CHAPTER 11

Areas Related to Circles

CLASS 10TH

NCERT EXERCISE AND SOLUTIONS - MATHEMATICS

EXERCISE 11.1

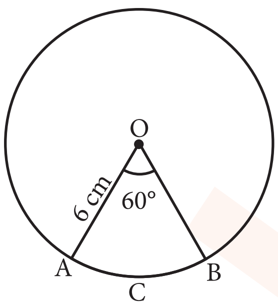
Unless stated otherwise, use $\pi = \frac{22}{7}$.

1. Find the area of a sector of a circle with radius 6 cm if angle of the sector is 60°.

SOLUTION:

Given: Radius of circle = 6cm and central angle = 60°.

Let OACB be a sector of circle making 60° angle at centre O of circle.



Area of sector of angle $\theta = \frac{\theta}{360^\circ} \times \pi r^2$

So, area of sector

$$OACB = \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times (6)^2$$

$$= \frac{1}{6} \times \frac{22}{7} \times 6 \times 6 = \frac{132}{7} \text{cm}^2$$

So, area of sector of circle making 60° at centre of circle is $\frac{132}{7}$ cm^2.

2. Find the area of a quadrant of a circle whose circumference is 22 cm.

SOLUTION:

Given: Circumference of circle = 22 cm.

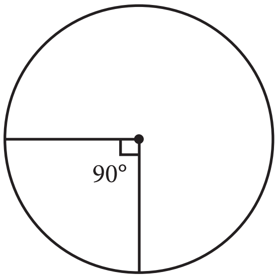
Let radius of circle be r.

Circumference = 22 cm

$2\pi r = 22$

$r = \frac{22}{2\pi}$

$r = \frac{11}{\pi}$



Quadrant of circle will subtend 90° angle at centre of circle.

So area of such quadrant of circle $= \frac{90^\circ}{360^\circ} \times \pi \times r^2$

$= \frac{1}{4} \times \pi \times \left(\frac{11}{\pi}\right)^2$

$= \frac{121}{4\pi} = \frac{121 \times 7}{4 \times 22}$

$= \frac{77}{8}\ \text{cm}^2$

3. **The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.**

SOLUTION:

Given: length of the minute hand of a clock = 14 cm.

We know that in 1 hour (i.e. 60 minutes) minute hand rotates 360°.

So in 5 minutes, minute hand will rotate $= \frac{360^\circ}{60} \times 5 = 30^\circ$

So area swept by minute hand in 5 minutes will be the area of a sector of 30° in a circle of 14 cm radius.

Area of sector of angle $\theta = \frac{\theta}{360^\circ} \times \pi r^2$

Area of sector of 30° $= \frac{30^\circ}{360^\circ} \times \frac{22}{7} \times 14 \times 14$

$= \frac{22}{12} \times 2 \times 14$

$= \frac{11 \times 14}{3}$

$= \frac{154}{3}\text{cm}^2$

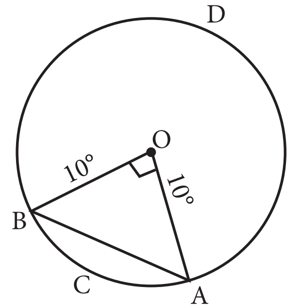
So, area swept by minute hand in 5 minutes is $\frac{154}{3}$ cm^2.

4. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding:

(i) Minor segment

(ii) Major sector. (Use $\pi = 3.14$)

SOLUTION:



Given: Radius of circle = 10 cm

Let AB the chord of circle subtending 90° angle at centre O of circle.

(i) Area of minor sector OACB

$$= \frac{90^\circ}{360^\circ} \times \pi r^2$$

$$= \frac{1}{4} \times 3.14 \times 10 \times 10$$

$$= \frac{314}{4} = 78.5 \text{ cm}^2$$

Area of $\Delta OAB = \frac{1}{2} \times OA \times OB = \frac{1}{2} \times 10 \times 10 = 50 \text{ cm}^2$

Area of minor segment ACB = Area of minor sector OACB – Area of $\triangle$OAB

= 78.5 – 50 = 28.5 cm^2

(ii) Area of Major sector OADB $= \left(\frac{360^\circ - 90^\circ}{360^\circ}\right) \times \pi r^2 = \left(\frac{270^\circ}{360^\circ}\right) \pi r^2$

$$= \frac{3}{4} \times 3.14 \times 10 \times 10$$

$$= \frac{942}{4} \text{ cm}^2 = 235.5 \text{ cm}^2$$

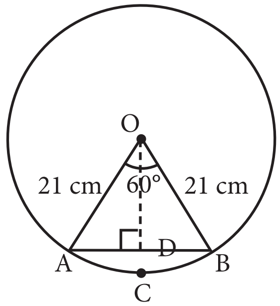
5. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find:

(i) The length of the arc

(ii) Area of the sector formed by the arc

(iii) Area of the segment formed by the corresponding chord

SOLUTION:



Given: Radius of circle = 21cm and central angle = 60°

Radius (r) of circle = 21 cm

Angle subtended by given arc = 60°

Length of an arc of a sector of angle $\theta = \frac{\theta}{360^\circ} \times 2\pi r$

Length of arc ACB $= \frac{60^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 21$

$= \frac{1}{6} \times 2 \times 22 \times 3$

= 22 cm

(ii) Area of sector OACB $= \frac{60^\circ}{360^\circ} \times \pi r^2$

$= \frac{1}{6} \times \frac{22}{7} \times 21 \times 21$

$= 231 \text{ cm}^2$

(iii) Now in $\triangle OAB$

$\angle OAB = \angle OBA$ as OA = OB

$\angle OAB + \angle AOB + \angle OBA = 180^\circ$

$2\angle OAB + 60^\circ = 180^\circ$

$\angle OAB = 60^\circ$

So, $\triangle OAB$ is an equilateral triangle.

Area of $\Delta OAB = \frac{\sqrt{3}}{4} \times (\text{side})^2$

$= \frac{\sqrt{3}}{4} \times (21)^2 = \frac{441\sqrt{3}}{4} \text{ cm}^2$

Area of segment ACB = Area of sector OACB – Area of $\triangle OAB$

$= \left(231 - \frac{441\sqrt{3}}{4}\right) \text{ cm}^2$

6. **A chord of a circle of radius 15 cm subtends an angle of 60°at the centre. Find the areas of the corresponding minor and major segments of the circle.**

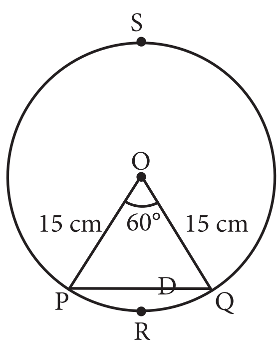
(Use $\pi = 3.14$ and $\sqrt{3} = 1.73$).

SOLUTION:

Given: Radius of circle = 15cm and central angle = 60°

Radius (r) of the circle = 15

Area of sector OPRQ $= \frac{60^\circ}{360^\circ} \times \pi r^2$



$= \frac{1}{6} \times 3.14 \times 15^2$

$= \frac{706.5}{6}$

$= 117.75 \text{ cm}^2$

In $\triangle OPQ$

$\angle OPQ = \angle OQP$ (Since OP = OQ)

$\angle OPQ + \angle OQP + \angle POQ = 180°$

$\therefore 2\angle OPQ = 120°$

$\therefore \angle OPQ = 60°$

$\triangle OPQ$ is an equilateral triangle.

Area of side

Area of $\Delta OPQ = \frac{\sqrt{3}}{4} \times (\text{ side})^2 = \frac{\sqrt{3}}{4} \times 15^2 = \frac{225 \times 1.73}{4} = 97.3125 cm^2$

Area of segment PRQ = Area of sector OPRQ – Area of $\triangle OPQ$

$= 117.75 - 97.3125$

$= 20.4375 \text{ cm}^2$.

Area of major segment PSQ = Area of circle Area of segment PRQ

$= 15^2 \pi - 20.4375$

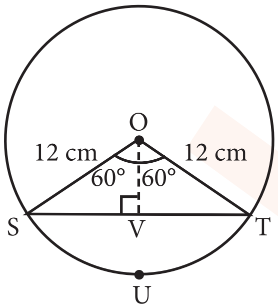
$=3.14 \times 225 - 20.4375$

$= 686.0625 \text{ cm}^2$

7. A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle.
(Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)

SOLUTION:

Given: Radius of circle = 12cm and central angle = 120°



Draw a perpendicular OV on chord ST. It will bisect the chord ST.

SV = VT

In ΔOVS

$\frac{OV}{OS} = \cos 60^\circ$

$\frac{OV}{OS} = \frac{1}{2}$

$OV = \frac{OS}{2} = \frac{12}{2} = 6\,\text{cm}$

$\frac{SV}{SO} = \sin 60^\circ = \frac{\sqrt{3}}{2}$

$\frac{SV}{12} = \frac{\sqrt{3}}{2}$

$SV = 6\sqrt{3}$

$ST = 2SV = 2\times 6\sqrt{3} = 12\sqrt{3}$

Area of $\triangle$OST $= \frac{1}{2}\times ST\times OV$

$= \frac{1}{2}\times 12\sqrt{3}\times 6$

$= 36\sqrt{3}$

= 36 × 1.73

= 62.28

Area of sector OSUT $= \frac{120^\circ}{360^\circ}\times \pi\times 12^2 = \frac{1}{3}\times 3.14\times 144$

= 150.72 cm^2

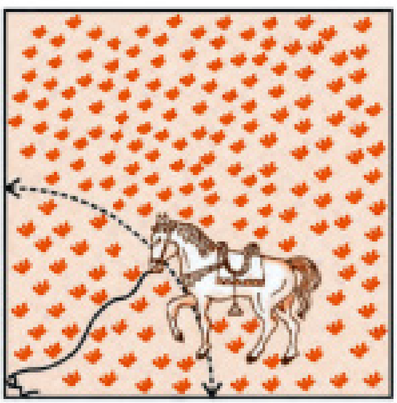
Area of segment SUT = Area of sector OSUT – Area $\triangle$OST

= 150.72 – 62.28 = 88.44 cm^2

8. A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope (see Fig.). Find:

(i) the area of that part of the field in which the horse can graze.

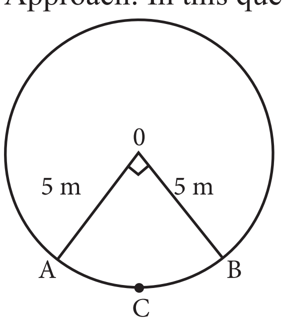
(ii) the increase in the grazing area if the rope were 10 m long instead of 5 m . (Use π = 3.14)



SOLUTION:

Given: Length of filed =15cm and length of rope=5cm.

Approach: In this question we have to find the area of quadrant which is made by horse.



The horse can graze a sector of 90° in a circle of 5 m radius.

(i) So, area that can be grazed by horse = area of sector OACB

$=\frac{90^\circ}{360^\circ}\times \pi r^2$

$=\frac{1}{4}\times 3.14\times 5^2$

$=19.625 = 19.625\ m^2$

(ii) Area that can be grazed by the horse when the length of rope is 10 m long

$=\frac{90^\circ}{360^\circ}\times \pi \times 10^2$

$=\frac{1}{4}\times 3.14\times 100$

$= 78.5\ m^2$

Change in grazing area = 78.5 – 19.625 = 58.875 m^2

9. A brooch is made with silver wire in the form of a circle with diameter 35 mm . The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in Fig. Find:

(i) the total length of the silver wire required.

(ii) the area of each sector of the brooch.

SOLUTION:

Given: Diameter of circle = 35 mm

Approach: For finding the length of wire, we have to find the circumference of brooch plus 5 times of diameter of brooch.

(i) Total length of wire required will be length of 5 diameters and circumference of brooch.

Radius of circle $=\frac{35}{2}$ mm

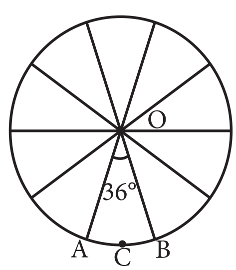
Circumference of brooch = $2\pi r$

$=2\times\frac{22}{7}\times\left(\frac{35}{2}\right)$

= 110 mm

Length of wire required = 110 + 5 × 35

= 110 + 175 = 285 mm



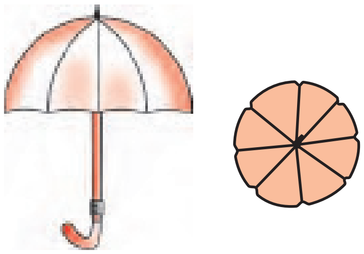
(ii) Each of 10 sectors of circle is subtending 36° at centre of circle.

So, area of each sector $= \frac{36^\circ}{360^\circ} \times \pi r^2$

$= \frac{1}{10} \times \frac{22}{7} \times \left(\frac{35}{2}\right) \times \left(\frac{35}{2}\right)$

$= \frac{385}{4}$ mm^2

10. An umbrella has 8 ribs which are equally spaced (see Fig.). Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella.



SOLUTION:

Given: Radius of circle = 45 cm

There are 8 ribs in umbrella. The area between two consecutive ribs is subtending an angle of $\frac{360^\circ}{8} = 45^\circ$ at centre of assumed flat circle.

So area between two consecutive ribs of circle $= \frac{45^\circ}{360^\circ} \times \pi r^2$

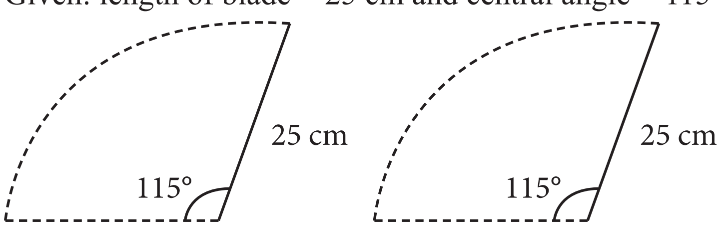
$= \frac{1}{8} \times \frac{22}{7} \times (45)^2$

$= \frac{11}{28} \times 2025 = \frac{22275}{28}$ cm^2

11. A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades.

SOLUTION:

Given: length of blade = 25 cm and central angle = 115°



The figure shows that each blade of the wiper will sweep an area of a sector of in a circle of 25 cm radius.

Area of such sector $= \frac{115^\circ}{360^\circ} \times \pi \times (25)^2$

$= \frac{23}{72} \times \frac{22}{7} \times 25 \times 25$

$= \frac{158125}{252}\ \text{cm}^2$

Area swept by 2 blades $= 2 \times \frac{158125}{252}$

$= \frac{158125}{126}\ \text{cm}^2$

12. To warn ships for underwater rocks, a lighthouse spreads a red coloured light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships are warned. (Use π = 3.14)

SOLUTION:

Given: lighthouse spreads light over a distance = 16.5 km and central angle = **80°**.

To find : Area of Sector formed by light spread by lighthouse.

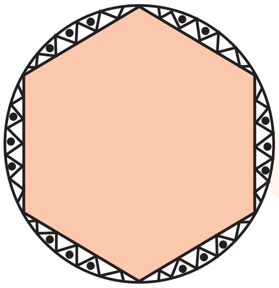
Lighthouse spreads light like a sector of angle 80° in a circle of 16.5 km radius

Area of sector $OACB = \frac{80^\circ}{360^\circ} \times \pi r^2$

$= \frac{2}{9} \times 3.14 \times 16.5 \times 16.5$

$= 189.97 = 189.97\ \text{km}^2$

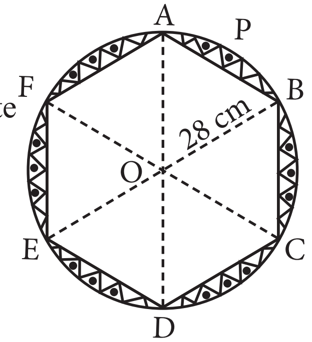
13. A round table cover has six equal designs as shown in Fig. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of ₹0.35 per cm. (Use $\sqrt{3} = 1.7$)



SOLUTION:

Given: Radius of cover = 28 cm.

Approach: First we have to find the area of minor segment and then we have to multiply it with the cost of design.



Designs are segments of circle.

Consider segment APB. Chord AB is a side of hexagon. Each chord will substitute $\frac{360^\circ}{6} = 60^\circ$ at centre of circle.

In △OAB

∠OAB = ∠OBA

∠AOB = 60°

∠OAB + ∠OBA + ∠AOB = 180°

∠OAB = 180° – 60° = 120°

$\angle OAB = 60°$

So $\triangle OAB$ is an equilateral triangle

Area of $\Delta OAB = \dfrac{\sqrt{3}}{4} \times (\text{side})^2$

$= \dfrac{\sqrt{3}}{4} \times (28)^2 = 196\sqrt{3}\,\text{cm}^2 = 333.2\,\text{cm}^2$

Area of sector OAPB $= \dfrac{60^\circ}{360^\circ} \times \pi r^2$

$= \dfrac{1}{6} \times \dfrac{22}{7} \times 28 \times 28$

$= \dfrac{1232}{3} = 410.6667\ \text{cm}^2$

Area of segment APB = Area of sector OAPB – Area of ΔOAB

= 410.6667 – 333.2

= 77.4667 cm^2

So, area of designs = 6 × 77.4667 = 464.8 cm^2

Cost occurred in making 1 cm^2 designs = ₹0.35

Cost occurred in making 464.8 cm^2 designs = 464.8 × ₹0.35 = ₹162.68

So, Cost of making such designs is ₹162.68.

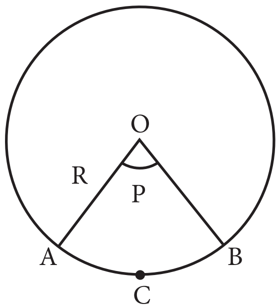
14. Tick the correct answer in the following: Area of a sector of angle *p* (in degrees) of a circle with radius *R* is

(A) $\dfrac{p}{180} \times 2\pi R$ **(B)** $\dfrac{p}{180} \times \pi R^2$ **(C)** $\dfrac{p}{360} \times 2\pi R$ **(D)** $\dfrac{p}{720} \times 2\pi R^2$

SOLUTION:

Given: Radius of circle = R and central angle = $p°$.

We know that area of sector of angle $\theta = \dfrac{\theta}{360^\circ} \pi R^2$



Area of sector of angle $p = \dfrac{p}{360^\circ}\left(\pi R^2\right)$

$= \left(\dfrac{p}{720^\circ}\right)\left(2\pi R^2\right)$

Hence correct option (D).