CHAPTER 10

Circles

CLASS 10TH

NCERT EXERCISE AND SOLUTIONS - MATHEMATICS

EXERCISE 10.1

1. **How many tangents can a circle have?**

SOLUTION:

There can be **infinite** tangents to a circle. A circle is made up of infinite points which are at an equal distance from a point. Since there are infinite points on the circumference of a circle, infinite tangents can be drawn from them.

2. **Fill in the blanks.**

(i) **A tangent to a circle intersects it in .... point(s).**

(ii) **A line intersecting a circle in two points is called a ....**

(iii) **A circle can have parallel tangents at the most.**

(iv) **The common point of a tangent to a circle and the circle is called ....**

SOLUTION:

(i) A tangent to a circle intersects it in **one** point(s).

(ii) A line intersecting a circle in two points is called a **secant.**

(iii) A circle can have **two** parallel tangents at the most.

(iv) The common point of a tangent to a circle and the circle is called the **point of contact.**

3. **A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Length PQ is:**

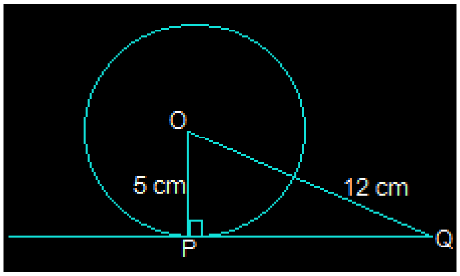
(A) **12 cm**

(B) **13 cm**

(C) **8.5 cm**

(D) **$\sqrt{119}$ cm**

SOLUTION:



In the above figure, the line that is drawn from the centre of the given circle to the tangent PQ is perpendicular to PQ. And so, OP $\perp$ PQ

Using Pythagoras' theorem in triangle ΔOPQ, we get,

$OQ^2 = OP^2 + PQ^2$

$(12)^2 = 5^2 + PQ^2$

$PQ^2 = 144c - 25$

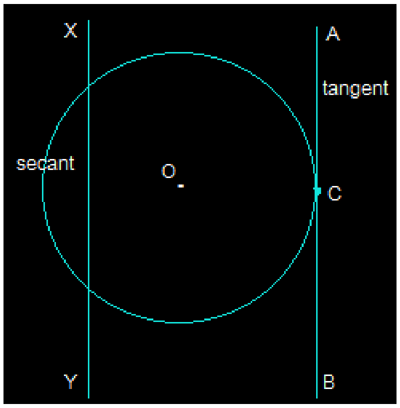
$PQ^2 = 119$

$PQ = \sqrt{119}$ cm

So, **option D,** i.e., $\sqrt{119}$ cm, is the length of PQ.

4. **Draw a circle and two lines parallel to a given line such that one is a tangent and the other, a secant to the circle.**

SOLUTION:



In the above figure, XY and AB are two parallel lines. Line segment AB is the tangent at point C, while line segment XY is the secant.

EXERCISE 10.2

In Q.1 to 3, choose the correct option and give a justification.

1. **From point Q, the length of the tangent to a circle is 24 cm, and the distance of Q from the centre is 25 cm. The radius of the circle is**

(A) **7 cm**

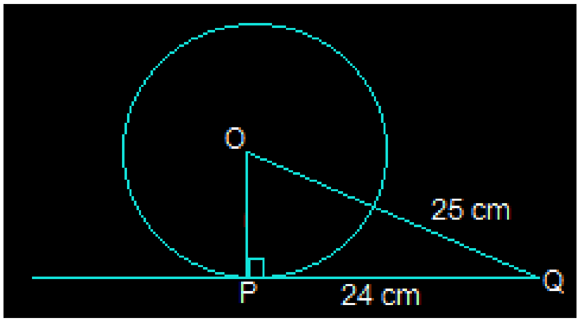
(B) **12 cm**

(C) **15 cm**

(D) **24.5 cm**

SOLUTION:

First, draw a perpendicular from the centre O of the triangle to a point P on the circle, which is touching the tangent. This line will be perpendicular to the tangent of the circle.



So, OP is perpendicular to PQ, i.e., OP ⊥ PQ

From the above figure, it is also seen that ΔOPQ is a right-angled triangle. It is given that

OQ = 25 cm and PQ = 24 cm

By using Pythagoras' theorem in ΔOPQ,

$OQ^2 = OP^2 + PQ^2$

$(25)^2 = OP^2 + (24)^2$

$OP^2 = 625 - 576$

$OP^2 = 49$

OP = 7 cm

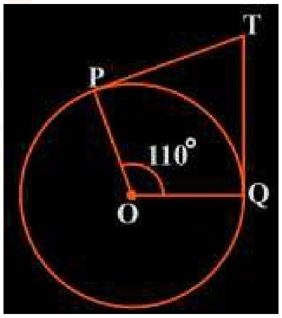
So, option A, i.e., 7 cm, is the radius of the given circle.

2. In Fig. 10.11, if TP and TQ are the two tangents to a circle with centre O so that ∠POQ = 110°, then ∠PTQ is equal to

(A) 60° (B) 70° (C) 80° (D) 90°

SOLUTION:

From the question, it is clear that OP is the radius of the circle to the tangent PT, and OQ is the radius to the tangent TQ.



So, OP ⊥ PT and TQ ⊥ OQ

∴ ∠OPT = ∠OQT = 90°

Now, in the quadrilateral POQT, we know that the sum of the interior angles is 360°. So, ∠PTQ + ∠POQ + ∠OPT + ∠OQT = 360°

Now, by putting the respective values, we get

∠PTQ + 90° + 110° + 90° = 360°

∠PTQ = 70°

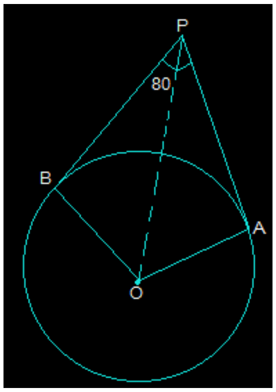
So, ∠PTQ is 70° which is option B.

3. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at an angle of 80°, then ∠ POA is equal to

(A) 50° (B) 60° (C) 70° (D) 80°

SOLUTION:

First, draw the diagram according to the given statement.



Now, in the above diagram, OA is the radius to tangent PA, and OB is the radius to tangent PB. So, OA is perpendicular to PA, and OB is perpendicular to PB, i.e., OA ⊥ PA and OB ⊥ PB. So, ∠OBP = ∠OAP = 90°

Now, in the quadrilateral AOBP,

The sum of all the interior angles will be 360°. So, ∠AOB + ∠OAP + ∠OBP + ∠APB = 360°

Putting their values, we get

∠AOB + 260° = 360°

∠AOB = 100°

Now, consider the triangles ΔOPB and ΔOPA. Here, AP = BP (Since the tangents from a point are always equal)

OA = OB (Which are the radii of the circle)

OP = OP (It is the common side)

Now, we can say that triangles OPB and OPA are similar using SSS congruency.

∴ $\triangle OPB \cong \triangle OPA$ So, $\angle POB = \angle POA$

$\angle AOB = \angle POA + \angle POB$ $2(\angle POA) = \angle AOB$

By putting the respective values, we get

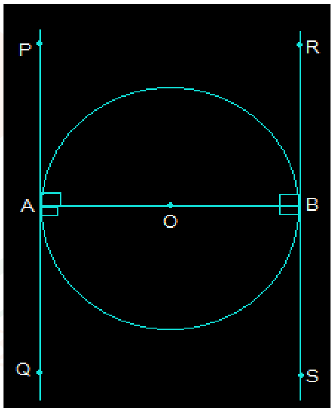
$\Rightarrow \angle POA = 100°/2 = 50°$

As the angle $\angle POA$ is 50°,

option A is the correct option.

4. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

SOLUTION:



First, draw a circle and connect two points, A and B, such that AB becomes the diameter of the circle. Now, draw two tangents, PQ and RS, at points A and B, respectively.

Now, both radii, i.e. AO and OB, are perpendicular to the tangents. So, OB is perpendicular to RS, and OA is perpendicular to PQ.

So, $\angle OAP = \angle OAQ = \angle OBR = \angle OBS = 90°$

From the above figure, angles OBR and OAQ are alternate interior angles.

Also, $\angle OBR = \angle OAQ$ and $\angle OBS = \angle OAP$ (Since they are also alternate interior angles) So, it can be said that line PQ and line RS will be parallel to each other (Hence Proved).

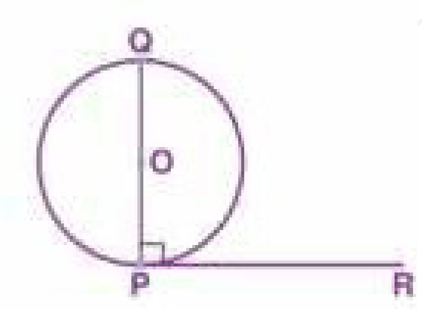
5. Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

SOLUTION:

Let, O is the centre of the given circle.

A tangent PR has been drawn touching the circle at point P.

Draw $QP \perp RP$ at point P, such that point Q lies on the circle.



$\angle OPR = 90°$ (radius ⊥ tangent) Also, $\angle QPR = 90°$ (Given)

∴ $\angle OPR = \angle QPR$

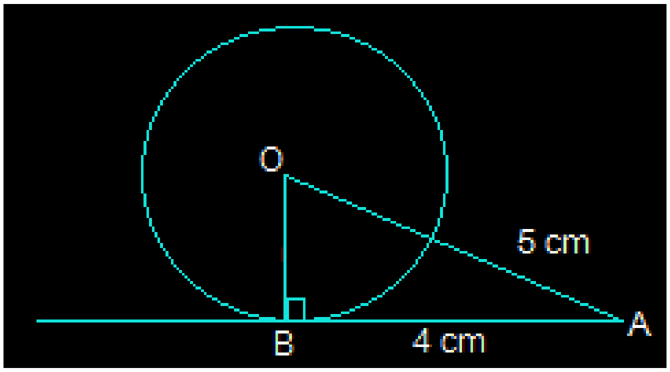
Now, the above case is possible only when centre O lies on the line QP.

Hence, perpendicular at the point of contact to the tangent to a circle passes through the centre of the circle.

6. **The length of a tangent from point A at a distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.**

SOLUTION:

Draw the diagram as shown below.



Here, AB is the tangent that is drawn on the circle from point A. So, the radius OB will be perpendicular to AB, i.e., $OB \perp AB$ We know, OA = 5cm and AB = 4 cm

Now, In ΔABO,

$OA^2 = AB^2 + BO^2$ (Using Pythagoras' theorem)

$5^2 = 4^2 + BO^2$

$BO^2 = 25 - 16$

$BO^2 = 9$

$BO = 3$

So, the radius of the given circle, i.e., BO, is 3 cm.

7. **Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.**

SOLUTION:

Draw two concentric circles with the centre O. Now, draw a chord AB in the larger circle, which touches the smaller circle at a point P, as shown in the figure below.

From the above diagram, AB is tangent to the smaller circle to point P.

$\therefore OP \perp AB$

Using Pythagoras' theorem in triangle OPA,

$OA^2 = AP^2 + OP^2$

$5^2 = AP^2 + 3^2$

$AP^2 = 25 - 9$

$AP = 4$

Now, as $OP \perp AB$,

Since the perpendicular from the centre of the circle bisects the chord, AP will be equal to PB.

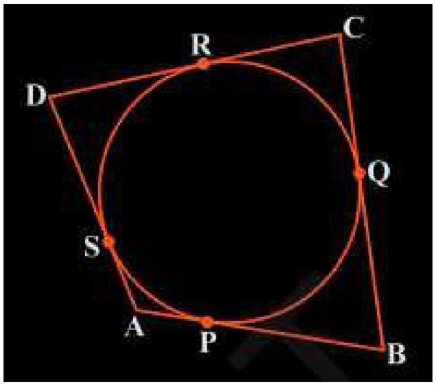
So, $AB = 2AP = 2 \times 4 = 8$ cm

So, the length of the chord of the larger circle is 8 cm.

8. **A quadrilateral ABCD is drawn to circumscribe a circle (see Fig. 10.12). Prove that AB + CD = AD + BC**

SOLUTION:

The figure given is:



From the figure, we can conclude a few points, which are

(i) DR = DS

(ii) BP = BQ

(iii) AP = AS

(iv) CR = CQ

Since they are tangents on the circle from points D, B, A, and C, respectively. Now, adding the LHS and RHS of the above equations, we get, DR + BP + AP + CR = DS + BQ + AS + CQ

By rearranging them, we get,

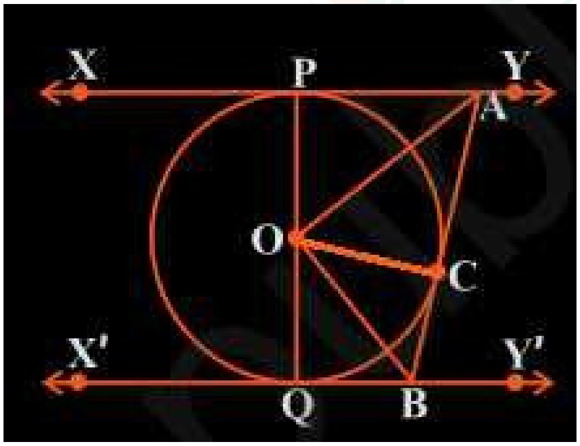
(DR + CR) + (BP + AP) = (CQ + BQ) + (DS + AS)

By simplifying, AD + BC = CD + AB

9. In Fig. 10.13, XY and X′Y′ are two parallel tangents to a circle with centre O and another tangent AB with the point of contact C intersecting XY at A and X′Y′ at B. Prove that ∆ AOB = 90°.

ANSWER:

From the figure given in the textbook, join OC. Now, the diagram will be as



Now, the triangles △OPA and △OCA are similar using SSS congruency as

(i) OP = OC They are the radii of the same circle

(ii) AO = AO It is the common side

(iii) AP = AC These are the tangents from point A

So, $\triangle OPA \cong \triangle OCA$

Similarly,

$\triangle OQB \cong \triangle OCB$

So,

$\angle POA = \angle COA$ … (Equation i)

And, $\angle QOB = \angle COB$ … (Equation ii)

Since the line POQ is a straight line, it can be considered as the diameter of the circle. So, $\angle POA + \angle COA + \angle COB + \angle QOB = 180°$

Now, from equations (i) and equation (ii), we get, $2\angle COA + 2\angle COB = 180°$

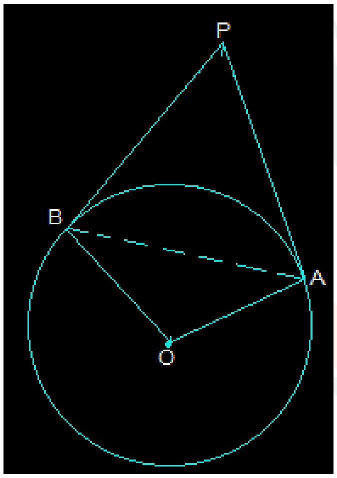
$\angle COA + \angle COB = 90°$

$\therefore \angle AOB = 90°$

10. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.

SOLUTION:

First, draw a circle with centre O. Choose an external point P and draw two tangents, PA and PB, at point A and point B, respectively. Now, join A and B to make AB in a way that subtends $\angle AOB$ at the centre of the circle. The diagram is as follows:



From the above diagram, it is seen that the line segments OA and PA are perpendicular. So, $\angle OAP = 90°$

In a similar way, the line segments $OB \perp PB$ and so, $\angle OBP = 90°$

Now, in the quadrilateral OAPB,

$\therefore \angle APB + \angle OAP + \angle PBO + \angle BOA = 360°$ (since the sum of all interior angles will be 360°) By putting the values, we get,

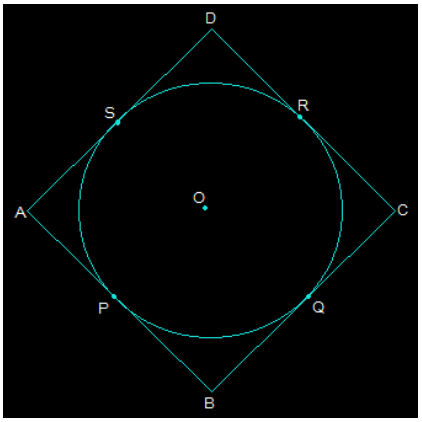
$\angle APB + 180° + \angle BOA = 360°$

So, $\angle APB + \angle BOA = 180°$ (Hence proved).

11. Prove that the parallelogram circumscribing a circle is a rhombus.

SOLUTION:

Consider a parallelogram ABCD which is circumscribing a circle with a centre O. Now, since ABCD is a parallelogram, AB = CD and BC = AD.



From the above figure, it is seen that,

(i) DR = DS

(ii) BP = BQ

(iii) CR = CQ

(iv) AP = AS

These are the tangents to the circle at D, B, C, and A, respectively. Adding all these, we get

DR + BP + CR + AP = DS + BQ + CQ + AS

By rearranging them, we get (BP + AP) + (DR + CR) = (CQ + BQ) + (DS + AS)

Again by rearranging them, we get

AB + CD = BC + AD

Now, since AB = CD and BC = AD, the above equation becomes 2AB = 2BC

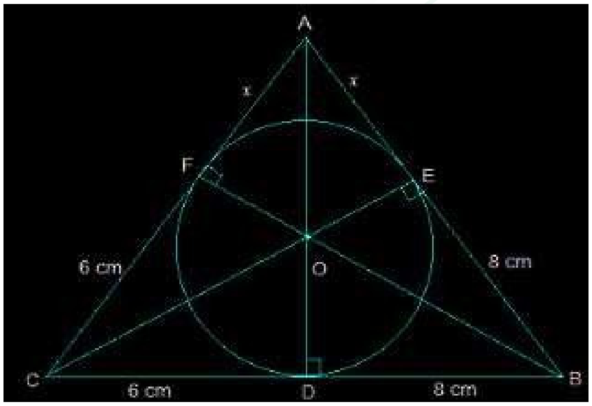
∴ AB = BC

Since AB = BC = CD = DA, it can be said that ABCD is a rhombus.

12. A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm, respectively (see Fig. 10.14). Find the sides AB and AC.

SOLUTION:

The figure given is as follows:



Consider the triangle ABC,

We know that the length of any two tangents which are drawn from the same point to the circle is equal. So,

(i) CF = CD = 6 cm

(ii) BE = BD = 8 cm

(iii) AE = AF = x

Now, it can be observed that,

(i) AB = EB + AE = 8 + x

(ii) CA = CF + FA = 6 + x

(iii) BC = DC + BD = 6 + 8 = 14

Now the semi-perimeter "s" will be calculated as follows

2s = AB + CA + BC

By putting the respective values, we get, 2s = 28+2x

s = 14 + x

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$ By solving this, we get,

$= \sqrt{(14+x)48x}$....................(i)

Again, the area of △ABC = 2 × area of (△AOF + △COD + △DOB)

= 2 × [(½ × OF × AF) + (½ × CD × OD) + (½ × DB × OD)]

= 2 × ½ (4x + 24 + 32) = 56 + 4x.....................(ii)

Now from (i) and (ii), we get,

$= \sqrt{(14+x)48x} = 56+4x$ Now, square both sides, $48x\,(14 + x) = (56 + 4x)^2$

$48x = [4\,(14 + x)]^2/(14 + x)$

$48x = 16\,(14 + x)$

$48x = 224 + 16x$

$32x = 224$

$x = 7$ cm

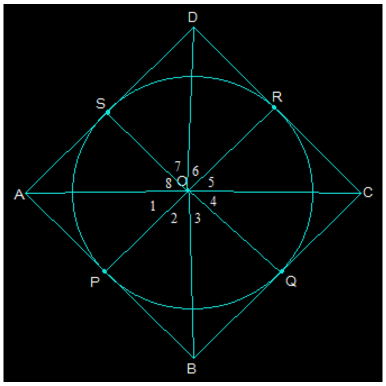
So, AB = 8 + x

i.e. AB = 15 cm

And, CA = x + 6 =13 cm.

13. **Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.**

SOLUTION:



First, draw a quadrilateral ABCD which will circumscribe a circle with its centre O in a way that it touches the circle at points P, Q, R, and S. Now, after joining the vertices of ABCD, we get the following figure:

Now, consider the triangles OAP and OAS.

AP = AS (They are the tangents from the same point A) OA = OA (It is the common side)

OP = OS (They are the radii of the circle)

So, by SSS congruency ΔOAP ≅ ΔOAS

So, ∠POA = ∠AOS

Which implies that $\angle 1 = \angle 8$ Similarly, other angles will be

$\angle 4 = \angle 5$

$\angle 2 = \angle 3$

$\angle 6 = \angle 7$

Now by adding these angles, we get

$\angle 1+\angle 2+\angle 3 +\angle 4 +\angle 5+\angle 6+\angle 7+\angle 8 = 360°$

Now by rearranging, $(\angle 1+\angle 8)+(\angle 2+\angle 3)+(\angle 4+\angle 5)+(\angle 6+\angle 7) = 360°$

$2\angle 1+2\angle 2+2\angle 5+2\angle 6 = 360°$

Taking 2 as common and solving, we get $(\angle 1+\angle 2)+(\angle 5+\angle 6) = 180°$

Thus, $\angle AOB+\angle COD = 180°$

Similarly, it can be proved that $\angle BOC+\angle DOA = 180°$

Therefore, the opposite sides of any quadrilateral which is circumscribing a given circle will subtend supplementary angles at the centre of the circle.