sqrt{3} = BD + DC$$

$$AB\sqrt{3} = \frac{AB}{\sqrt{3}} + DC[\text{ From}(i)]$$

$$DC = AB\sqrt{3} - \frac{AB}{\sqrt{3}} = AB\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) = \frac{2AB}{\sqrt{3}}$$

Time taken by car to travel distance DC $\left(= \frac{2AB}{\sqrt{3}}\right) = 6$ seconds

Time taken by car to travel distance $DB\left(= \frac{AB}{\sqrt{3}}\right) = \frac{6}{\frac{2AB}{\sqrt{3}}} \times \frac{AB}{\sqrt{3}} = 3$ seconds.